Fluid Mechanics-II

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Introduction to Basic Concepts Open Channels

Unit II

Module 3

Topics to be covered

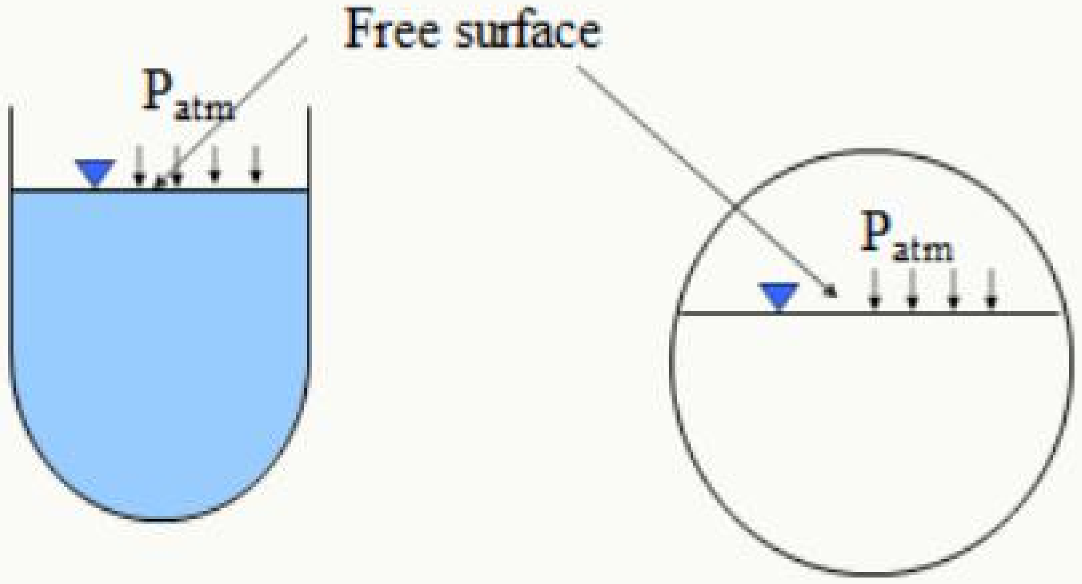
- *Basic Concepts*
- *Open channel v/s pipe flow*
- *Classification of open channel*
- *Definitions*
- *Conservation Laws*
- *Critical Flows*
- *Gradually Varied Flows*
- *Rapidly Varied Flows*

Basic Concepts

- Open Channel flows deal with flow of water in open channels
- Open channel may be define as a passage in which liquid flows with its upper surface exposed to atmosphere.
- Pressure is atmospheric at the water surface and the pressure is equal to the depth of water at any section
- Pressure head is the ratio of pressure and the specific weight of water
- Elevation head or the datum head is the height of the section under consideration above a datum
- Velocity head ($=v^2/2g$) is due to the average velocity of flow in that vertical section

OPEN-CHANNEL FLOW

- Open-channel flow is a flow of liquid (basically water) in a conduit with a free surface.
- That is a surface on which pressure is equal to local atmospheric pressure.



Types of Channels

Natural channel : irregular sections of varying shapes, developed in natural way

Artificial channel : cross sections with regular geometric shapes (remain width and length)

Open channel : A channel without any cover at its top (canals rivers streams water falls)

Covered or closed channel : cover at its top (tunnel, water supply, sewerage lines)

Prismatic channel : A channel with constant bed slope and same cross section along its length is known as a prismatic channel (ex. Triangular , trapezoidal and circular channel)

Kinds of Open Channel

- Canal
- Flume
- Chute
- Drop
- Culvert
- Open-Flow Tunnel

Kinds of Open Channel

- CANAL is usually a long and mild-sloped channel built in the ground.

Kinds of Open Channel

- FLUME is a channel usually supported on or above the surface of the ground to carry water across a depression.

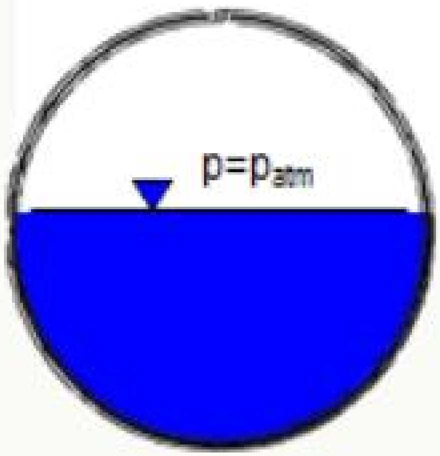
Kinds of Open Channel

- CULVERT is a covered channel flowing partly full, which is installed to drain water through highway and railroad embankments.

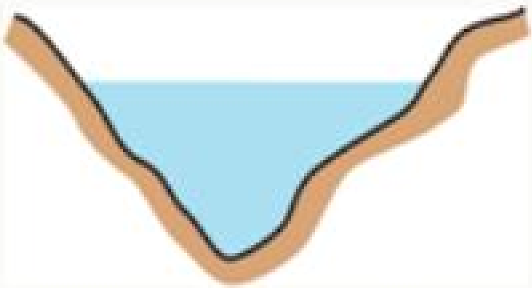
Classification of Open-Channel Flows

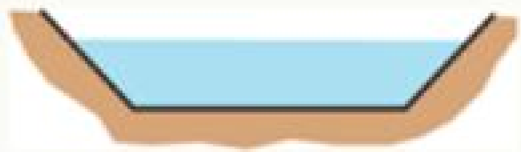
Open-channel flows are characterized by the presence of a liquid-gas interface called the *free surface*.

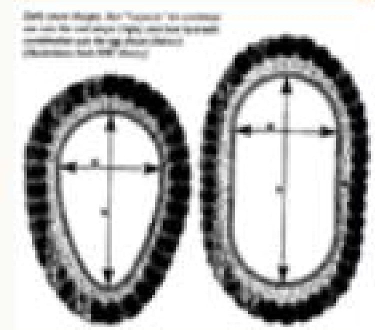
- Natural flows: rivers, creeks, floods, etc.
- Human-made systems: fresh-water aquaducts, irrigation, sewers, drainage ditches, etc.

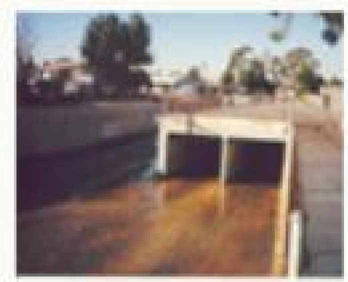


- Open channels
  - Natural channels
  - Artificial channels
    - Open cross section
    - Covered cross section









Comparison of Open Channel Flow & Pipe Flow

Open Channel Flow	Pipe Flow
1) OCF must have a free surface	1) No free surface in pipe flow
2) A free surface is subject to atmospheric pressure	2) No direct atmospheric pressure, hydraulic pressure only.
3) The driving force is mainly the component of gravity along the flow direction.	3) The driving force is mainly the pressure force along the flow direction.
4) HGL is coincident with the free surface.	4) HGL is (usually) above the conduit
5) Flow area is determined by the geometry of the channel plus the level of free surface, which is likely to change along the flow direction and with as well as time.	5) Flow area is fixed by the pipe dimensions The cross section of a pipe is usually circular..

