# * FERMI DIRAC STATISTICS:-

→ The Fermi Dirac distribution function gives the statistical behaviour of free electrons in metals and heavily doped semiconductors.

→ The particles that obey the Fermi Dirac statistics are sometimes called "fermions".

→ For a system of N indistinguishable non-interacting particles obeying Pauli Exclusion principle with corresponding energies as $E_1, E_2, \ldots E_N$ and the multiplicity or the degeneracy of the energy level as $g_i$.

The number of ways in which $N_i$ particles are arranged in $g_i$ quantum states is given by

$$g_i C_{N_i} = \frac{g_i!}{N_i!(g_i - N_i)!} \quad \ldots (1)$$

Each quantum states can accomodate only one particle in accordance with Pauli's Exclusion principle.

$$F = \prod_{i=1} \frac{g_i!}{N_i!(g_i - N_i)!} \quad \ldots (2)$$

F = no. of ways of distributing $N_1, N_2, \ldots N_n$ particle energy levels

$\Pi$ = product

Applying Stirling's theorem in the above equation:

$$\ln F = \sum_{i=1}^{n} \left[ g_i(\ln g_i - 1) - N_i(\ln N_i - 1) - (g_i - N_i) \left\{ \ln(g_i - N_i) - (g_i - N_i) \right\} \right]$$

$$= \sum_{i=1}^{n} \left[ g_i \ln g_i - N_i \ln N_i - (g_i - N_i) \ln(g_i - N_i) \right] \quad \ldots (3)$$

Since the system is in equilibrium, F or ln F must be maximised. The total number of particles N and the total energy are constants.

i.e. $N = \sum_{i=1}^{n} N_i = \text{constant} \quad \ldots (4)$

$$U = \sum_{i=1}^{n} N_i E_i = \text{constant} \quad \ldots (5)$$

Applying Lagrange's method of undetermined multiplier to obtain (3), (4) and (5).

$$\ln(g_i - N_i) - \ln N_i + \alpha + \beta E_i = 0.$$

$\alpha, \beta$ are constants.

$$f(E_i) = \frac{N_i}{g_i} = \frac{1}{1 + e^{-\alpha - \beta E_i}} \quad \ldots (6)$$

This is the Fermi Dirac Distribution function.

$\beta = \frac{-1}{K_B T}$ ; where $K_B$ = Boltzmann constant
$T$ = absolute temperature

$\alpha = \frac{E_F}{K_B T}$ , $E_F$ = Fermi Energy.

The FD distribution function takes the form

$$f(E_i) = \frac{1}{1 + e^{(E_i - E_F)/K_B T}} \quad \ldots (7)$$

→ FD statistics is the probability that a quantum states of energy $E_i$ is occupied.

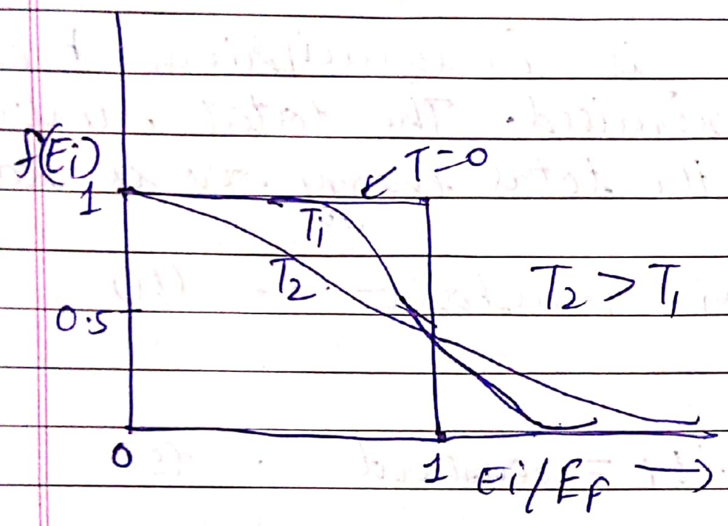


Fig: Plot of the Fermi distribution function $f(E_i)$ against normalised energy $E_i/E_F$ at different temperatures.

