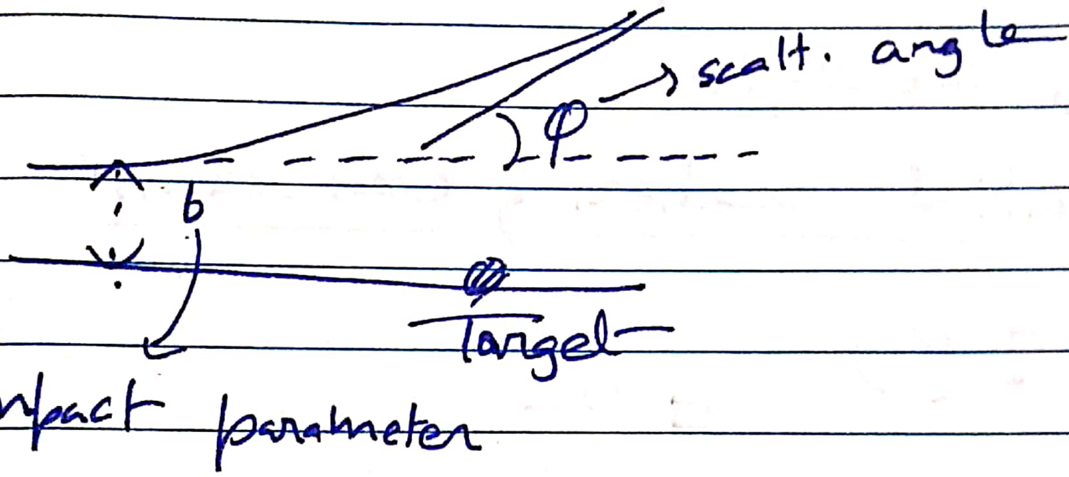
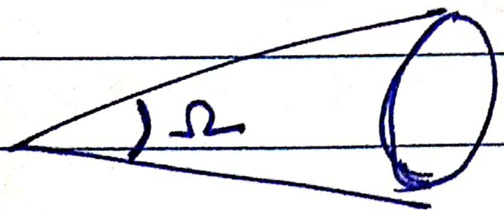
Scattering

- Classical theory → a beam of particle hits the target and emerges at same scattering angle — is known as "scattering"



It is understood that the smaller the impact parameter 'b', larger the scattering angle 'φ'.

- Concept of solid angle → it is a 3D angle (can imagine of an ~~ice~~ ice-cream cone, the angle within is an exp. of solid angle)



Solid angle : $d\Omega = \sin\theta \, d\theta \, d\phi$

Hence, particle incident within an infinitesimal patch of cross-sectional area '$d\sigma$' will scatter into a corresponding solid angle '$d\Omega$'

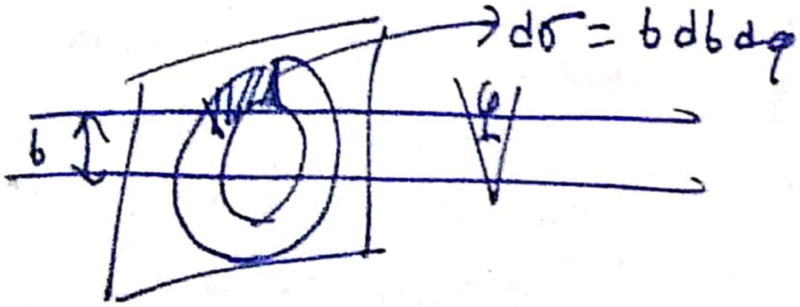
Thus, we define,

"<u>differential scattering cross-section</u>"

as, $$D(\theta) = \frac{d\sigma}{d\Omega}$$

(Cross-section gives a measure of probability of interaction or scattering.)