Comparision of Open Channel Flow & Pipe Flow

6) The cross section may be of any from circular to irregular forms of natural streams, which may change along the flow direction and as well as with time.

6) The cross section of a pipe is usually circular

7) Relative roughness changes with the level of free surface

7) The relative roughness is a fixed quantity.

8) The depth of flow, discharge and the slopes of channel bottom and of the free surface are interdependent.

8) No such dependence.

Definitions

- Depth of flow (D) : vertical distance between lowest point of channel section from free surface
- Top width (B) : it is the width of channel section at the free surface
- Wetted area (A) : cross section area of the flow section of channel
- Wetted perimeter (p): length of channel boundary in contact with the flowing water at any section
- Hydraulic radius (R) : it is the ratio of cross sectional area of flow to wetted perimeter. It is also called hydraulic mean depth (m).

$$R = A/P$$

Basic Concepts Cont…

Total head = $p/\bullet + v^2/2g + z$

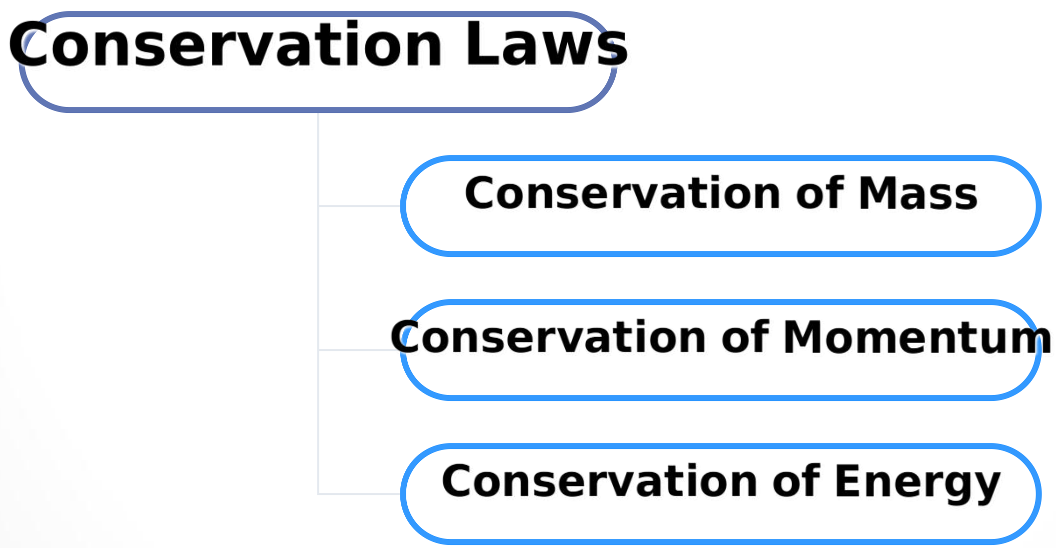
- Pressure head = $p/\bullet$
- Velocity head = $v^2/2g$
- Datum head = z

- The flow of water in an open channel is mainly due to head gradient and gravity

- Open Channels are mainly used to transport water for irrigation, industry and domestic water supply

Conservation Laws

The main conservation laws used in open channels are



Conservation of Mass

Conservation of Mass

In any control volume consisting of the fluid (water) under consideration, the net change of mass in the control volume due to inflow and out flow is equal to the the net rate of change of mass in the control volume

� This leads to the classical continuity equation balancing the inflow, out flow and the storage change in the control volume.

� Since we are considering only water which is treated as incompressible, the density effect can be ignored

Conservation of Momentum and energy

Conservation of Momentum

This law states that the rate of change of momentum in the control volume is equal to the net forces acting on the control volume

� Since the water under consideration is moving, it is acted upon by external forces

� Essentially this leads to the Newton's second law

Conservation of Energy

This law states that neither the energy can be created or destroyed. It only changes its form.

Conservation of Energy

� Mainly in open channels the energy will be in the form of potential energy and kinetic energy

� Potential energy is due to the elevation of the water parcel while the kinetic energy is due to its movement

� In the context of open channel flow the total energy due these factors between any two sections is conserved

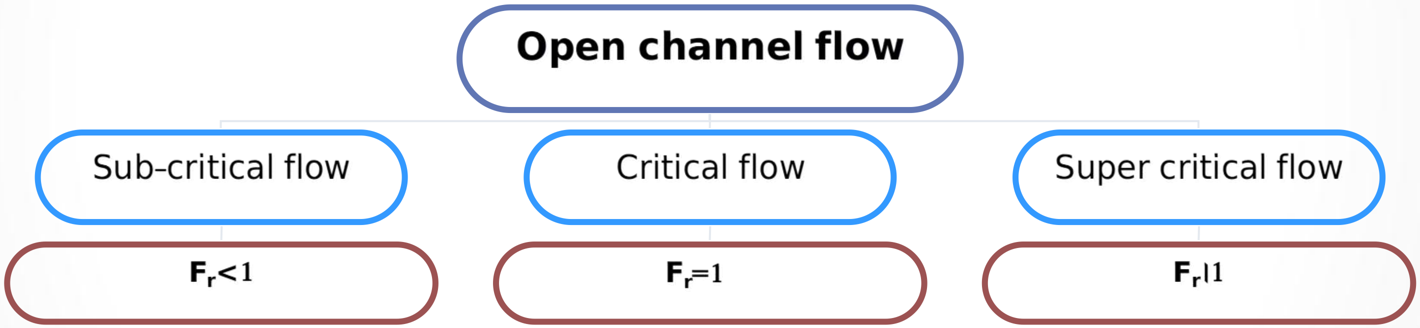
� This conservation of energy principle leads to the classical Bernoulli's equation

