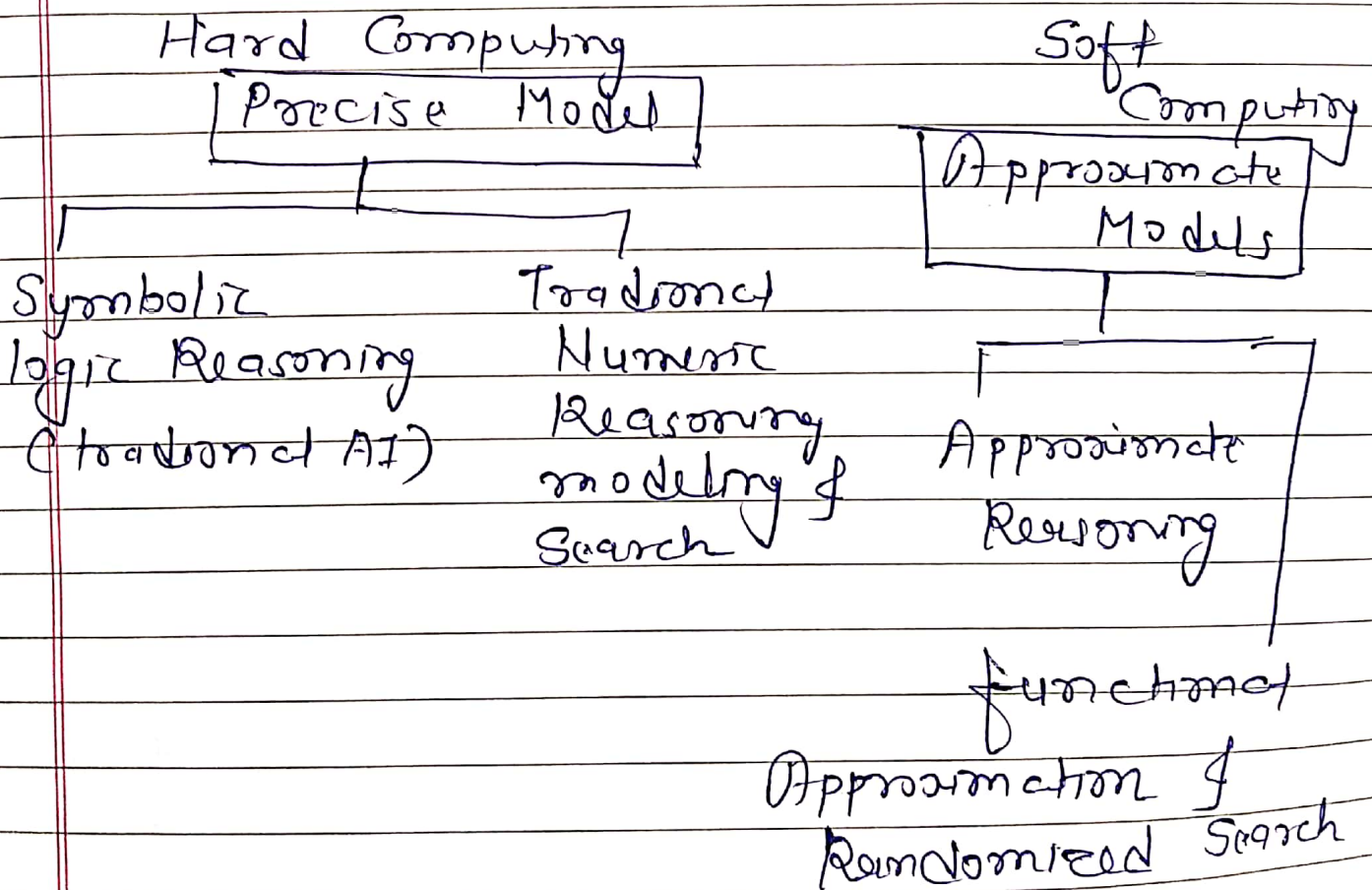


## Soft Computing :

↳ Introduced by Lotfi Zadeh

Goal :- able to emulate human brain  
= as closely as possible.  
i.e. self learning

Combination of Genetic Algorithms,  
Neural Network & fuzzy logic  
along with evolutionary  
computing



## ⇒ Artificial Neural Network (ANN)

① Neural Network is a processing device either an algorithm or an actual NW whose design was inspired by design and functioning of animal brain of components that of.

→ have ability to learn by example  
flexible & powerful.

→ no need to device an algo, to perform specific task.

## ② Human Brain:-

how it works is a mystery

→ most basic element is neuron which does not regenerate.

→ provide us ability to

↳ think

↳ learn (apply experiences)

↳ Remember.

## ③ Artificial Neural Network

defined as an information processing model i.e. inspired by how biological nervous system works, such as the brain process information.

→ replicates the most basic function of brain

→ learn by example

## (\*) Advantages of Neural n/w

1) Adaptive learning  
↳ ability to learn how to do task based on given data.

2) Self Organization  
↳ Can create its own representation of information it receives during learning time

3) Real Time Operation  
↳ computations may be carried out in parallel.  
↳ special hw needed

4) Fault tolerance via redundant information coding

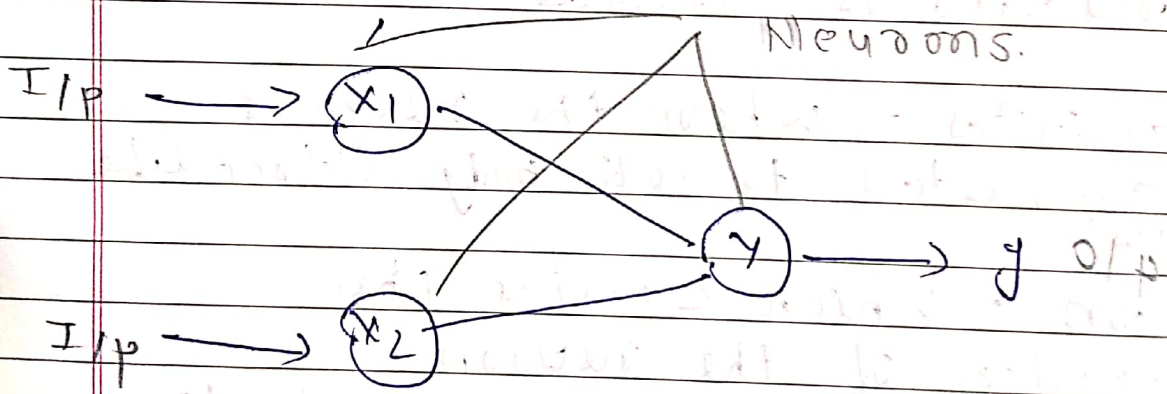
① Objective of Neural Network

To develop a computational device for modeling the brain to perform various computational task at a faster rate than a traditional system.

② Neuron :- Processing element in ANN

→ Each neuron has an internal state of its own, called the activation or activity level of neuron, which is a function of i/p the neuron receives

→ The activation signal of neuron can be transmitted to other neurons. (One signal at a time)



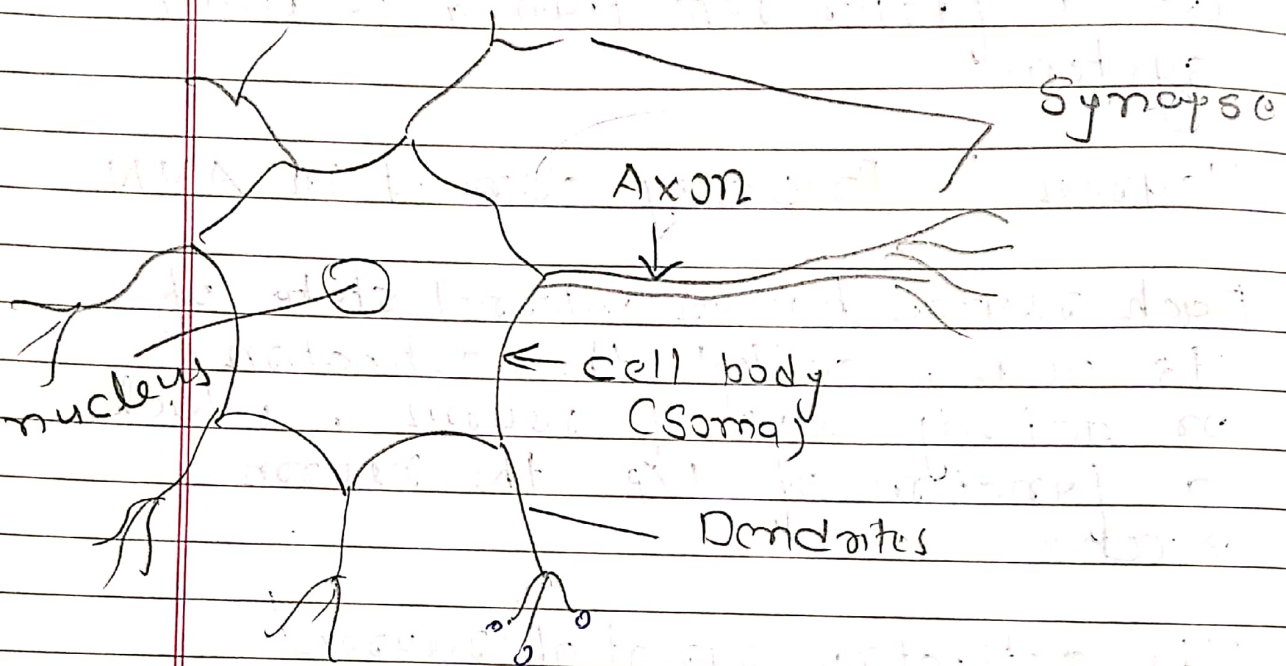
$$y_m = a_1 w_1 + a_2 w_2$$

$$y = f(y_m)$$

O/p = function of out. i/p calculated

## (\*) Biological Neural nu

Brain consists of approximately  $10^{11}$  neurons, with numerous interconnections



- 1) Soma / Cell body - where the cell nucleus is located
- 2) Dendrites - where the nerve is connected to cell body (tree like nu)
- 3) Axon - which carries the impulses of the neuron  
 → single, long connection extending the cell body area carrying
- 4) The end of the Axon & Dendrites ~~splits~~ splits in to fine strands  
 ⇒ carries Synapses

It is found that each strand terminates into a small bulb like organ called Synapse.

→ Through Synapse neuron can introduce signals to one by neuron.

⇒ Electric impulses are passed between synapse of dendrites

If electrical potential inside the body reaches a threshold then the receiving cells fires an impulse or action of fixed strength & duration, which sent through the axon to the synaptic junction of the other cells.