1) $T=0$, $f(E_i)=1$ for $E_i < E_f$ and $f(E_i)=0$ for $E_i > E_f$.

→ At absolute zero of temperature, the Fermi level represents the highest occupied energy level.

2) $T>0$, $f(E_i)$ is close to unity for $E_i \ll E_f$ and approaches zero for $E_i \gg E_f$.

3) If the temperature is not very large $f(E_i)$ varies rapidly from about unity to about zero over an energy range of few times $K_B T$ around $E_f$.

→ Thus ~~at~~ the Fermi level is that energy level for which the probability of occupation at $T > 0$ is $\frac{1}{2}$.

→ At low temperatures when $f(E_i)$ is nearly a step function the distribution function is said to be strongly degenerate.

→ At very high temperatures it is said to be nearly non degenerate.

* BOSE - EINSTEIN STATISTICS.

→ Bose-Einstein statistics gives the statistical behaviour of an ensemble of indistinguishable particles which do not follow Pauli Exclusion Principle. Examples are photons and phonons.

→ The particles that obey the BE statistics are sometimes called Bosons.

The total number of ways V of distributing $N_1, N_2, \ldots N_n$ particles in an energy level is the product of the terms represented by

$$M = \frac{(N_i + g_i - 1)!}{N_i!\,(g_i - 1)!}, \text{ where } g_i = \text{states} \quad \ldots (8)$$

is given by

$$V = \prod_{i=1} \frac{(N_i + g_i - 1)!}{N_i!\,(g_i - 1)!} \quad \ldots (9)$$

Applying Stirling's theorem in the above equation and then applying ~~log~~ Lagrange's method of undetermined multipliers as did in the FD distribution process we get

$$\sum_{i=1}^{n} [\ln(N_i + g_i - 1) - \ln N_i + \alpha + \beta E_i] \delta N_i = 0 \quad \cdots (10)$$

$\alpha, \beta \rightarrow$ constants

The Bose Einstein distribution function yields

$$f(E_i) = \frac{N_i}{g_i} = \frac{1}{e^{-\alpha - \beta E_i} - 1} \quad \cdots (11)$$

→ At very high temperatures, the particles are distributed over a wide energy range so that the number of particles in any energy range would be much smaller than the number of quantum states in that range i.e $g_i >> N_i$

$$\therefore \ln g_i - \ln N_i + \alpha + \beta E_i = 0 \quad \cdots (12)$$

The MB distribution function is then reduced to

$$f(E_i) = e^{\alpha + \beta E_i} \quad \cdots (13)$$

$$\beta = \frac{-1}{K_B T} \quad \therefore \quad f(E_i) = \frac{1}{e^{-\alpha} e^{E_i/K_B T} - 1}$$

At high temperatures $\alpha$ must be very large negative numbers. As the temperature approaches zero, $\alpha \rightarrow 0$

$\therefore$

$$f(E_i) = \frac{1}{e^{E_i/K_B T} - 1} \quad \cdots (14)$$

→ At the absolute zero of temperature the particles tend to occupy the lowest energy state. This phenomenon is known as Bose-condensation.

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* Comparision of Bose-Einstein and Fermi-Dirac distribution functions.

| FERMI DIRAC | BOSE-EINSTEIN. |
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| 1) It holds for indistinguishable particles obeying the Pauli's Exclusion Principle | It holds for distinguishable particles not obeying Pauli's Exclusion Principle. |
| 2) $f(E) = \frac{1}{e^{(E-E_F)/K_BT}+1}$ | $f(E) = \frac{1}{e^{-\alpha}e^{E/K_BT}-1}$ |
| 3) It applies to electron gas in metals, protons, muons, neutrons, electrons, positrons etc | It applies to photon gas, phonon gas, pions, and the particles with integral or zero spin. |