$$P/\gamma + v^2/2g + z = Constant$$

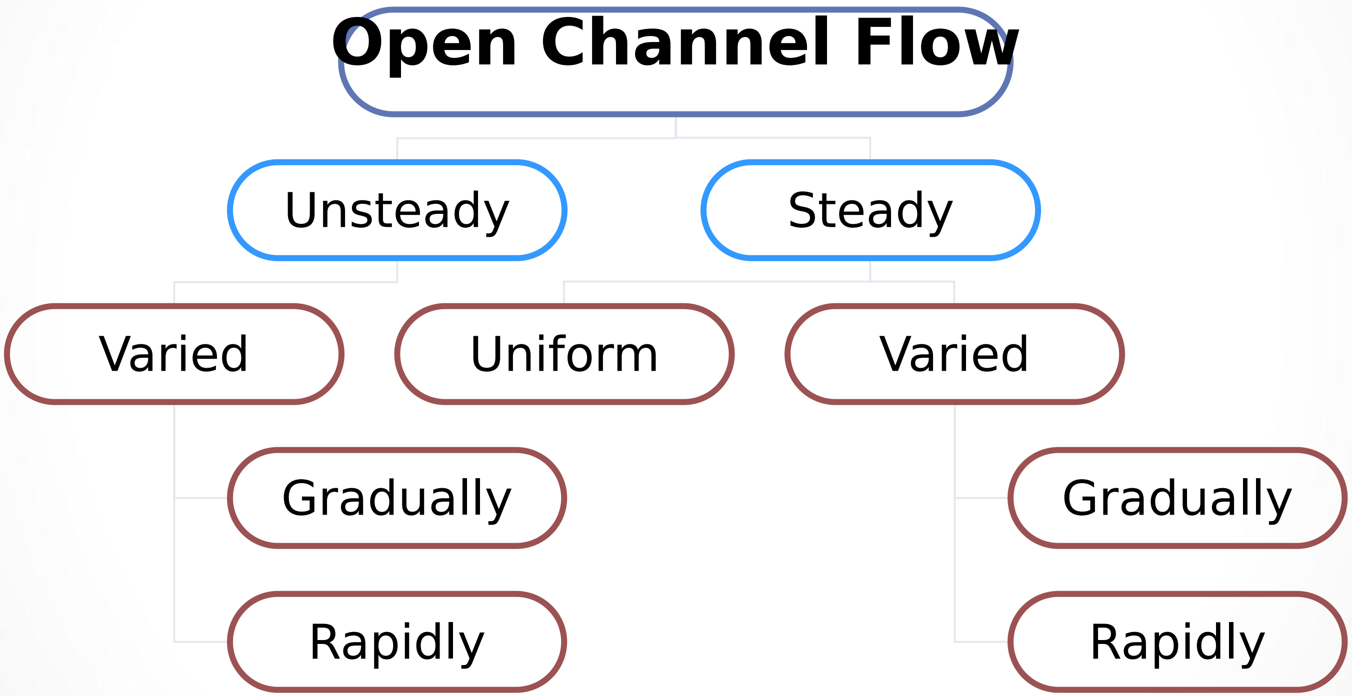
� When used between two sections this equation has to account for the energy loss between the two sections which is due to the resistance to the flow by the bed shear etc.

Types of Open Channel Flows

Depending on the Froude number (F_r) the flow in an open channel is classified as Sub critical flow, Super Critical flow, and Critical flow, where Froude number can be defined as



Types of Open Channel Flow Cont…



Types of Open Channel Flow Cont…

- ***Steady Flow***

Flow is said to be steady when discharge does not change along the course of the channel flow

- ***Unsteady Flow***

Flow is said to be unsteady when the discharge changes with time

- ***Uniform Flow***

Flow is said to be uniform when both the depth and discharge is same at any two sections of the channel

Types of Open Channel Cont…

- ***Gradually Varied Flow***

Flow is said to be gradually varied when ever the depth changes gradually along the channel

- ***Rapidly varied flow***

Whenever the flow depth changes rapidly along the channel the flow is termed rapidly varied flow

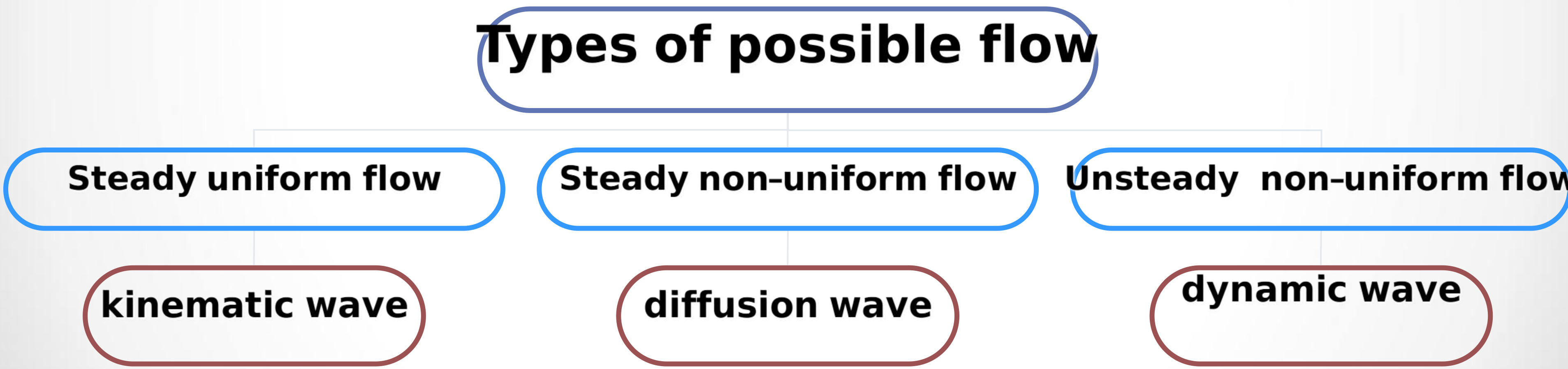
- ***Spatially varied flow***

Whenever the depth of flow changes gradually due to change in discharge the flow is termed spatially varied flow

cont...

- ***Unsteady Flow***

Whenever the discharge and depth of flow changes with time, the flow is termed unsteady flow



Definitions

Specific Energy

It is defined as the energy acquired by the water at a section due to its depth and the velocity with which it is flowing

- Specific Energy E is given by, $E = y + v^2/2g$

Where y is the depth of flow at that section and v is the average velocity of flow

- Specific energy is minimum at critical condition

Definitions

Specific Force

It is defined as the sum of the momentum of the flow passing through the channel section per unit time per unit weight of water and the force per unit weight of water

$$F = Q^2/gA + yA$$

- The specific forces of two sections are equal provided that the external forces and the weight effect of water in the reach between the two sections can be ignored.
- At the critical state of flow the specific force is a minimum for the given discharge.

Critical Flow

Flow is critical when the specific energy is minimum. Also whenever the flow changes from sub critical to super critical or vice versa the flow has to go through critical condition