### ⊗ Artificial Neuron:-

Biological Neuron      Artificial Neuron

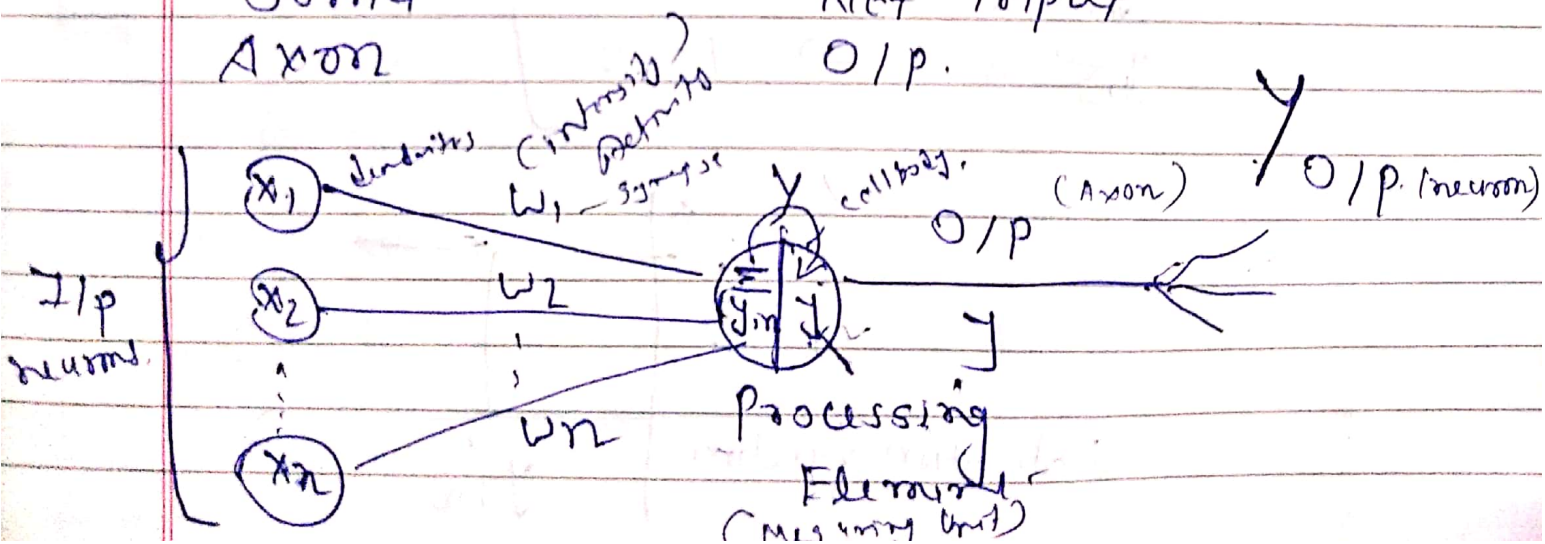
Cell

Dendrites

Soma

Axon

Neuron.  
 (Synapse)  
 Weights or interconnection  
 Net input  
 O/p.



net input

$$y_{in} = a_1 w_1 + a_2 w_2 + \dots + a_n w_n$$

$$y = \sum_{j=1}^n a_j w_j$$

where  $j$  is the processing element.

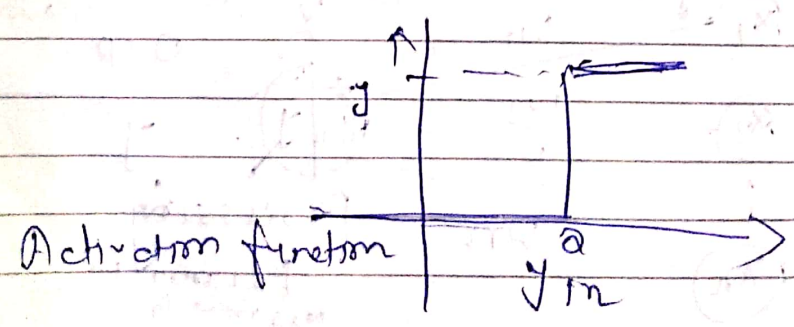
→ The activation function is to apply over processing elements to calculate the o/p

→ The weights represents strength of synapse connecting the i/p & o/p neuron

→ A positive weight corresponds to excitatory synapse =  $w$

A negative weight corresponds to inhibitory synapse =  $w$

$$\left. \begin{aligned} y_{in} \geq \theta &\Rightarrow y = 1 \\ y_{in} < \theta &\Rightarrow y = 0 \end{aligned} \right\} \theta \text{ - Threshold}$$



ANN - Artificial Neural Network

BNN - Biological Neural Network

(BNN)

classmate

Date \_\_\_\_\_

Page \_\_\_\_\_

(\*) Comparison between biological Neuron & Artificial Neuron. (ANN)

(1) Speed -

Biological Neuron is slower than ANN because of massive parallel architecture.

BNN -  $10^3$  seconds

ANN -  $10^{-9}$  seconds

(2)

Processing :-

BNN is faster than ANN

because of parallel processing.

(3) Storage Capacity :-

BNN - information is stored in synapse strength

ANN - information stored in contiguous memory allocation

(Memory overloading can be a problem)

In BNN new information is added by extending interconnections by adjusting strength without destroying older information.

In ANN information get lost or corrupted if connections disconnected.



#### ④ Size of Complexity

Total No. of neurons in brain is about  $10^{11}$

Total no of interconnections:  $10^{15}$

So. BNN Complexity is higher than ANN

→ Size of Complexity of ANN is based on application

#### ⑤ Tolerance :-

(b'ce of 11<sup>th</sup> nature)

BNN has fault tolerance capability while ANNs are not fault tolerant

#### ⑥ Control Mechanism :-

AN modeled using computer, there is control unit present in CPU.

There is no control unit present in BNN as such an ANN, channels of brain control the informations through chemical actions.

Control mechanism of AN is very simpler than BNN

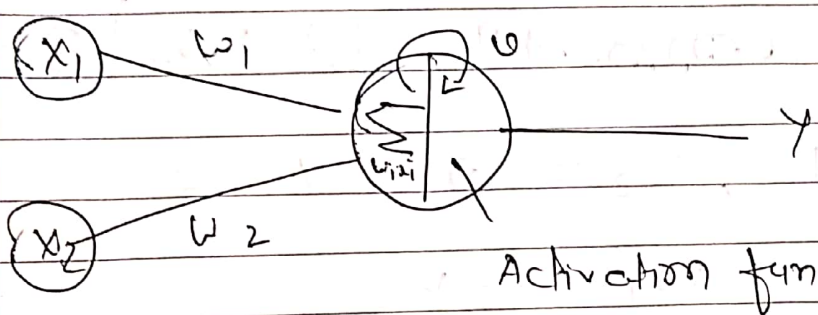
Ex Implement Simple AND function with Neuron.

$x_1$	$x_2$	$Y$	$\theta$	$U$
0	0	0	1	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	2

Conclusion

$U > 1 \Rightarrow Y = 1$   
 $U \leq 1 \Rightarrow Y = 0$

- Assume  $w_1 = w_2 = 1$



Activation function

→ Activation function

$U > \theta \Rightarrow Y = 1$   
 $U \leq \theta \Rightarrow Y = 0$

$U = \sum a_i v_i + b$   
 $U > \theta \Rightarrow Y = 1$   
 $U \leq \theta \Rightarrow Y = 0$

→  $U = \sum a_i w_i$

① for  $x_1 = 0, x_2 = 0, Y = 0$

$$U = x_1 w_1 + x_2 w_2$$

$$= (0 \times 1) + (0 \times 1) = 0 \quad 0 \leq \theta$$

$Y = 0$

now  $U \leq \theta \Rightarrow Y = 0$

$\theta > 0$  ~~0.2 0.3 ...~~ let's take  $\theta = 1.5$

②  $x_1 = 0, x_2 = 1, y = 0$

$$U = -x_1 w_1 + x_2 w_2$$

$$= (0 \times 1) + (1 \times 1)$$

$$= 1$$

$y = 0$

$U \leq \theta \Rightarrow y = 0$

$1 \leq \theta \Rightarrow y = 0$

$\theta > 1$  ~~same as~~ let take  $\theta = 1$

③  $x_1 = 1, x_2 = 0, y = 0$

$$U = x_1 w_1 + x_2 w_2$$

$$= (1 \times 1) + (0 \times 1)$$

$$= 1$$

$U = 1$  so  $\theta \geq 1$  same as ②

④  $x_1 = 1, x_2 = 1, y = 1$

$$U = x_1 w_1 + x_2 w_2$$

$$= (1 \times 1) + (1 \times 1)$$

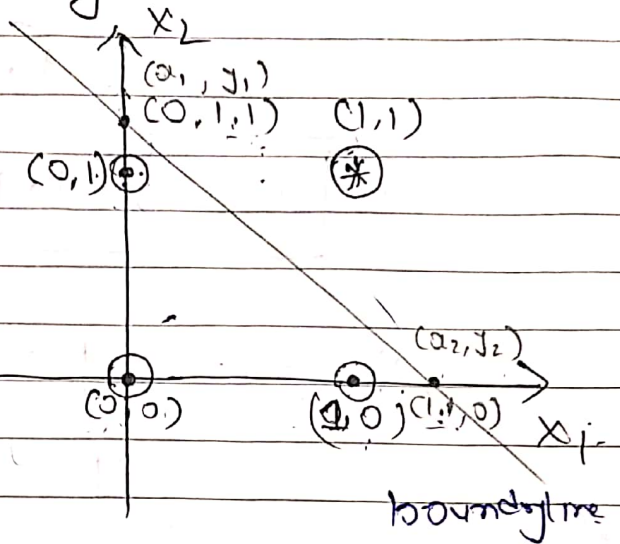
$$= 2$$

~~$U > \theta \Rightarrow y = 1$~~   
 ~~$2 > \theta \Rightarrow y = 1$~~  }  $\theta < 2$

take  $\theta = 1$

Ex-2 Using linear Separability concept  
Obtain the response for AND  
function (take binary i/p's)

$x_1$	$x_2$	$y$
0	0	0
0	1	0
1	0	0
1	1	1

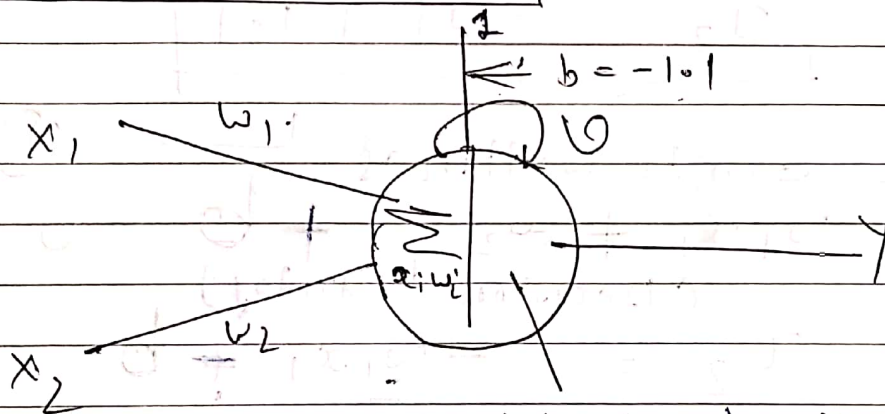


Note :-

no of lines needed  
to separate pattern  
= no of neurons  
needed in n/w

⊙ ⇒  $y = 0$

⊛ ⇒  $y = 1$



Activation function }  $v > 0 \Rightarrow y = 1$   
 $v \leq 0 \Rightarrow y = 0$

line equation

$$y = mx + c$$

$m = \frac{y_2 - y_1}{x_2 - x_1}$   
 $= \frac{0 - 1}{1 - 0}$   
 $= -1$

$$m = -1$$

$w_1 a_1 + b > 0$  for positive response  
 $w_1 a_1 + b < 0$  for negative response

$$y_1 = m a_1 + c$$

$$c = y_1 - m a_1$$

$$= 1.1 - ((-1) \times 0)$$

$$c = 1.1$$

now,  $y = m a + c$

$$y = (-1) a + 1.1$$

$$y = -a + 1.1$$

but we have  $x = a_1$  &  $y = a_2$

$$So, a_2 = -a_1 + 1.1$$

& this can be written as ①

$$w_1 a_1 + w_2 a_2 + b = 0$$

(learned boundary)

$$a_2 = -\frac{w_1 a_1 + b}{w_2}$$

$w = \frac{1}{2} w_1 + \frac{1}{2} w_2$

Activation function

$w \leq 0 \Rightarrow y = 0$   
 $w > 0 \Rightarrow y = 1$

Compare ① & ②

$$\frac{w_1}{w_2} = 1$$

$$b = -1.1$$

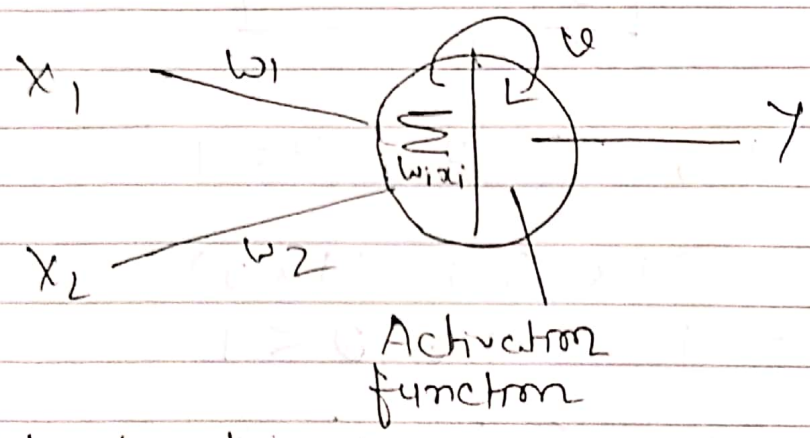
i.e.  $w_1 = 1$  |  $w_2 = 1$  |  $b = -1.1$

(i)  $7m - 0x1 + 0x2 = 7 < 0 \Rightarrow y = 0$   
 (ii)  $7m - 0x1 + 1x2 = 7 < 0 \Rightarrow y = 0$   
 (iii)  $-2 - 1.1 > 0 \Rightarrow y = 1$

Ex-3 Implement OR function with neuron.