So, $$d\sigma = D(\theta) \, d\Omega$$



Now, since, $d\sigma = b\,db\,d\varphi$

we can write,

$$D(\theta) = \frac{b\,db\,d\varphi}{\sin\theta\,d\theta\,d\varphi}$$

$$= \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$

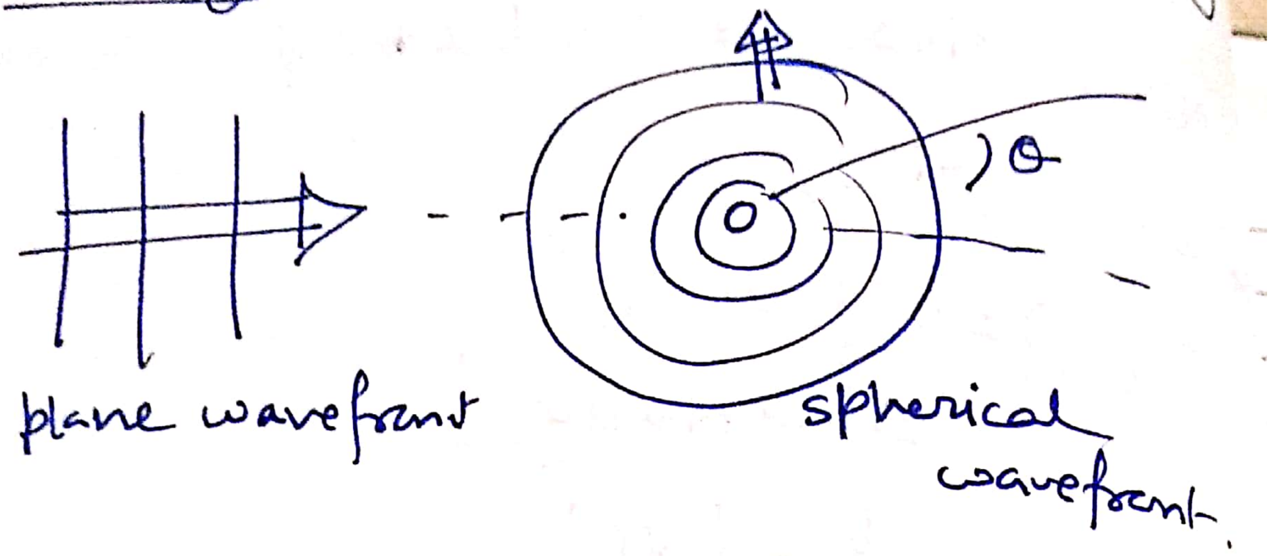
Total cross-section is given by,

$$\sigma = \int D(\theta)\,d\Omega$$

Exercise:- Find the impact paramete and scattering angle for hard spher Scattering. Also find the total cross-section 'σ'. (Radius is 'R')

{ Check that, $\sigma = \pi R^2$ }

Quantam Analysis of Scattering theory →



Here, we imagine that plane wave (e^{ikz}) in z dir^n interacts with the scattering potential or target, hence produces outgoing spherical waves.

If we solve Schrödinger eqn, the sol^n will take the general form of,

$$\psi(r,\theta) \simeq A\left\{ \underbrace{e^{ikz}}_{\text{plane wave}} + \underbrace{f(\theta)\,\frac{e^{ikr}}{r}}_{\text{sph. wave}} \right\}$$

$f(\theta)$ is known as the "scattering amplitude", which one needs to determine in the whole problem.

Now, the probability for incident wave is given in the form,

$$dP = |\psi_{inc.}|^2 dV$$

$$= A^2 (v\,dt)\, d\sigma$$

Similarly for scattered wave,

$$dP = |\psi_{scatt.}|^2 dV = \frac{A^2 f^2}{r^2} (v\,dt)\, r^2 d\Omega$$

$$\left(\text{where } d\sigma = r^2 \sin\theta\, d\theta\, d\phi = r^2 d\Omega \right)$$

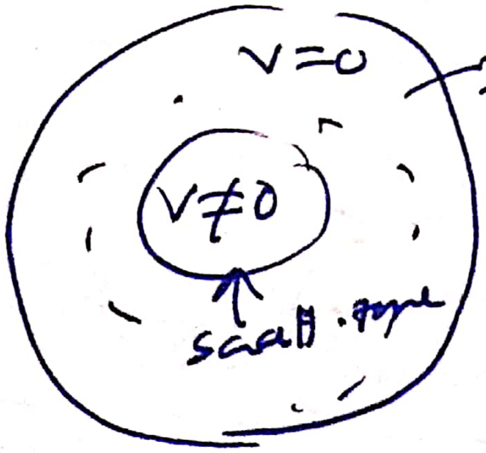
Now, $\boxed{D(\theta) = \frac{d\sigma}{d\Omega} = |f(\theta)|^2}$

Partial Wave analysis → D

Spherically symmetric potential gives rise to wave func.,

$$\psi(r,\theta,\varphi) = \underbrace{R(r)}_{\text{radial}} \, \underbrace{Y_l^m(\theta,\varphi)}_{\text{spherical harmonic}}$$

where, $R(r) \sim \frac{e^{ikr}}{r}$



→ intermediate zone.

$kr \gg 1$

In this assumption the radial eqn. becomes,

$$\frac{d^2u}{dr^2} - \frac{l(l+1)}{r^2}u = -k^2u$$

→ sol^n is combination of spherical Bessel functions $j_l(kr)$ and $\eta_l(kr)$

Again, $h_l^{(1)(2)} = j_l \pm i\eta_l$

Thus outside the scattering region ($V=0$) and we get,

$$\psi(r,\theta,\varphi) = A\left\{e^{ikz} + \sum C_{l,m} h_l^{(1)}(kr)\, Y_l^m(\theta,\varphi)\right\}$$

and $Y_l^0(\theta,\varphi) = \sqrt{\frac{2l+1}{4\pi}}\, P_l(\cos\theta)$

Legendre poly

Finally, we get,

$$f(\theta) = \sum_{l=0}^{\infty} (2l+1)\, a_l\, P_l(\cos\theta)$$

and $$\sigma = 4\pi \sum_{l=0}^{\infty} (2l+1)\, |a_l|^2$$