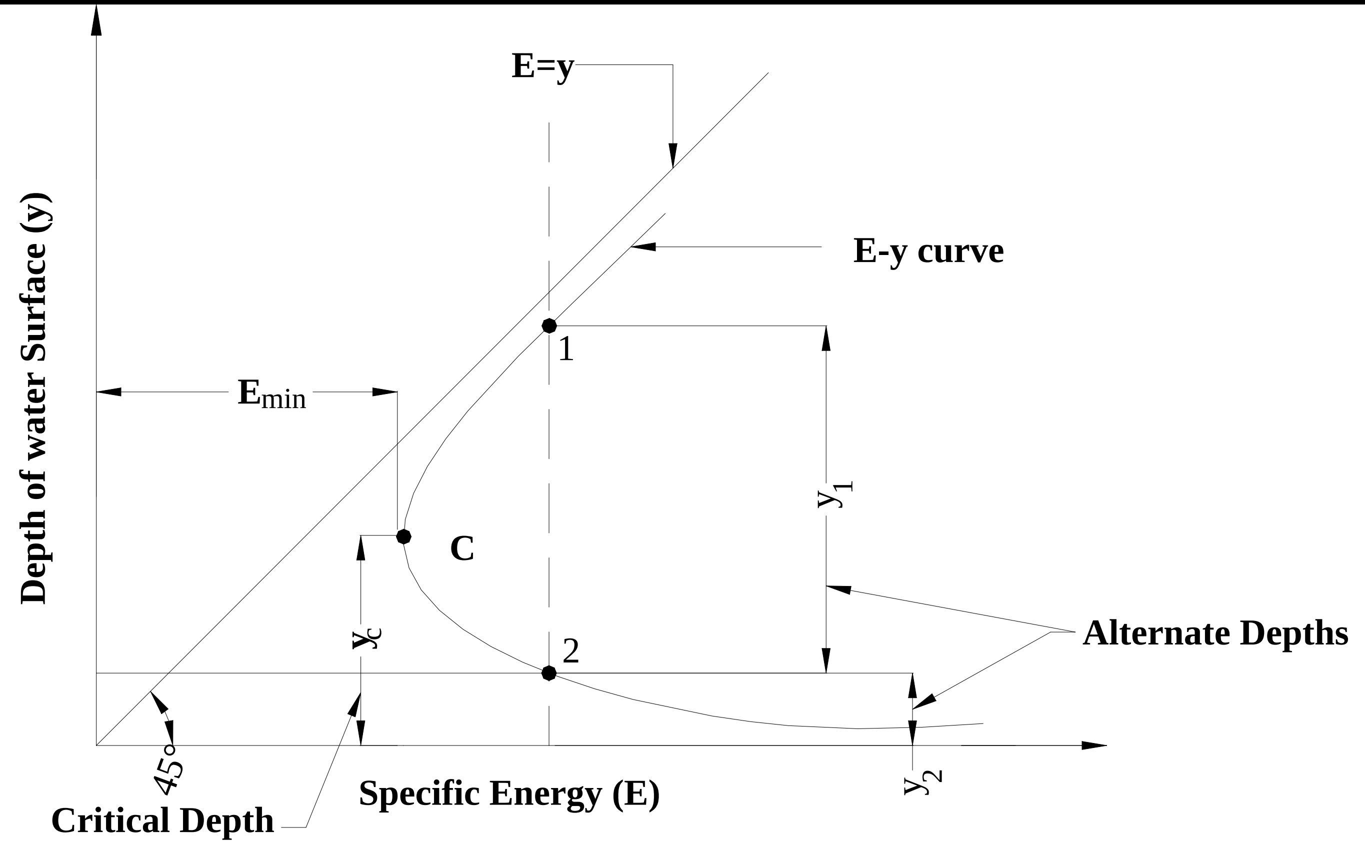
figure is shown in next slide

� **Sub-critical** flow-the depth of flow will be higher whereas the velocity will be lower.

� **Super-critical** flow-the depth of flow will be lower but the velocity will be higher

� **Critical flow**: Flow over a free over-fall

Specific energy diagram



Specific Energy Curve for a given discharge

Characteristics of Critical Flow

- For a rectangular channel $A_c / T_c = y_c$
- Following the derivation for a rectangular channel,
- The same principle is valid for trapezoidal and other cross sections
- Critical flow condition defines an unique relationship between depth and discharge which is very useful in the design of flow measurement structures

Uniform Flows

- This is one of the most important concept in open channel flows

- The most important equation for uniform flow is Manning's equation given by

Where R = the hydraulic radius = A/P

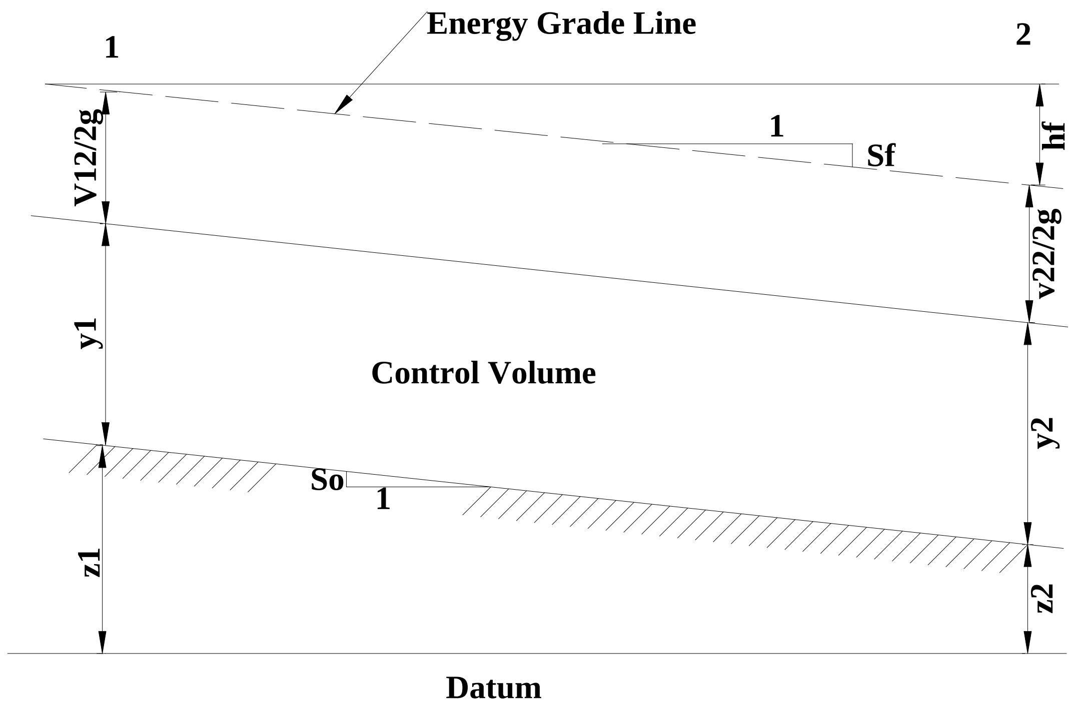
P = wetted perimeter = $f(y, S_0)$

Y = depth of the channel bed

S_0 = bed slope (same as the energy slope, S_f)