$x_1$	$x_2$	$y$	$u$	$\theta$
0	0	0	0	0.2
0	1	1	1	0.2
1	0	1	1	0.2
1	1	1	2	0.2

→ take  $w_1 = w_2 = 1$  (Assume)



Conclusion  
 $u > 0.2 \Rightarrow y = 1$   
 $u \leq 0.2 \Rightarrow y = 0$

Activation function

$$u > \theta \Rightarrow y = 1$$

$$u \leq \theta \Rightarrow y = 0$$

①  $x_1 = 0, x_2 = 0, y = 0$

$$u = x_1 w_1 + x_2 w_2$$

$$= (0 \times 1) + (0 \times 1)$$

$$= 0$$

So  $u \leq \theta \Rightarrow y = 0$   
 $0 \leq 0.2 \Rightarrow y = 0$

~~$\theta = 0$~~  Take  $\theta = 0.2$

(2)  $x_1 = 0, x_2 = 1, y = 1$

$$\begin{aligned} \theta &= w_1 x_1 + w_2 x_2 \\ &= (1 \times 0) + (1 \times 1) \\ &= 1 \end{aligned}$$

So  $\theta > \theta \Rightarrow y = 1$   
 $1 > \theta \Rightarrow y = 1$

$\theta < 1$   
 $\theta = [0 \dots 0.9]$

take  $\theta = 0.2$

(3)  $x_1 = 1, x_2 = 0, y = 1$

$$\begin{aligned} \theta &= (1 \times 1) + (1 \times 0) \\ &= 1 \quad \text{i.e. } \theta < 1 \end{aligned}$$

Same as (2), take  $\theta = 0.2$

(4)  $x_1 = 1, x_2 = 1, y = 1$

$$\begin{aligned} \theta &= (1 \times 1) + (1 \times 1) \\ &= 2 \end{aligned}$$

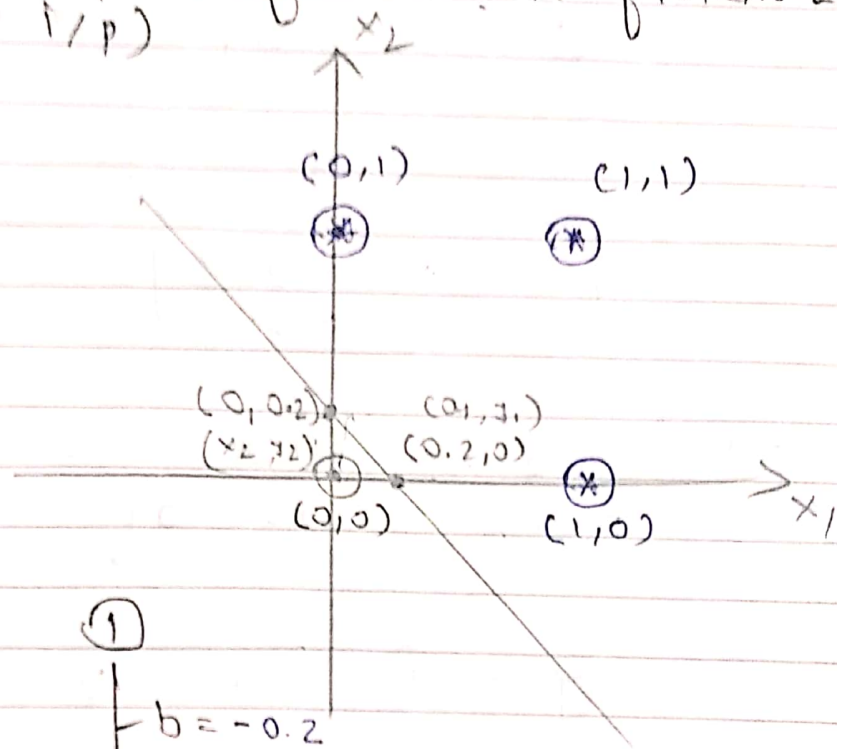
So  $2 > \theta \Rightarrow y = 1$

$\theta = [0 \dots 0.9]$

take  $\theta = 0.2$

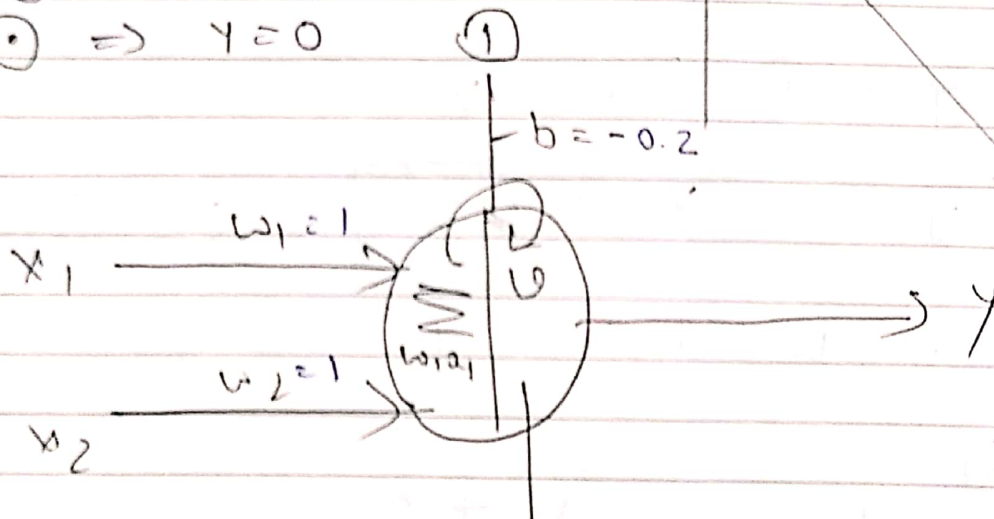
Ex-4 Using linear Separability concept  
 Obtain the response for OR function  
 (Use binary i/p)

$x_1$	$x_2$	$y$
0	0	0
0	1	1
1	0	1
1	1	1



$\odot \Rightarrow y = 0$

$\otimes \Rightarrow y = 1$



Activation function

$$\begin{cases} u \geq 0 \Rightarrow y = 1 \\ u \leq 0 \Rightarrow y = 0 \end{cases}$$

→ equation for line  $\Rightarrow y = mx + b$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{0.2 - 0}{0 - 0.2} = \boxed{-1}$$



$$\rightarrow y_1 = m a_1 + c$$

$$c = y_1 - m a_1$$

$$= 0 - [(-1) \times 0.2]$$

$$\boxed{c = 0.2}$$

$$\rightarrow y = m x + c$$

put values of  $m$  &  $c$

$$y = (-1)x + 0.2$$

$$\boxed{y = -x + 0.2}$$

①

$\rightarrow$  here we have  $y_2 = a_2$  &  $a_2 = x_1$

So

$$a_2 = -a_1 + 0.2$$

also we can write

$$w_1 a_1 + w_2 a_2 + b = 0 \quad (\text{decision boundary})$$

$$w_2 a_2 = -w_1 a_1 + b$$

$$\boxed{a_2 = \frac{-w_1 a_1 + b}{w_2}}$$

②

Compare ① & ②

$$U = a_1 u_1 + a_2 u_2 + b$$

$$w_1 = w_2 = 1$$

$$b = -0.2$$

Using Neuron.

Ex:- Implement a function to identify following objects are eatable or not. Take color & shape as i/p.  
(Blue) (round)

	Color (Blue=1)	Shape (round?)	eatable
blue berry	1	1	1
Violet	1	0	0
Golf ball	0	1	0
hot dog	0	0	0

1 = Yes  
0 = No

## (\*) McCulloch Pitts Neuron

(MCP / MP neuron)

→ Threshold with activation function should satisfy following condition

$$p - n \leq \theta$$

$w$  = excitatory  $w$

$p$  = inhibitory  $w$

$n$  = no. of i/p.

$$U \geq \theta \Rightarrow Y = 1$$

$$U < \theta \Rightarrow Y = 0$$

Ex:- AND function using MCP neuron (binary data)

$a_1$	$a_2$	$Y$
0	0	0
0	1	0
1	0	0
1	1	1

→ Assume  $w_1 = 1$ ,  $w_2 = 1$

(both ~~excitatory~~ excitatory  $w$ )

for (0,0)

$$\begin{aligned} U &= a_1 w_1 + a_2 w_2 \\ &= 0 \times 1 + 0 \times 1 \\ &= 0 \end{aligned}$$

$$\text{for } (0,1) = 0 \times 1 + 1 \times 1 = 1$$

$$(1,0) = 1 \times 0 + 0 \times 1 = 0$$

$$(1,1) = 1 \times 1 + 1 \times 1 = 2$$

here  $\theta = 2$  then  $\phi$  only the neuron fires.

This can also be obtained by:

$$\theta \geq nw - p$$

$$\theta \geq 2(1) - 0$$

$$\theta \geq 2$$

$$v \geq 2 \Rightarrow y = 1$$

$$v < 2 \Rightarrow y = 0$$

Activation function.

$$\phi [w_1 = w_2 = 1]$$

Ex-2 Implement AND-NOT function using Mc P neuron for inputs.

Case I

$$w_1 = w_2 = 1$$

$x_1$	$x_2$	$y$	
0	0	0	$(0,0), v = 0 \times 1 + 0 \times 1 = 0$
0	1	0	$(0,1), v = 0 \times 1 + 1 \times 1 = 1$
1	0	1	$(1,0), v = 1 \times 1 + 0 \times 1 = 1$
1	1	0	$(1,1), v = 1 \times 1 + 1 \times 1 = 2$

Assume

$$w_1 = w_2 = 1$$

→ for  $w_1, w_2 = 1$   $\phi$  from calculated output i/p ( $v$ ), it is not possible to fire the neuron for i/p (1,0) only

→ hence these weights are not suitable.

Case - II Assume one weight as excitatory & other as inhibitory.

i.e.  $w_1 = 1, w_2 = -1$

now for inputs.