n = the Manning's dimensional empirical constant

Uniform Flows



Steady Uniform Flow in an Open Channel

Uniform Flow

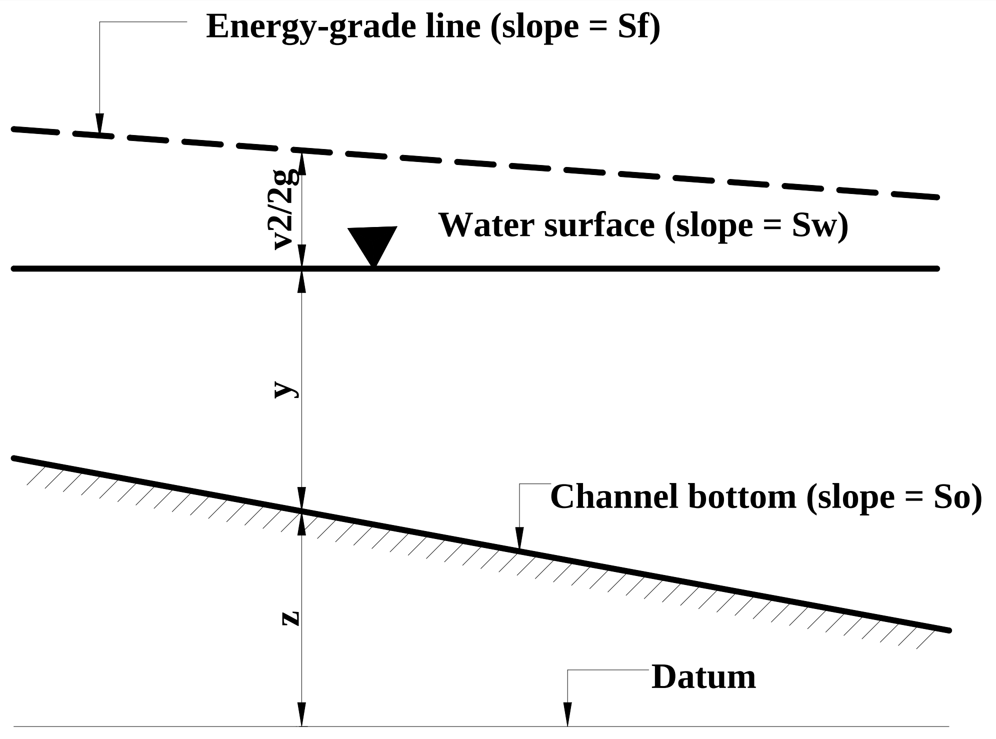
- Example : Flow in an open channel

- This concept is used in most of the open channel flow design

- The uniform flow means that there is no acceleration to the flow leading to the weight component of the flow being balanced by the resistance offered by the bed shear

- In terms of discharge the Manning’s equation is given by

Gradually Varied Flow



Total head at a channel section

Gradually Varied Flow

- Numerical integration of the gradually varied flow equation will give the water surface profile along the channel

- Depending on the depth of flow where it lies when compared with the normal depth and the critical depth along with the bed slope compared with the friction slope different types of profiles are formed such as M (mild), C (critical), S (steep) profiles. All these have real examples.

- M (mild)-If the slope is so small that the normal depth (Uniform flow depth) is greater than critical depth for the given discharge, then the slope of the channel is **mild**.

Gradually Varied Flow

- C (critical)–if the slope's normal depth equals its critical depth, then we call it a **critical** slope, denoted by C

- S (steep)–if the channel slope is so steep that a normal depth less than critical is produced, then the channel is **steep**, and water surface profile designated as S

Rapidly Varied Flow

- This flow has very pronounced curvature of the streamlines
- It is such that pressure distribution cannot be assumed to be hydrostatic
- The rapid variation in flow regime often take place in short span
- When rapidly varied flow occurs in a sudden–transition structure, the physical characteristics of the flow are basically fixed by the boundary geometry of the structure as well as by the state of the flow

Examples:

- Channel expansion and cannel contraction
- Sharp crested weirs
- Broad crested weirs

Unsteady flows

- When the flow conditions vary with respect to time, we call it unsteady flows.
- Some terminologies used for the analysis of unsteady flows are defined below:
- ***Wave***: it is defined as a temporal or spatial variation of flow depth and rate of discharge.
- ***Wave length***: it is the distance between two adjacent wave crests or trough
- ***Amplitude***: it is the height between the maximum water level and the still water level

Unit - 2

Basic Concepts of open channel

→ Empirical Relation for the Chezy's constant C

1) Bazin's (1897) :-

$$C = \frac{157.6}{1.81 + \frac{K}{\sqrt{m}}}$$

where, m : hydraulic Radius mean depth.

K : Bazin's Constant

2) Kutter's formula :-

$$C = \frac{23 + \frac{0.00155}{i} + \frac{1}{N}}{1 + \left(23 + \frac{0.00155}{i}\right)\frac{N}{\sqrt{m}}}$$

where, N : Kutter's Constant (Roughness Co-efficient)

i : Side slope / Slope of bed.

m : hydraulic mean depth.

3) Manning's formula :-

$$C = \frac{1}{N} m^{1/6}$$

where, N : Manning's Constant

m : hydraulic mean depth.

Ex.1 Find the velocity of flow and rate of flow of water through a rectangular channel of 6 m wide & 3 m deep, when it is running full. The channel is having bed slope as 1 in 2000. Take chezy's constant $C = 55$.

Soln ::

width of rectangular channel, $b = 6\,m$.

Depth of channel, $d = 3\,m$.

Area $A = 18\,m^2$

Bed slope $i = \frac{1}{2000}$

$C = 55$

→ Perimeter, $P = b + 2d$

$= 6 + (2 \times 3)$

$= 12\,m$

→ Hydraulic mean depth $m = \frac{A}{P}$

$= 1.5\,m.$

→ Velocity of flow is given by,

$V = C\sqrt{mi}$

$= 55\sqrt{1.5 \times \frac{1}{2000}}$

$= 1.506\,m/s$

$Q = A \times V$

$= 18 \times 1.506$

$= 27.10\,m^3/s$

* Discharge through open channel by Chezy's formula:-

→ Consider uniform flow of water in a channel. (velocity, depth of flow & area of flow will be constant for given l. of channel)

Consider Sections 1-1 & 2-2.

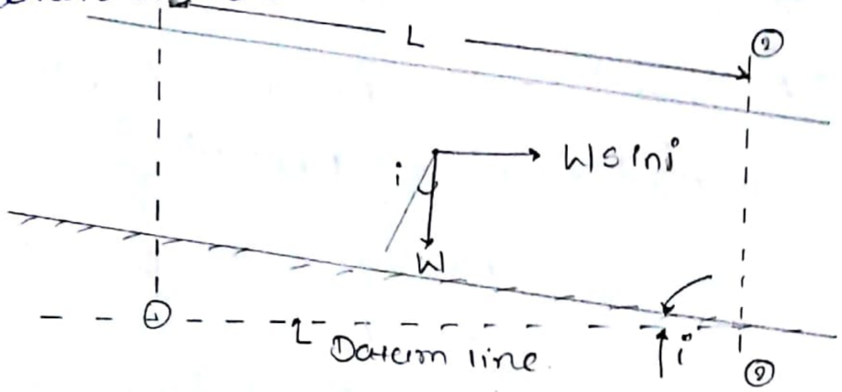


Fig:- Uniform flow in open channel.

Let,

l = length of channel

A = Area of flow of water

i = slope of bed

V = mean velocity of flow of water

P = wetted perimeter of c/s

f = frictional resistance per unit velocity per unit area.

The weight of water b/w section 1-1 & 2-2.

W = sp. weight of water × Vol of water

= w × A × L

→ Component of W along direction of flow

= W × sin i = wAL sin i —(i)

Friction resistance against motion of water

= f × surface area × (V)^n

Value of n is found experimentally = 2 & surface area = P × L

→ ∴ Frictional resistance against motion = f × P × L × V²

The forces acting on the water between section 1-1 & 2-2 are,

1. Component of wt of water along direction of flow.
2. Friction resistance against flow of water.
3. Pressure force at section 1-1 & 2-2.