$(0,0), v = 0 \times 1 + 0 \times (-1) = 0$   
 $(0,1), v = 0 \times 1 + 1 \times (-1) = -1$   
 $(1,0), v = 1 \times 1 + 0 \times (-1) = 1$   
 $(1,1), v = 1 \times 1 + 1 \times (-1) = 0$

from calculated i/p it is possible to get required o/p.

$Q >= n \cdot w - p$

$Q >= 2(1) - (1)$

$Q >= 1$

Activation function

$Q \geq 1 \Rightarrow y = 1$

$Q < 1 \Rightarrow y = 0$

$f \left[ \begin{array}{|c|} \hline w_1 = 1 \\ \hline \end{array} \right] \left[ \begin{array}{|c|} \hline w_2 = -1 \\ \hline \end{array} \right]$

Ex-3 Implement XOR function using Mc Culloch Pitts neuron.

for XOR		
$x_1$	$x_2$	$y$
0	0	0
0	1	1
1	0	1
1	1	0

This can not be represented by simple & single logic function

$$\text{here } y = \frac{a_1 \bar{a}_2}{z_1} + \frac{\bar{a}_1 a_2}{z_2}$$

$$y = z_1 \text{ OR } z_2$$

⇒ First function:  $z_1 = x_1 \bar{x}_2$

$x_1$	$x_2$	$\bar{x}_2$	$z_1 = x_1 \bar{x}_2$ (AND-NOT)
0	0	1	0
0	1	0	0
1	0	1	1
1	1	0	0

Case 1: Assume  $w_1 = 1, w_2 = 1$

for inputs,  $U_{z_1}$

$$(0,0), U_{z_1} = 0 \times 1 + 0 \times 1 = 0$$

$$(0,1), U_{z_1} = 0 \times 1 + 1 \times 1 = 1$$

$$(1,0), U_{z_1} = 1 \times 1 + 0 \times 1 = 1$$

$$(1,1), U_{z_1} = 1 \times 1 + 1 \times 1 = 2$$

→ It is not possible to obtain function  $z_1$  using these weights.

Case II :- take  $w_1 = 1$ ,  $w_2 = -1$

for inputs,  $U_{z1}$

$$(0, 0) \quad U_{z1} = 0 \times 1 + 0 \times (-1) = 0$$

$$(0, 1) \quad U_{z1} = 0 \times 1 + 1 \times (-1) = -1$$

$$(1, 0) \quad U_{z1} = 1 \times 1 + 0 \times (-1) = 1$$

$$(1, 1) \quad U_{z1} = 1 \times 1 + 1 \times (-1) = 0$$

$$0 \Rightarrow n w - p$$

$$0 > 2(1) - (1)$$

$$0 > 1 \quad \text{for } z_1 \text{ neuron.}$$

Activation function

$$U \geq 1 \Rightarrow Y = 1$$

$$U < 1 \Rightarrow Y = 0$$

⇒ Second function =  $Z_2 = \overline{x_1} x_2$

$x_1$	$\overline{x_1}$	$x_2$	$Z_2 = \overline{x_1} x_2$ (NOT-AND)
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0

Case I take  $w_1 = -1$ ,  $w_2 = 1$

for inputs,  $U_{Z_2}$

$$(0,0) \quad U_{Z_2} = -1 \times 0 + 1 \times 0 = 0$$

$$(0,1) \quad U_{Z_2} = -1 \times 0 + 1 \times 1 = 1$$

$$(1,0) \quad U_{Z_2} = -1 \times 1 + 1 \times 0 = -1$$

$$(1,1) \quad U_{Z_2} = -1 \times 1 + 1 \times 1 = 0$$

$$0 \geq nw - p$$

$$0 \geq 2(1) - 1$$

$$0 \geq 1$$

Activation function :-

$$U \geq 1 \Rightarrow Y = 1$$

$$U < 1 \Rightarrow Y = 0$$



⇒ function  $z = y = z_1 \text{ OR } z_2$

$z_1$	$z_2$	$y$
0	0	0
0	1	1
1	0	1
0	0	0

Case I :- take  $w_1 = 1$   $w_2 = 1$

for inputs,  $u$

$(0, 0)$   $u = 0 \times 1 + 0 \times 1 = 0$

$(0, 1)$   $u = 0 \times 1 + 1 \times 1 = 1$

$(1, 0)$   $u = 1 \times 1 + 0 \times 1 = 1$

$Q \geq 1 \Rightarrow P(1) = 0$

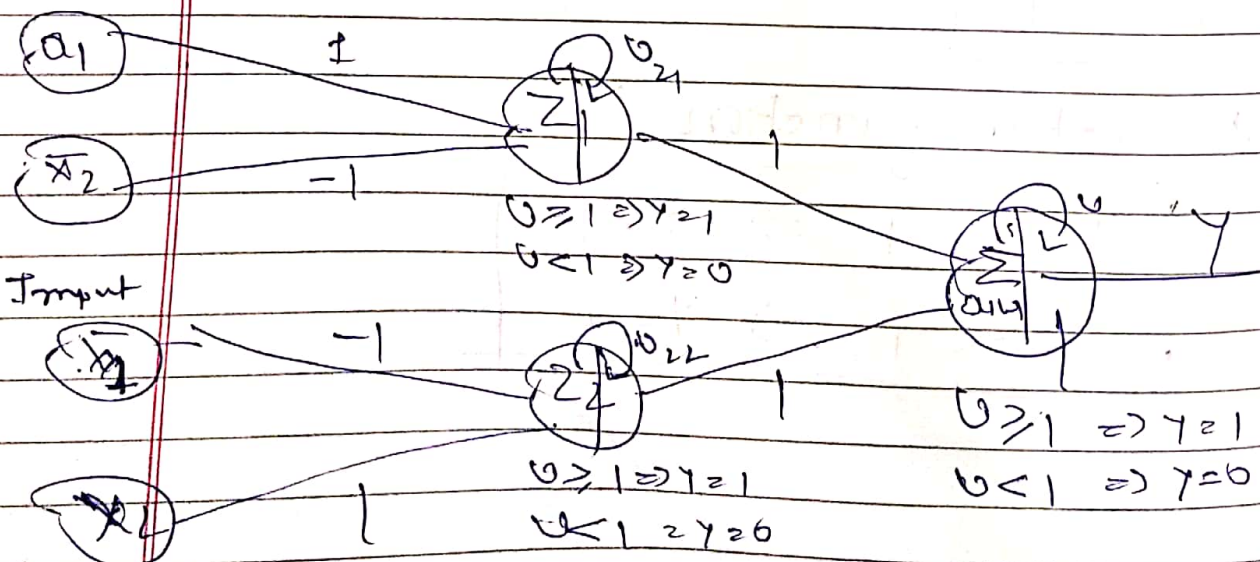
~~scribbles~~

~~scribbles~~

$u \geq 1 \Rightarrow y = 1$

$u < 1 \Rightarrow y = 0$

Input  $x_1 = 1$



## (\*) Linear Separability

- ANN does not give exact solution for a non linear problem. However it provides possible approximate solutions to a nonlinear problem.
- Linear separability is a concept where in the separation of i/p space into regions is based on whether the net response is positive or ~~not~~ negative.
- A decision line is drawn to separate positive & negative responses. also known as linear separable line or decision support line or decision making line.
- We need to classify the pattern based on o/p responses.
- The net i/p for n/w is calculated by
- $$J_{in} = b + \sum_{i=1}^n a_i w_i$$
- The decision boundary can be derived by
- $$b + \sum_{i=1}^n a_i w_i = 0$$

Suppose for two i/p's,

Decision boundary is

$$w_1 a_1 + w_2 a_2 + b = 0$$

$$\text{So, } a_2 = -\frac{w_1 a_1}{w_2} - \frac{b}{w_2}$$

if  $w_2 \neq 0$

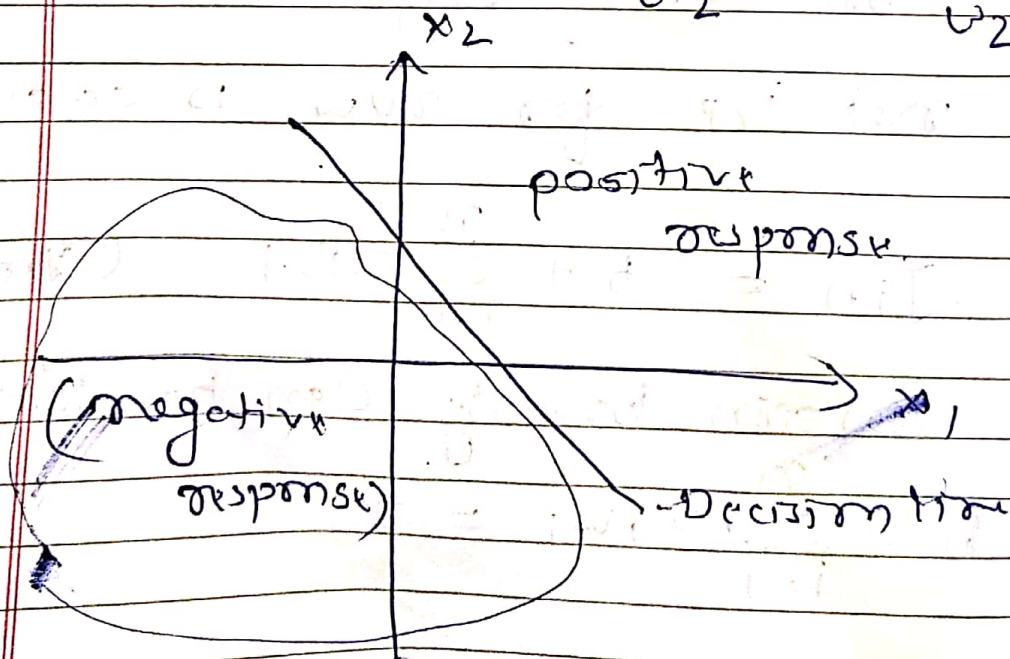
→ if Threshold is used,

$$\text{net i/p} > \theta \text{ (threshold)}$$

then the separating line eq<sup>n</sup> can be

$$w_1 a_1 + w_2 a_2 = \theta$$

$$a_2 = -\frac{w_1 a_1}{w_2} + \frac{\theta}{w_2}$$

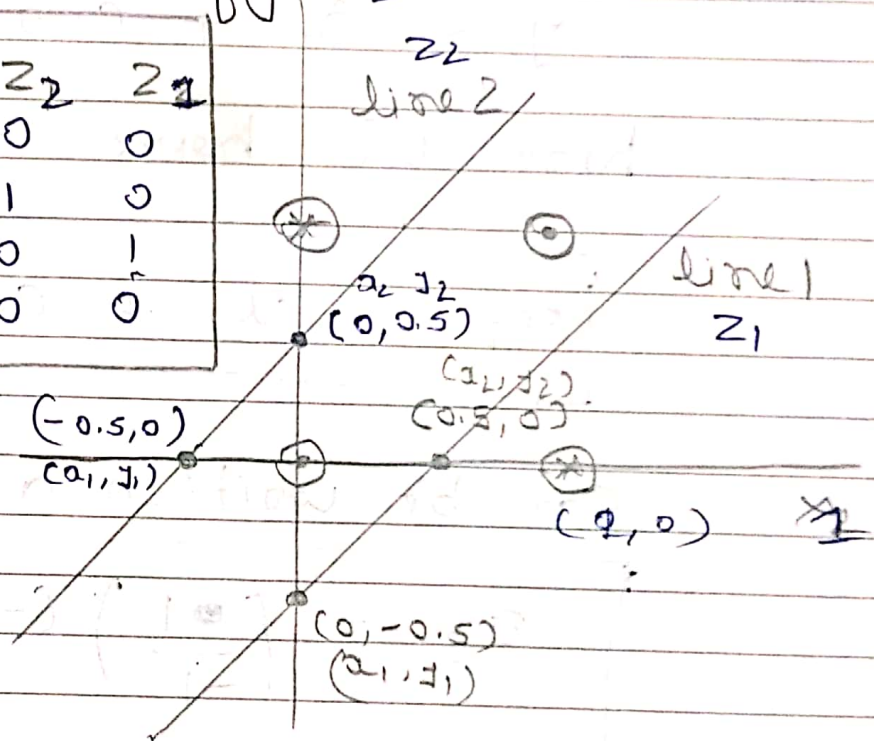


$$x_1, x_2 + x_1, x_2$$

Ex:- Using the linear Separability concept obtain the response for XOR function.

Note:- XOR is non linear or linearly non Separable Problem as its patterns can not be separated by a single line as shown in fig.

$x_1$	$x_2$	$y$	$z_2$	$z_1$
0	0	0	0	0
0	1	1	1	0
1	0	1	0	1
1	1	0	0	0



⊙ ⇒  $y = 0$   
⊗ ⇒  $y = 1$

⇒ for line 1, let's say 0/1 is  $z_1$  for inputs  $x_1$  &  $x_2$  every eq we have  $y = ma + c$  ①



$$m = \frac{z_2 - z_1}{x_2 - x_1} = \frac{0 - (-0.5)}{0.5 - 0} = 1$$

$m = 1$

$$c = y_1 - m a_1$$

$$= -0.5 - (1) \times 0$$

$$c = -0.5$$

→ put values of  $m$  &  $c$  in eqn ①

$$y = x + (-0.5)$$

here we have  $y = a_2$  &  $x = a_1$

$$a_2 = a_1 + (-0.5)$$

can be written as,

$$a_2 = \left( \begin{array}{c} 1 \\ -1 \end{array} \right) (-a_1) + \frac{(-0.5)}{(-1)}$$

②

→ equation for decision boundary is.

$$a_2 = -a_1 \frac{w_1}{w_2} - \frac{b}{w_2}$$

③

Compare (2) & (3)

$$\boxed{w_1 = 1} \quad \boxed{w_2 = -1} \quad \boxed{b = -0.5}$$

→ for i/p  $x_1$  &  $x_2$ , net input  $U_{z_1}$  & output  $z_1$  can be calculated as follows.

here we have,

Activation function

$$U_{z_1} > 0 \Rightarrow z_1 = 1$$

$$U_{z_1} \leq 0 \Rightarrow z_1 = 0$$

$$(0,0), U_{z_1} = 0 \times 1 + 0 \times (-1) - 0.5 = -0.5$$

$$\Rightarrow z_1 = 0$$

$$(0,1), U_{z_1} = 0 \times 1 + 1 \times (-1) - 0.5 = -1.5$$

$$\Rightarrow z_1 = 0$$

$$(1,0), U_{z_1} = 1 \times 1 + 0 \times (-1) - 0.5 = 0.5$$

$$\Rightarrow z_1 = 1$$

$$(1,1), U_{z_1} = 1 \times 1 + 1 \times (-1) - 0.5 = -0.5$$

$$\Rightarrow z_1 = 0$$

So,

$x_1$	$x_2$	$z_1$
0	0	0
0	1	0
1	0	1
1	1	0

⇒ for line 2 let's say o/p is  $z_2$

$$m = \frac{0.5 - 0}{0 - (-0.5)} = 1$$

$$m = 1$$

$$c = 0.5$$

i.e.  $y = x + 0.5$  (1)

i.e.  $x_2 = x_1 + 0.5$   
can be written as

$$x_2 = \frac{(-1)}{1} (x_1) - \frac{(-0.5)}{1}$$

(2)

Decision boundary eq<sup>n</sup> is

$$x_2 = \frac{w_1}{w_2} (x_1) - \frac{b}{w_2}$$

$$w_1 = -1$$

$$w_2 = 1$$

$$b = -0.5$$

Activation fun<sup>n</sup>.

$$w_{z_2} > 0 \Rightarrow z_2 = 1$$

$$w_{z_2} \leq 0 \Rightarrow z_2 = 0$$

for i/p's

$$(0,0), U_{z_2} = 0 \times (-1) + 0 \times 1 - 0.5 = -0.5$$

$$\Rightarrow Z_2 = 0$$

$$(0,1), U_{z_2} = 0 \times (-1) + 1 \times 1 - 0.5 = 0.5$$

$$\Rightarrow Z_2 = 1$$

$$(1,0), U_{z_2} = 1 \times (-1) + 1 \times 0 - 0.5 = -1.5$$

$$\Rightarrow Z_2 = 0$$

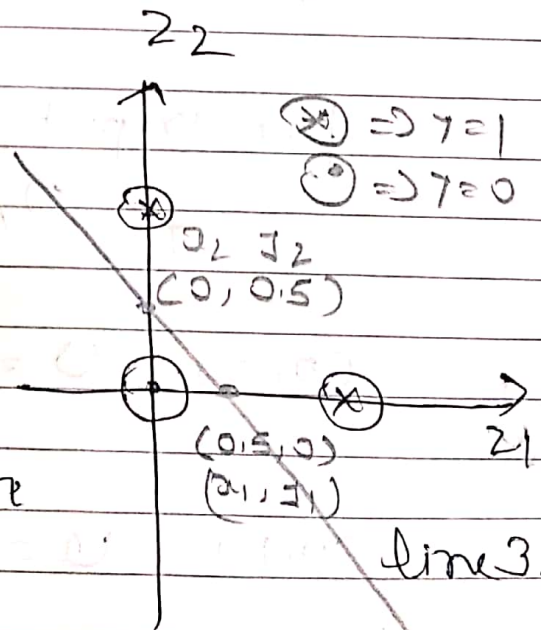
$$(1,1), U_{z_2} = 1 \times (-1) + 1 \times 1 - 0.5 = -0.5$$

$$\Rightarrow Z_2 = 0$$

$x_1$	$x_2$	$z_2$
0	0	0
0	1	1
1	0	0
1	1	0

$$\Rightarrow$$

$x_1$	$x_2$	$z_1$	$z_2$	$y$
0	0	0	0	0
0	1	0	1	1
1	0	1	0	1
1	1	0	0	0



for line 3, inputs are  
 $z_1$  &  $z_2$  & o/p is  $y$

$$m = -1$$

$$C = 0 - (-1)(0.5)$$

$$= 0.5$$

$$C = 0.5$$



$$y = -a + 0.5$$

$$\text{P.e. } a_2 = -a_1 - (-0.5)$$

↳ ①

Decision boundary eq<sup>n</sup>.