∴ Resolving all forces in the direction of flow, we get

$$w A L \sin i - f \times P \times L \times V^2 = 0$$

$$w A L \sin i = f \times P \times L \times V^2$$

$$V^2 = \frac{w A L \sin i}{f \times P \times L}$$

$$= \frac{w}{f} \times \frac{A}{P} \times \sin i$$

where, $A/P = m$

$\sqrt{w/f} = C$ = chezy's Const.

$$\therefore V = \sqrt{\frac{w}{f}} \times \sqrt{A/P \times \sin i}$$

$$\therefore V = C\sqrt{m \sin i}$$

for small values of i = $\sin i = \tan i = i$

$$\therefore V = C\sqrt{mi}$$

∴ Discharge $Q = A \times V$

$= A \times C\sqrt{mi}$

$$\therefore Q = AC\sqrt{mi}$$

Q.2. Find the slope of bed of a rectangular channel of width 5m when depth of water is 2m and rate of flow is 20 m^3/s. Take Chezy's constant, C = 50.

Soln:

Width of channel $b = 5\,m$

Depth of water. $d = 2\,m$

Rate of flow $Q = 20\,m^3/s$

$C = 50$

Find bed slop $i = ?$

→ Area = $b \times d$

$= 5 \times 2 = 10\,m^2$

→ Hydraulic mean depth $m = \frac{A}{P}$

$= \frac{10}{(b+2d)}$

$= 1.11\,m.$

→ $Q = A \times C\sqrt{mi}$

$20 = 10 \times 50\sqrt{1.11 \times i}$

$i = 0.04$

$0.04 = \sqrt{1.11\,i}$

$1.6 \times 10^{-3} = 1.11\,i$

$i = 0.0014$

Ex 3 Find the discharge through a rectangular channel 2.3m wide, depth of water 1.5 m a bed slope as 1 in 2000. Take the value of $k = 2.36$ in bazin's formula.

Soln.

width of channel, $b = 2.5$ m

$d = 1.5$ m.

Area $A = b \times d$

$= 3.75\ m^2$

wetted perimeter $P = b + 2d$

$= 5.5$ m.

→ Hydraulic mean depth $m = \frac{A}{P}$

$= 0.682$ m.

Bed slope $i^\circ = \frac{1}{2000}$

Using Bazin's formula,

$$C = \frac{157.6}{1.81 + \frac{k}{\sqrt{m}}}$$

$$= \frac{157.6}{1.81 + \frac{2.36}{\sqrt{0.682}}} = 33.76$$

Discharge $Q = AC\sqrt{mi}$

$$= 3.75 \times 33.76 \times \sqrt{0.682 \times \frac{1}{2000}}$$

$= 2.337\ m^3/s$

Q.4 Find the discharge through a rectangular channel 4 m wide, having depth of water 3 m and bed slope 1 in 1500. Take the value of N = 0.03 in Kutter's formula.

Soln:

Width of channel $b = 4\,m$

Depth of water, $d = 3\,m$

Bed slope, $i = \frac{1}{1500}$

Kutter's Const $N = 0.03$.

→ Area of flow $A = b \times d$

$= 12\,m^2$

→ Wetted perimeter $P = b + 2d$

$= 10\,m$.

→ Hydraulic mean depth, $m = \frac{A}{P}$

$= 1.2\,m$

→ Using Kutter's Eqn,

$$C = \frac{23 + \frac{0.00155}{i} + \frac{1}{N}}{1 + \left(23 + \frac{0.00155}{i}\right)\frac{N}{\sqrt{m}}}$$

$= 32.01$

→ Discharge Q, $= AC\sqrt{mi}$

$= 12 \times 32.01 \times \sqrt{1.2 \times \frac{1}{1500}}$

$= 10.867\ m^3/s$.

16.5 MOST ECONOMICAL SECTION OF CHANNELS

A section of a channel is said to be most economical when the cost of construction of the channel is minimum. But the cost of construction of a channel depends upon the excavation and the lining. To keep the cost down or minimum, the wetted perimeter, for a given discharge, should be minimum. This condition is utilized for determining the dimensions of a economical sections of different form of channels.

Most economical section is also called the best section or most efficient section as the discharge, passing through a most economical section of channel for a given cross-sectional area (A), slope of the bed (i) and a resistance co-efficient, is maximum. But the discharge, Q is given by equation (16.5) as

$$Q = AC\sqrt{mi} = AC\sqrt{\frac{A \times i}{P}} \qquad \left(\because \; m = \frac{A}{P}\right)$$

For a given A, i and resistance co-efficient C, the above equation is written as

$$Q = K\frac{1}{\sqrt{P}}, \quad \text{where } K = AC\sqrt{Ai} = \text{constant}$$

Hence the discharge, Q will be maximum, when the wetted perimeter P is minimum. This condition will be used for determining the best section of a channel *i.e.*, best dimensions of a channel for a given area.

The conditions to be most economical for the following shapes of the channels will be considered :

1. Rectangular Channel, 2. Trapezoidal Channel, and 3. Circular Channel.

16.5.1 Most Economical Rectangular Channel.

The condition for most economical section, is that for a given area, the perimeter should be minimum. Consider a rectangular channel as shown in Fig. 16.9

Let b = width of channel,

d = depth of the flow,

$\therefore$ Area of flow, $A = b \times d$ ...(*i*)

Wetted perimeter, $P = d + b + d = b + 2d$ ...(*ii*)

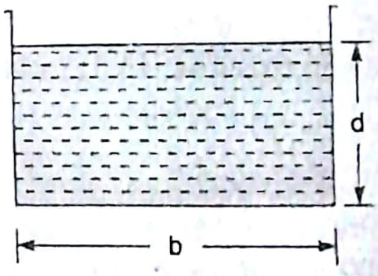


Fig. 16.9 *Rectangular channel*

From equation (*i*), $b = \dfrac{A}{d}$

Substituting the value of b in (*ii*),

$$P = b + 2d = \frac{A}{d} + 2d \qquad \text{...(iii)}$$

For most economical section, P should be minimum for a given area.

$$\frac{dP}{d(d)} = 0$$

Differentiating the equation (*iii*) with respect to d and equating the same to zero, we get

$$\frac{d}{d(d)}\left[\frac{A}{d} + 2d\right] = 0 \quad \text{or} \quad -\frac{A}{d^2} + 2 = 0 \quad \text{or} \quad A = 2d^2$$

But $A = b \times d$, $\therefore$ $b \times d = 2d^2$ or $b = 2d$ ...(16

Now hydraulic mean depth, $m = \dfrac{A}{P} = \dfrac{b \times d}{b + 2d}$ $(\because \; A = bd, P = b + $

Vel. of fluid a c/s of channel do not change from pt to pt

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$$= \frac{2d \times d}{2d + 2d} \qquad (\because \quad b = 2d)$$

$$m = \frac{2d^2}{4d} = \frac{d}{2} \qquad \ldots(16.10)$$

From equations (16.9) and (16.10), it is clear that rectangular channel will be most economical when:

(*i*) Either $b = 2d$ means width is two times depth of flow.

(*ii*) Or $m = \frac{d}{2}$ means hydraulic depth is half the depth of flow.

∴ Increase in discharge

16.5.2 Most Economical Trapezoidal Channel.

The trapezoidal section of a channel will be most economical, when its wetted perimeter is minimum. Consider a trapezoidal section of channel as shown in Fig. 16.10.

Let b = width of channel at bottom,

d = depth of flow,

θ = angle made by the sides with horizontal.