$$a_2 = \frac{w_1}{w_2} (-a_1) - \frac{b}{w_2}$$

↳ ②

Compare ① & ②

$$w_1 = 1$$

$$w_2 = 1$$

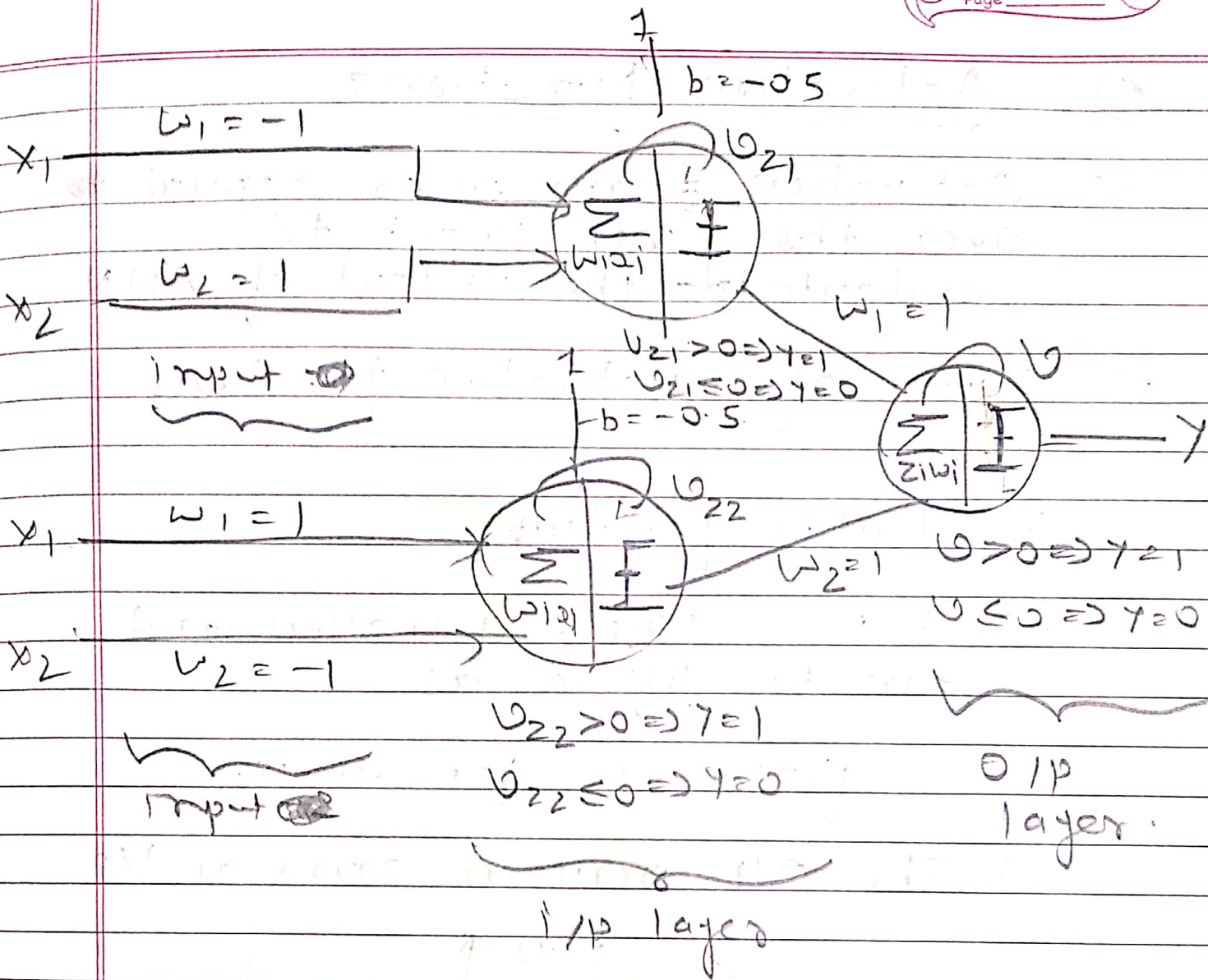
$$b = -0.5$$

for input  $z_1, z_2$ , net input  $u$  of o/p  $y$  is calculated as

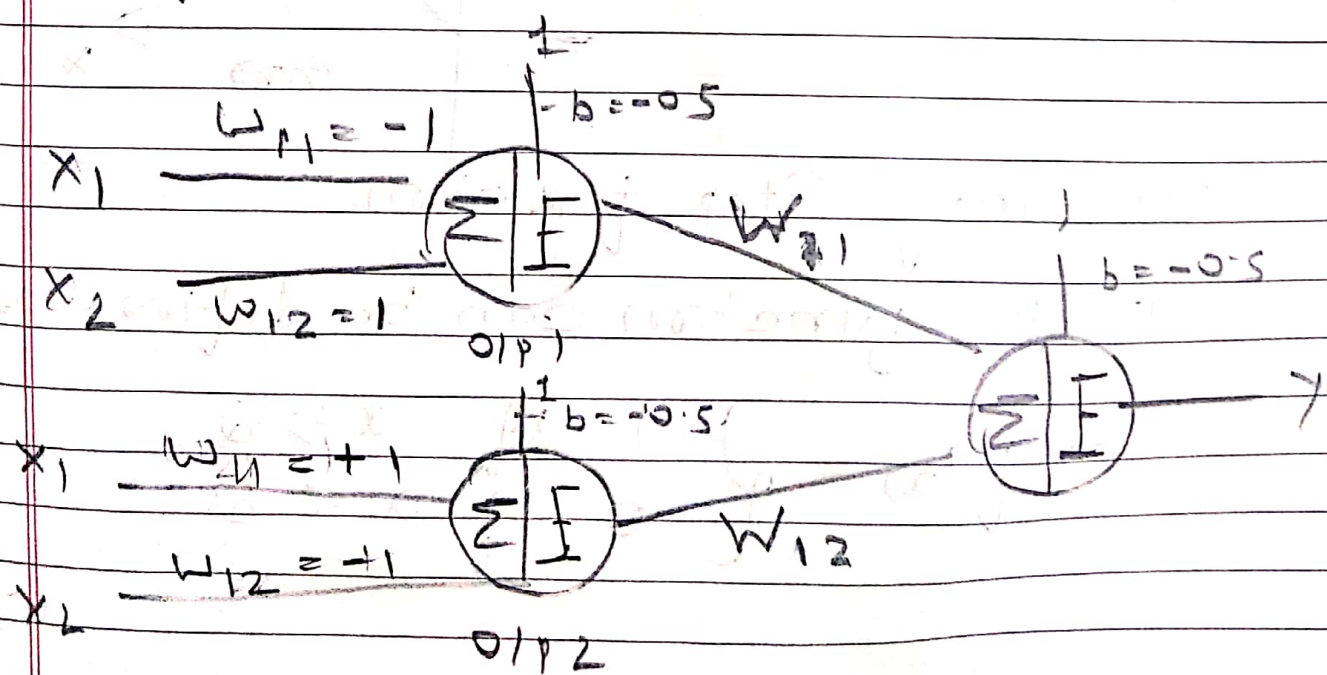
$$(0,0) \quad u = 0 \times 1 + 0 \times 1 - 0.5 = -0.5 \Rightarrow y = 0$$

$$(0,1) \quad u = 0 \times 1 + 1 \times (1) - 0.5 = 0.5 \Rightarrow y = 1$$

$$(1,0) \quad u = 1 \times 1 + 0 \times (1) - 0.5 = 0.5 \Rightarrow y = 1$$



Weights can be represented as  $w_{ij}$



## (\*) Activation functions

→ Activation function is applied to over the net input to calculate the output of ANN

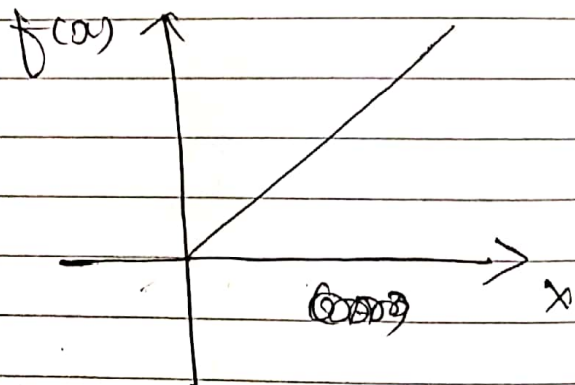
→ Types of Activation functions.

### ① Identity function

→ It is a linear function and can be defined as

$$f(x) = x \text{ for all } x.$$

→ The 0/1 remain same as 1/0



### ② Binary Step function

→ This function can be defined as

$$f(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

→ widely used in single-layer nets to convert net i/p to o/p. i.e. (1 or 0) binary. where  $\theta$  = Threshold

### ③ Bipolar Step function.

$$f(x) = \begin{cases} (+1) & \text{if } x \geq \theta \\ (-1) & \text{if } x < \theta \end{cases}$$

where  $\theta$  = Threshold.

- used in single layer nets to convert net input to output i.e. bipolar (+1 or -1)

### ④ Sigmoidal functions

- widely used in back propagation nets because the relation between the value of functions at a point and the value of the derivative at that point

- which reduces computational burden during training.

- Sigmoidal function have 2 types

(A) Binary Sigmoid

(B) Bipolar Sigmoid.

(A) Binary Sigmoid function

- known as logistic sigmoid or unipolar

- can be defined as

$$f(x) = \frac{1}{1 + e^{-\lambda x}}$$

$\lambda = \text{constant}$

$$f'(x) = \lambda f(x) [1 - f(x)]$$

range is 0 to 1

(B) Bipolar sigmoid function

$$f(x) = \frac{2}{1 + e^{-\lambda x}} - 1$$

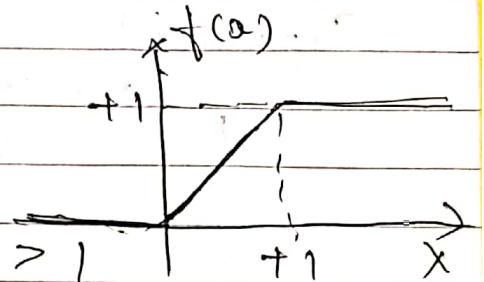
$$= \frac{1 - e^{-\lambda x}}{1 + e^{-\lambda x}}$$

range is  $-1$  to  $+1$

$$f'(a) = \frac{\lambda}{2} [1 + f(a)] [1 - f(a)]$$

⑤ Ramp function.

$$f(a) = \begin{cases} 1 & \text{if } a > 1 \\ a & \text{if } 0 \leq a \leq 1 \\ 0 & \text{if } a < 0 \end{cases}$$



⇒ graphical representation

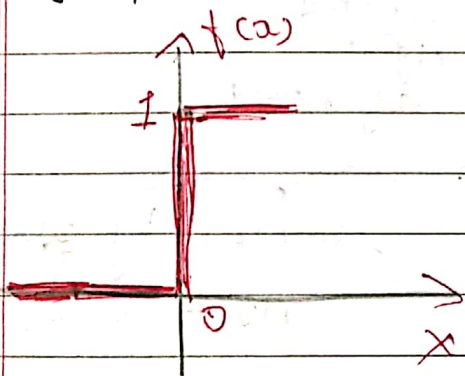


fig binary step fun<sup>m</sup>

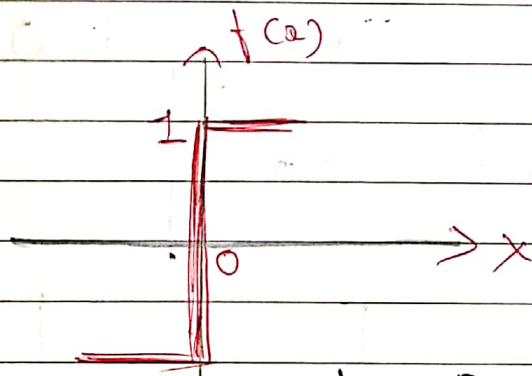


fig Bipolar step fun<sup>m</sup>

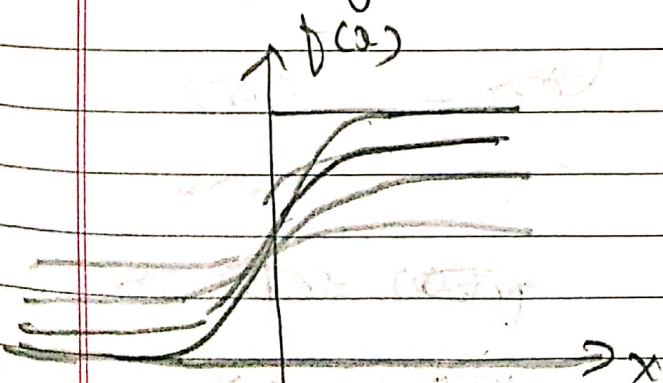


fig. binary sigmoid fun<sup>m</sup>

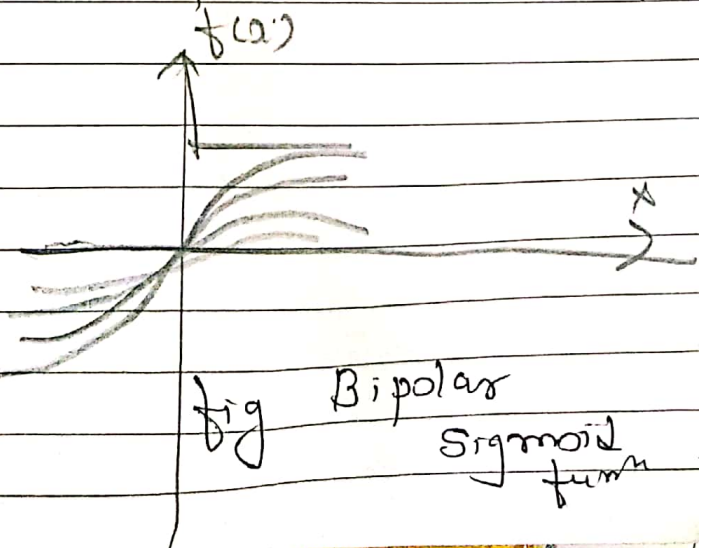
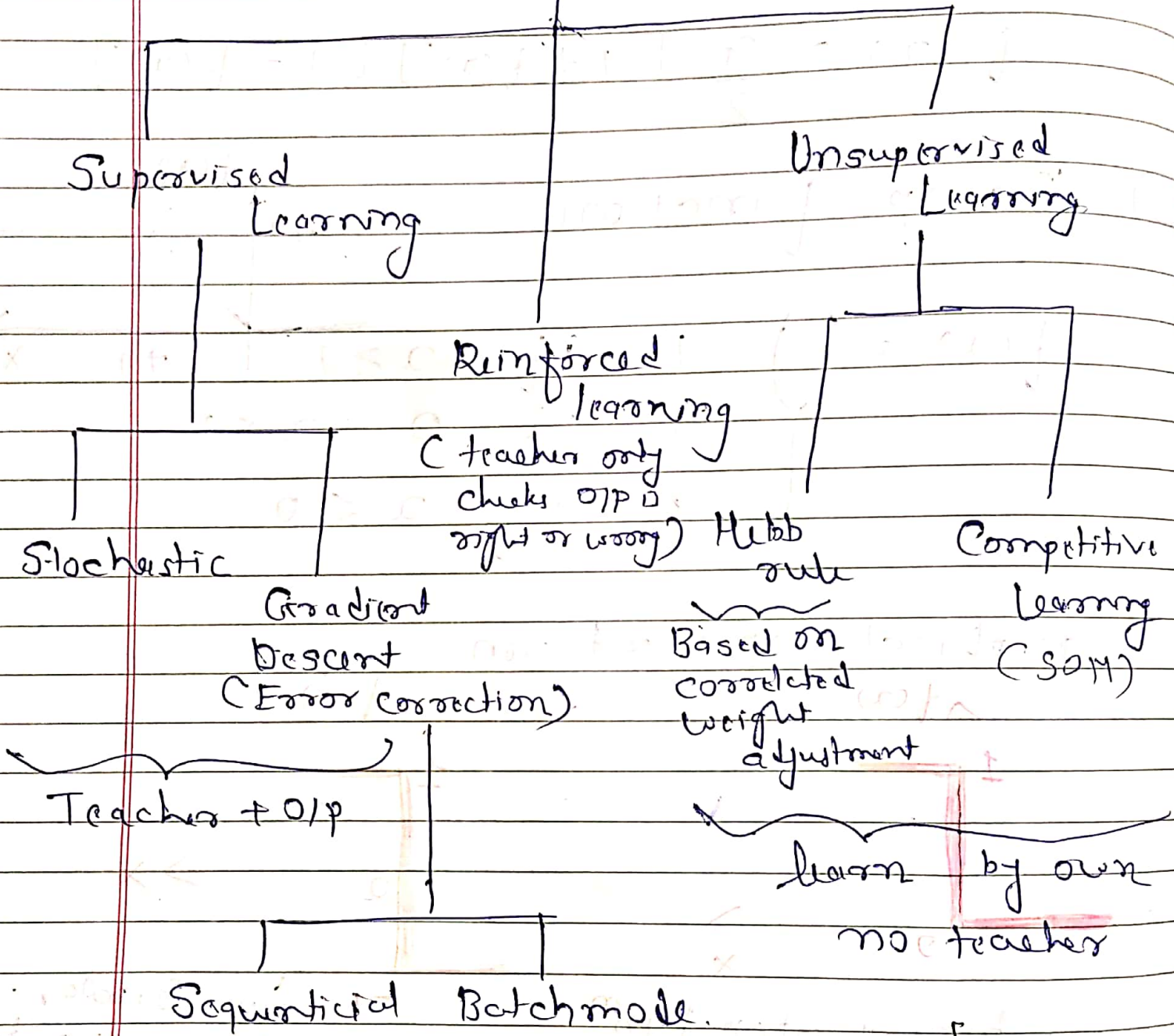


fig Bipolar sigmoid fun<sup>m</sup>

# Techniques

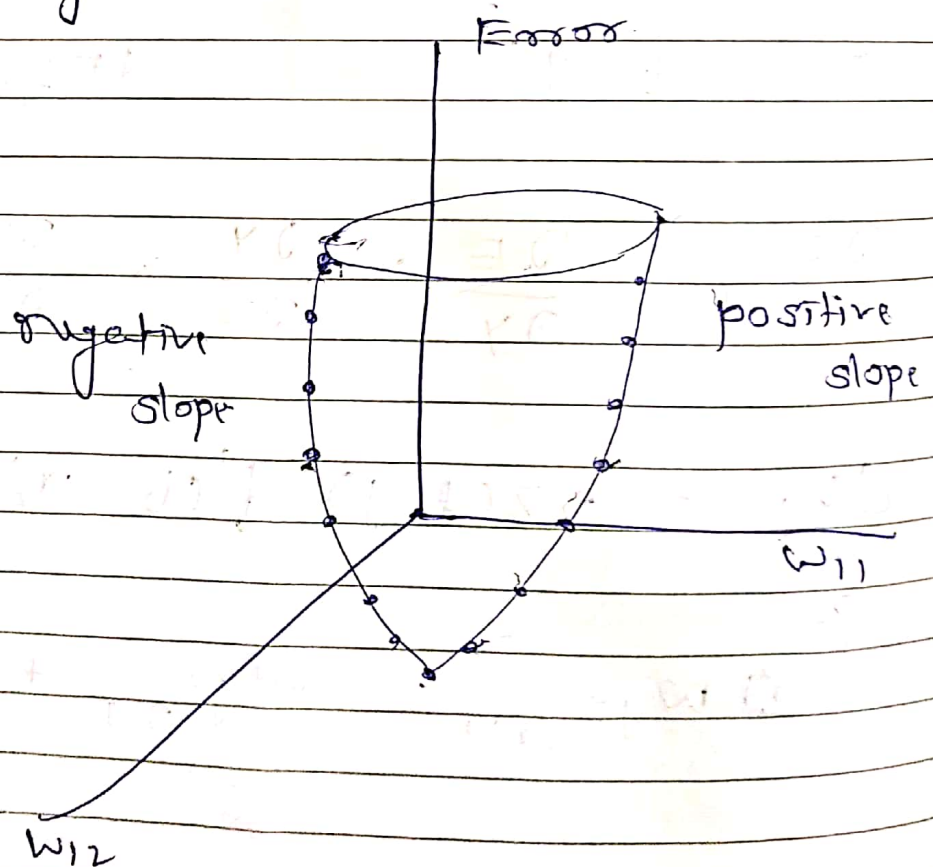
④

## Learning (Process) in Neural Network



## (\*) Gradient Descent Method. (Steepest Descent)

- A gradient measures how much an O/P function changes if you change in an I/P
- Gradient is a partial derivative w.r.t its learning
- Higher the gradient, steeper the slope of faster the model.
- But if slope is 0, a model stops learning.





# Widrow-Hoff rule

classmate

Date  
Page

OR

Here

$$\Delta w_{ji} = \alpha (t - y) x_i$$

OR

where  $\alpha$  is a

learning rate

$\Rightarrow \alpha$  (learning rate) ranges from  $[0 \text{ to } 1]$

Note: If initial weights are taken as 0.

that is  $y^m = 0 \Rightarrow y = 0$

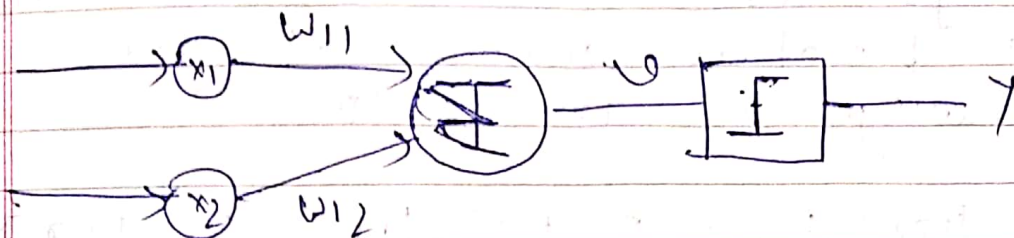
$$\Delta w_{ji} = \alpha (t - y) x_i$$

$$\Delta w_{ji} = \alpha t x_i$$

for single output  $j=1$

$$\Delta w_1 = \alpha t x_i$$

(\*) Finding the equation of  $\Delta w_{ji}$  (Error) in respect of

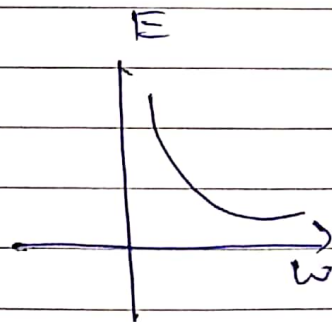


$$u = \sum_1^n w_{ji} x_j \quad \frac{\partial u}{\partial w_{ji}} = x_j$$

$$y = f(u) \quad \frac{\partial y}{\partial u} = f'(u)$$

$$E = (1 - y)^2 \quad \frac{\partial E}{\partial y} = -2(1 - y)$$

$$\frac{\partial E}{\partial w_{ji}} = \frac{\partial E}{\partial y} \times \frac{\partial y}{\partial u} \times \frac{\partial u}{\partial w_{ji}}$$



$$\Delta w_{ji} = -2(1 - y) f'(u) x_j$$

So

$$w_{ji}^{new} = w_{ji}^{old} + \Delta w_{ji}$$

# (X) Learning Process in Biological Neuron

OR  
learning in BNN

OR : (Perception Learning Rule)  
Hebb learning / Hebb rule. (1949)

→ "When an axon of cell A is near enough to excite cell B and repeatedly or persistently takes place in firing it, some growth process or metabolic change takes place in one or both of the cells, such that A's efficiency, as one of the cell firing B, is increased."

→ According to Hebb rule

The weight vector is found to increase proportionally to the product of the input of learning signal.

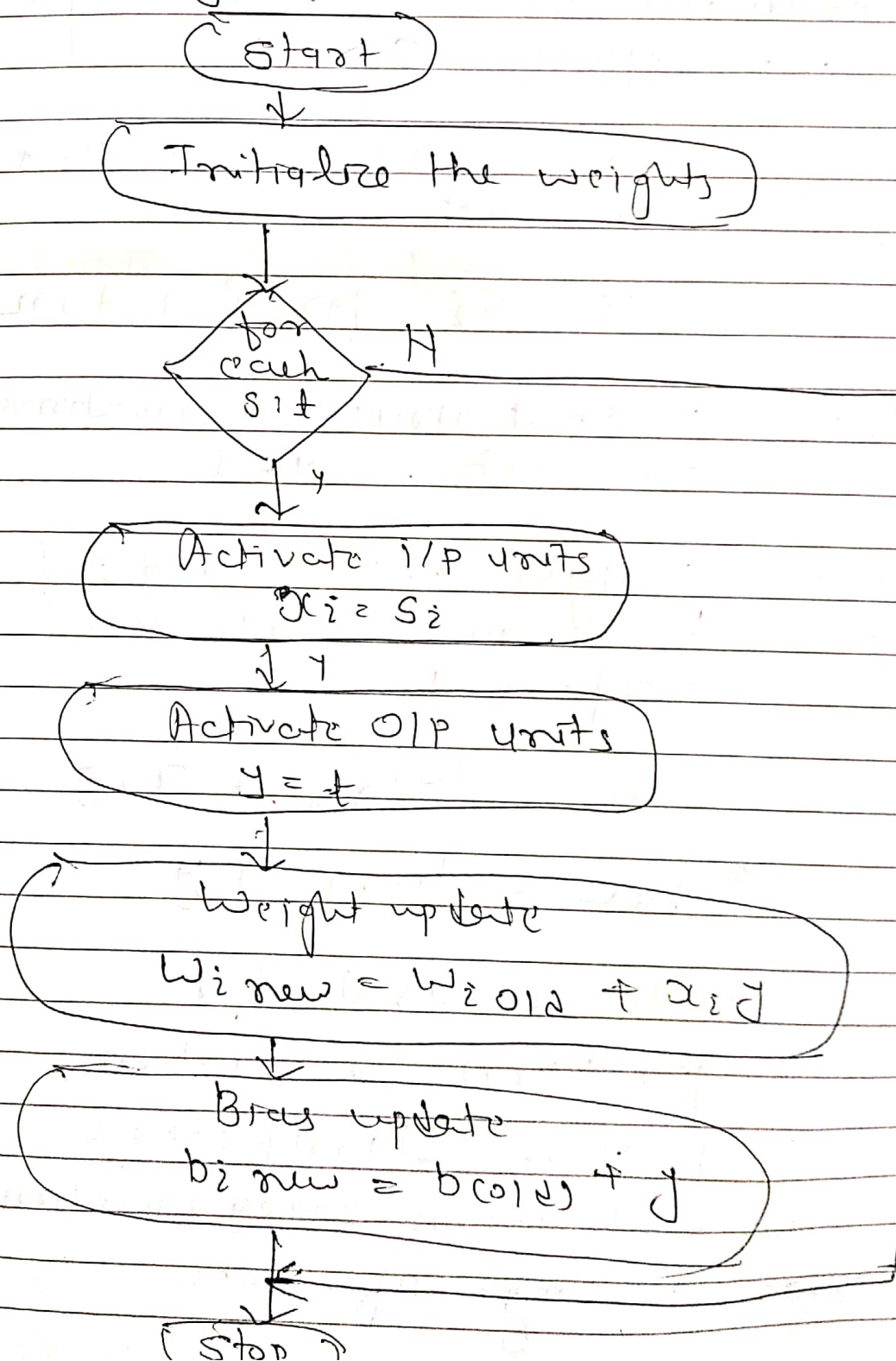
→ Here the learning signal is equals to neuron's o/p

→ here the weights can be increased by modification in the synaptic gap.

→ The weight update in hebb rule is given by

$$W_i(\text{new}) = W_i(\text{old}) + x_i d$$

⇒ Training algorithm (Hebb training)



⇒ Algorithm for Hebb training.