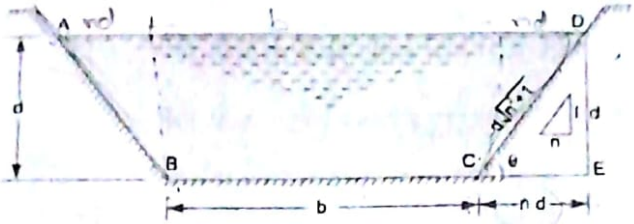

Fig. 16.10 *Trapezoidal section.*

(i) The side slope is given as 1 vertical to n horizontal.

∴ Area of flow, $A = \frac{(BC + AD)}{2} \times d = \frac{b + (b + 2nd)}{2} \times d$ $(\because \quad AD = b + 2nd)$

$$A = \frac{2b + 2nd}{2} \times d = (b + nd) \times d \quad \text{...(i)}$$

∴ $\frac{A}{d} = b + nd$

∴ $b = \frac{A}{d} - nd$ ...(ii

Now wetted perimeter, $P = AB + BC + CD = BC + 2CD$ $(\because \quad AB = CD$

$$= b + 2\sqrt{CE^2 + DE^2} = b + 2\sqrt{n^2d^2 + d^2} = b + 2d\sqrt{n^2 + 1} \quad \text{...(iia}$$

Substituting the value of b from equation (ii), we get

$$P = \frac{A}{d} - nd + 2d\sqrt{n^2 + 1} \quad \text{...(ii}$$

For most economical section, P should be minimum or $\frac{dP}{d(d)} = 0$

∴ Differentiating equation (iii) with respect to d and equating it equal to zero, we get

$$\frac{d}{d(d)}\left[\frac{A}{d} - nd + 2d\sqrt{n^2 + 1}\right] = 0$$

r $$-\frac{A}{d^2} - n + 2\sqrt{n^2 + 1} = 0$$ $(\because \quad n$ is constar

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or $$\frac{A}{d^2} + n = 2\sqrt{n^2+1}$$

Substituting the value of A from equation (i) in the above equation,

$$\frac{(b+nd)d}{d^2} + n = 2\sqrt{n^2+1} \quad \text{or} \quad \frac{b+nd}{d} + n = 2\sqrt{n^2+1}$$

or $$\frac{b+nd+nd}{d} = \frac{b+2nd}{d} = 2\sqrt{n^2+1} \quad \text{or} \quad \frac{b+2nd}{2} = d\sqrt{n^2+1} \qquad \text{...(16.11)}$$

But from Fig. 16.8, $\frac{b+2nd}{2}$ = Half of top width

and $d\sqrt{n^2+1} = CD$ = one of the sloping side .

Equation (16.11) is the required condition for a trapezoidal section to be most economical, which can be expressed as half of the top width must be equal to one of the sloping sides of the channel.

(*ii*) **Hydraulic mean depth**

Hydraulic mean depth, $m = \frac{A}{P}$

Value of A from (i), $A = (b + nd) \times d$

Value of P from (iia), $P = b + 2d\sqrt{n^2+1} = b + (b + 2nd)$ ($\because$ From equation (16.11) $b + 2nd = 2d\sqrt{n^2+1}$)

$$= 2b + 2nd = 2(b + nd)$$

$\therefore$ Hydraulic mean depth, $$m = \frac{A}{P} = \frac{(b+nd)d}{2(b+nd)} = \frac{d}{2} \qquad \text{...(16.12)}$$

Hence for a trapezoidal section to be most economical hydraulic mean depth must be equal to half the depth of flow.

(*iii*) The three sides of the trapezoidal section of most economical section are tangential to the semi-circle described on the water line. This is proved as :

Let Fig. 16.11 shows the trapezoidal channel of most economical section.

Let θ = angle made by the sloping side with horizontal, and

O = the centre of the top width, AD.

Draw OF perpendicular to the sloping side AB.

ΔOAF is a right-angled triangle and angle $OAF = \theta$

$\therefore$ $$\sin\theta = \frac{OF}{OA} \qquad \therefore \quad OF = AO \sin\theta \qquad \text{...(iv)}$$

In ΔAEB, $$\sin\theta = \frac{AE}{AB} = \frac{d}{\sqrt{d^2+n^2d^2}}$$

$$= \frac{d}{d\sqrt{1+n^2}} = \frac{1}{\sqrt{1+n^2}}$$

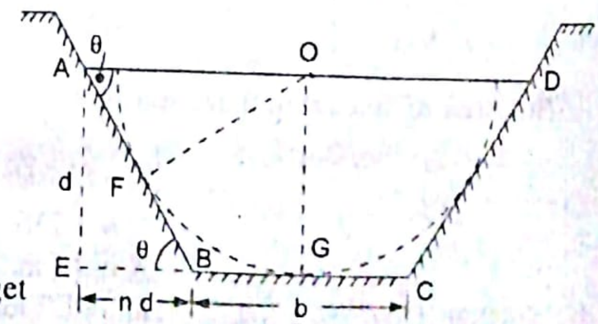


Fig. 16.11

Substituting $\sin\theta = \frac{1}{\sqrt{1+n^2}}$ in equation (iv), we get

$$OF = AO \times \frac{1}{\sqrt{1+n^2}} \quad ...(v)$$

But $$AO = \text{half of top width}$$

$$= \frac{b+2nd}{2} = d\sqrt{n^2+1} \text{ from equation (16.11)}$$

Substituting this value of AO in equation (v),

$$OF = \frac{d\sqrt{n^2+1}}{\sqrt{n^2+1}} = d \text{ depth of flow} \quad ...(16.1$$

Thus, if a semi-circle is drawn with O as centre and radius equal to the depth of flow d, the thr sides of most economical trapezoidal section will be tangential to the semi-circle.

Hence the conditions for the most economical trapezoidal section are:

1. $$\frac{b+2nd}{2} = d\sqrt{n^2+1}$$

2. $$m = \frac{d}{2}$$

3. A semi-circle drawn from O with radius equal to depth of flow will touch the three sides of t channel.

$$= 3.75 \times 33.76 \times \sqrt{0.682 \times \frac{1}{2000}} = \mathbf{2.337\ m^3/s.}\ \text{Ans.}$$

Problem 16.9 *Find the discharge through a rectangular channel 14 m wide, having depth of water 3 m and bed slope 1 in 1500. Take the value of N = 0.03 in the Kutter's formula.*

Solution. Given :

Width of channel, $b = 4$ m

Depth of water, $d = 3$ m

Bed slope, $i = \frac{1}{1500} = 0.000667$

Kutter's constant, $N = 0.03$

Area of flow, $A = b \times d = 4 \times 3 = 12\ m^2$

Wetted perimeter, $P = d + b + d = 3 + 4 + 3 = 10$ m

∴ Hydraulic mean depth, $m = \frac{A}{P} = \frac{12}{10} = 1.2$ m

Using Kutter's formula given by equation (16.7), as

$$C = \frac{23 + \frac{.00155}{i} + \frac{1}{N}}{1 + \left(23 + \frac{.00155}{i}\right) \times \frac{N}{\sqrt{m}}} = \frac{23 + \frac{.00155}{.000667} + \frac{1}{.03}}{1 + \left(23 + \frac{.00155}{.000667}\right) \times \frac{.03}{\sqrt{1.20}}}$$

$$= \frac{23 + 2.3238 + 33.33}{1 + (23 + 2.3238) \times .03286} = \frac{58.633}{1.832} = 32.01$$