Step 0:- Initialize the weights

Step 1 :- Step 2-4 have to be performed for each IP training vector and target O/P pair. S:t.

Step 2:- Input units activations  $x_i$  are set

$$x_i = s_i \quad \text{for } i = 1 \text{ to } n$$

Step 3 Output units activations are set,  $y = t$

Step 4 Weight adjustments & bias adjustments are performed.

$$w_{ij}^{\text{new}} = w_{ij}^{\text{old}} + x_i y_j$$

$$b_j^{\text{new}} = b_j^{\text{old}} + y_j$$

→ Hebb rule is used for

- ↳ pattern association
- ↳ pattern classification
- ↳ pattern categorization

over a range of data

Ex: 
$$\begin{array}{ccc} \text{I} & & \text{O} \\ * & * & * \\ 0 & * & 0 \\ * & * & * \end{array}$$

$$\begin{array}{l} * = 1 \\ 0 = 0 \end{array}$$

Pattern	$a_1$	$a_2$	$a_3$	$a_4$	$a_5$	$a_6$	$a_7$	$a_8$	$a_9$	O/P
I	1	1	1	-1	1	-1	1	1	1	1
0	1	1	1	1	-1	1	1	1	1	-1

→ Initialize all weights  $w_1$  to  $w_9 = 0$   
 $b = 0$

① for input pattern I change in  $w$

$$\Delta w_j = a_j \cdot d$$

So,

$$\Delta w_1 = 1 \times 1 = 1$$

$$\Delta w_2 = 1 \times 1 = 1$$

$$\Delta w_3 = 1 \times 1 = 1$$

$$\Delta w_4 = -1 \times 1 = (-1)$$

$$\Delta w_5 = 1 \times 1 = 1$$

$$\Delta w_6 = -1 \times 1 = (-1)$$

$$\Delta w_7 = 1 \times 1 = 1$$

$$\Delta w_8 = 1 \times 1 = 1$$

$$\Delta w_9 = 1 \times 1 = 1$$

$$\Delta b = d = 1$$

→ Calculate new weights.

$$w_j^{\text{new}} = w_j^{\text{old}} + \Delta w_j$$

$$w_{1 \text{ row}} = 0 + 1 = 1$$

$$w_{2 \text{ row}} = 1$$

$$w_{3 \text{ row}} = 1$$

$$w_{4 \text{ row}} = (-1)$$

$$w_{5 \text{ row}} = +1$$

$$w_{6 \text{ row}} = (-1)$$

$$w_{7 \text{ row}} = 1$$

$$w_{8 \text{ row}} = 1$$

$$w_{9 \text{ row}} = 1$$

$$B(\text{row}) = 1$$

So

$$w_{\text{row}} = [1 \ 1 \ 1 \ -1 \ 1 \ -1 \ 1 \ 1 \ 1 \ 1]$$

⇒ ② for second pattern 0, weight change is

$$\Delta w_j = a_j y$$

$$y = -1 \quad w_{\text{row}} = [1 \ 1 \ 1 \ -1 \ 1 \ -1 \ 1 \ 1 \ 1 \ 1]$$

$$w_j^{\text{new}} = w_j^{\text{old}} + \Delta w_j$$

$$w_{1 \text{ row}} = 1 + 1 \times (-1) = 0$$

$$w_{2 \text{ row}} = 1 + 1 \times (-1) = 0$$

$$w_{3 \text{ row}} = 1 + 1 \times (-1) = 0$$

$$W_4 \text{ row} = -1 + 1 \times (-1) = -2$$

$$W_5 \text{ row} = 1 + (-1) \times (-1) = 2$$

$$W_6 \text{ row} = -1 + 1 \times (-1) = -2$$

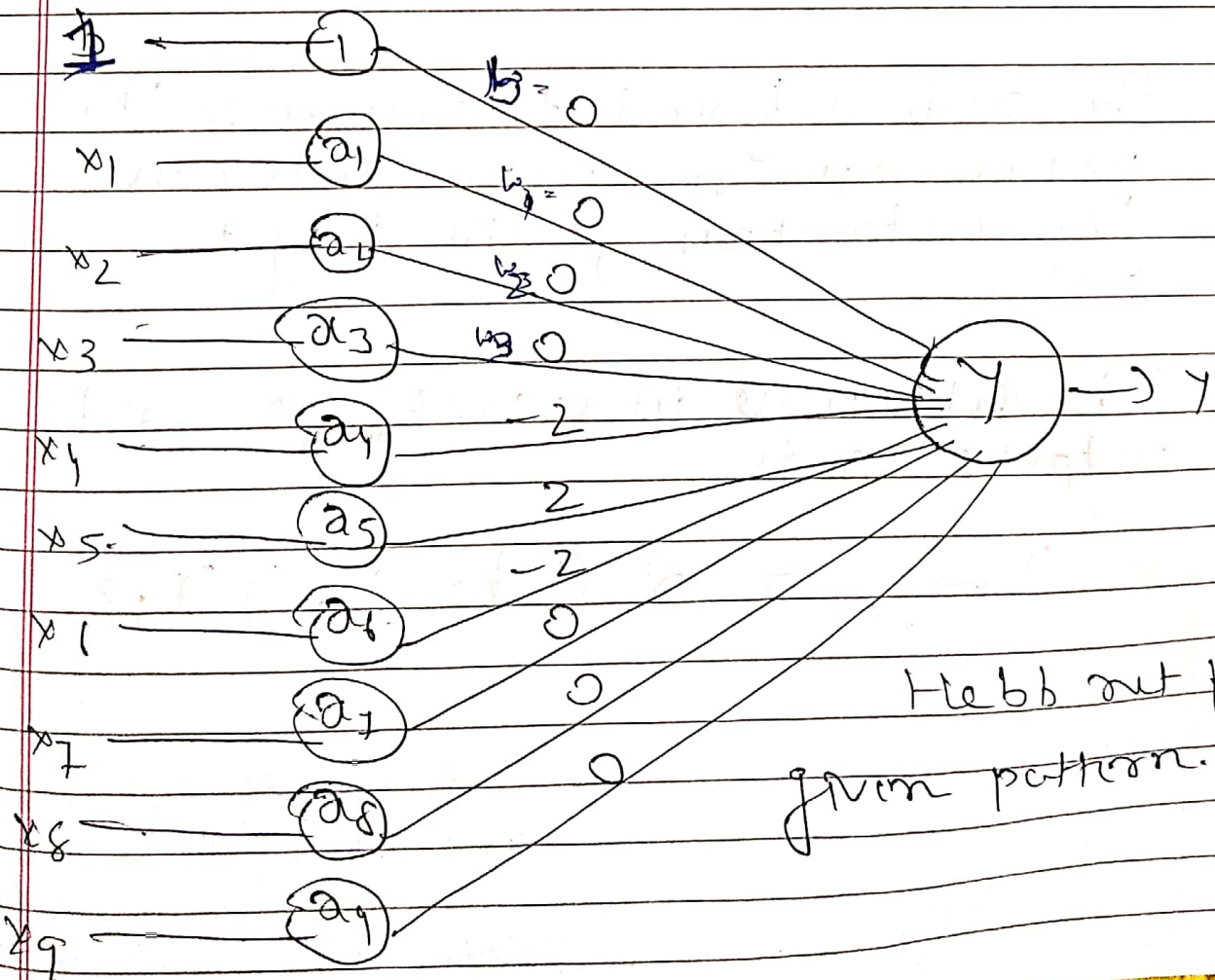
$$W_7 \text{ row} = 1 + 1 \times (-1) = 0$$

$$W_8 \text{ row} = 1 + 1 \times (-1) = 0$$

$$W_9 \text{ row} = 1 + 1 \times (-1) = 0$$

$$b \text{ row} = 1 + 1 \times (-1) = 0$$

$$W_{\text{row}} = [0 \ 0 \ 0 \ -2 \ 2 \ -2 \ 0 \ 0 \ 0 \ 0]$$



Hebb out for given pattern.

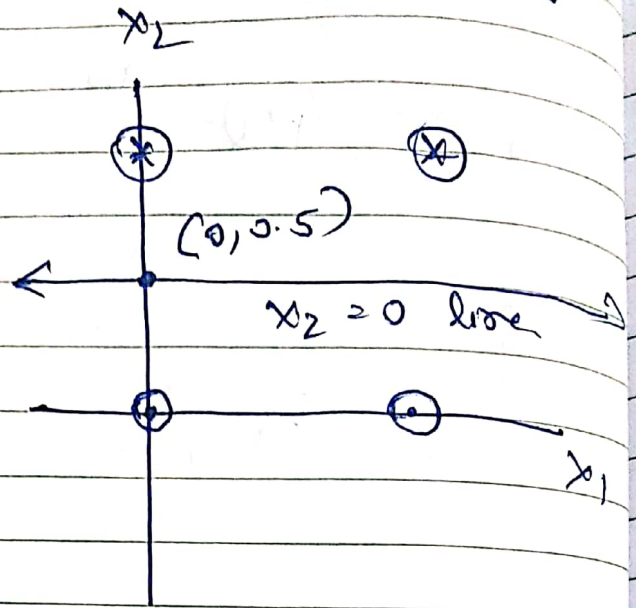


Ex: Find Weight & bias for given function using Linear Separability concept.

$x_1$	$x_2$	$y$
0	0	0
0	1	0
1	0	1
1	1	1

$y = 0 \Rightarrow \odot$

$y = 1 \Rightarrow \otimes$



$$y x_1 + a x_2 = 0.5 - 0$$

$\downarrow$                        $\downarrow$   
 $x_1$                        $x_2$

here  $m = 0$

$$C = y_1 - m a_1$$

$$= 0.5 - 0$$

$$= 0.5$$

Decision boundary Line is

$$w_1 a_1 + w_2 a_2 + b = 0$$

here bias is  $x_2 = 0$

So, Decision boundary Line is.

$$w_1 a_1 + b = 0$$

$$a_1 = -b/w_1$$

$b = 0.5$  &  $w_1 = 1, w_2 = 0$

Test:-

$$V(0,0) = 0 \times 1 + 0 \times 0 - 0.5 < 0$$

$$\Rightarrow y = 0$$

$$v(0,1) = 0 \times 1 + 1 \times 0 - 0.5 < 0$$

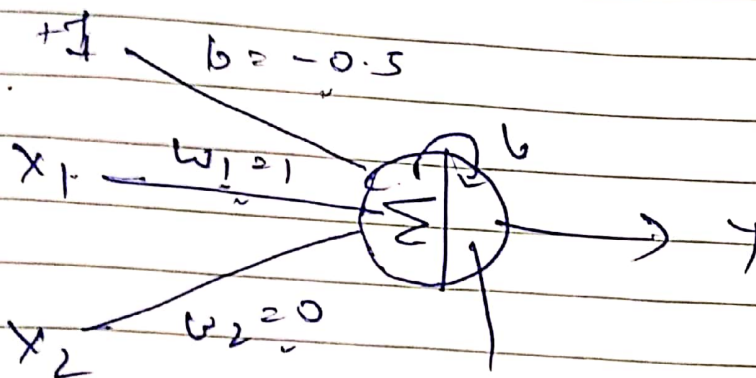
$$\Rightarrow \gamma = 0$$

$$v(1,0) = 1 \times 1 + 0 \times 0 - 0.5 > 0$$

$$\Rightarrow \gamma = 1$$

$$v(1,1) = 1 \times 1 + 0 \times 1 - 0.5 > 0$$

$$\Rightarrow \gamma = 1$$



$$v \leq 0 \Rightarrow \gamma = 0$$

$$v > 0 \Rightarrow \gamma = 1$$