Discharge, Q is given by equation (16.5), as

$$Q = AC\sqrt{mi} = 12 \times 32.01 \times \sqrt{12 \times .000667} = \mathbf{10.867\ m^3/s.\ Ans.}$$

Problem 16.10 *Find the discharge through a rectangular channel of width 2 m, having a bed slope of 4 in 8000. The depth of flow is 1.5 m and take the value of N in Manning's formula as 0.012.*

Solution. Given :

Width of the channel, $b = 2$ m

Depth of the flow, $d = 1.5$ m

Area of flow, $A = b \times d = 2 \times 1.5 = 3.0\ m^2$

etted perimeter,

$$P = b + d + d = 2 + 1.5 + 1.5 = 5.0 \text{ m}$$

$\therefore$ Hydraulic mean depth, $m = \frac{A}{P} = \frac{3.0}{5.0} = 0.6$

Bed slope,

$$i = 4 \text{ in } 8000 = \frac{4}{8000} = \frac{1}{2000}$$

Value of N

$$= 0.012$$

Using Manning's formula, given by equation (16.8), as

$$C = \frac{1}{N} m^{1/6} = \frac{1}{0.012} \times 0.6^{1/6} = 76.54$$

Discharge, Q is given by equation (16.5), as

$$Q = AC\sqrt{mi}$$

$$= 3.0 \times 76.54 \sqrt{0.6 \times \frac{1}{2000}} \text{ m}^3\text{/s} = 3.977 \text{ m}^3\text{/s. Ans.}$$

Problem 16.11 *Find the bed slope of trapezoidal channel of bed width 4 m, depth of water 3 m and side slope of 2 horizontal to 3 vertical, when the discharge through the channel is 20 m³/s. Take Manning's $N = 0.03$ in Manning's formula $C = \frac{1}{N} m^{1/6}$.*

Solution. Given :

Bed width, $b = 4$ m

Depth of flow, $d = 3$ m

Side slope $= 2$ hor. to 3 vert.

Discharge, $Q = 20.0 \text{ m}^3\text{/s}$

Manning's, $N = 0.03$

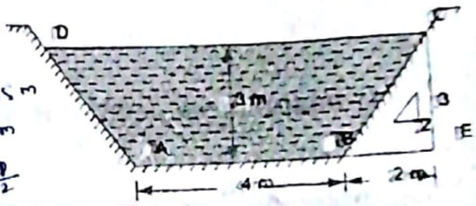


Fig. 16.7

From Fig. 16.7, we have

Distance, $BE = d \times \frac{2}{3} = 3 \times \frac{2}{3} = 2$ m

$\therefore$ Top width, $CD = AB + 2BE$

$$= 4 + 2 \times 2 = 8.0 \text{ m}$$

$\therefore$ Area of flow, A = Area of trapezoidal section $ABCD$

$$= \frac{(AB + CD)}{2} \times d = \frac{(4 + 8)}{2} \times 3 = 18 \text{ m}^2$$

Wetted perimeter, $P = AD + AB + BC = AB + 2BC$ $\quad (\because AD = BC$

$$= 4.0 + 2\sqrt{BE^2 + EC^2} = 4.0 + 2\sqrt{2^2 + 3^2} = 4.0 + 2 \times \sqrt{13} = 11.21$$

$\therefore$ Hydraulic mean depth, $m = \frac{A}{P} = \frac{18}{11.21} = 1.6057$

Using Manning's formula, $C = \frac{1}{N} m^{1/6} = \frac{1}{0.03} \times (1.6057)^{1/6} = 36.07$

$$i = \left(\frac{20.0}{822.71}\right)^2 = 0.0005909 = \frac{}{1692}. \text{ Ans.}$$

Q.4

Problem 16.12 *Find the diameter of a circular sewer pipe which is laid at a slope of 1 in 8000 and carries a discharge of 800 litres/s when flowing half full. Take the value of Manning's N = 0.020.*

Solution. Given :

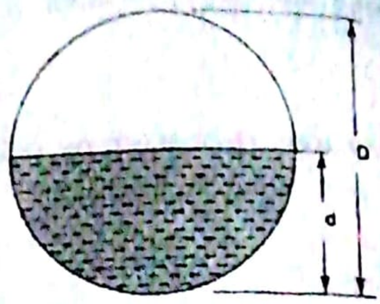


Fig. 16.8

Slope of pipe, $i = \frac{1}{8000}$

Discharge, $Q = 800$ litres/s $= 0.8\ m^3/s$

Manning's, $N = 0.020$

Let the dia. of sewer pipe, $= D$

Depth of flow, $d = \frac{D}{2}$

∴ Area of flow, $A = \frac{\pi}{4} D^2 \times \frac{1}{2} = \frac{\pi D^2}{8}$

Wetted perimeter, $P = \frac{\pi D}{2}$

∴ Hydraulic mean depth, $m = \frac{A}{P} = \frac{\frac{\pi D^2}{8}}{\pi D / 2} = \frac{D}{4}$

Using Manning's formula given by equation (16.8), $C = \frac{1}{N} m^{1/6}$

The discharge, Q through pipe is given by equation (16.6), as

$$Q = AC\sqrt{mi}$$

$$= \frac{\pi D^2}{8} \times \frac{1}{N} m^{1/6} \sqrt{mi}$$

$$0.80 = \frac{\pi}{8} D^2 \times \frac{1}{.020} \times m^{1/6} \times m^{1/2} \times \sqrt{i}$$

0.0118

$$= \frac{\pi}{8} D^2 \times \frac{1}{.020} m^{(1/6 + 1/2)} \times \sqrt{\frac{1}{8000}} = \frac{\pi}{8} D^2 \times \frac{1}{.020} \times m^{2/3} \times 0.0[illegible]$$

$$= 0.2195 \times D^2 \times \left(\frac{D}{4}\right)^{2/3} \qquad \left(\because m = \right.$$

$$= \frac{.2195}{4^{2/3}} \times D^2 \times D^{2/3} = 0.0871\ D^{8/3}$$

$$D^{8/3} = \frac{0.80}{.0871} = 9.1848$$

$$D = (9.1848)^{3/8} = (9.1848)^{0.375} = \mathbf{2.296\ m.\ Ans.}$$

∴ ≈ 2.30 m.

channel.

Problem 16.16 *A trapezoidal channel has side slopes of 1 horizontal to 2 vertical and the slope of the bed is 1 in 1500. The area of the section is 40 m². Find the dimensions of the section if it is most economical. Determine the discharge of the most economical section if C = 50.*

Solution. Given :

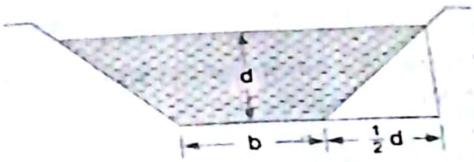


Fig. 16.12

Side slope, $n = \frac{\text{Horizontal}}{\text{Vertical}} = \frac{1}{2}$

Bed slope, $i = \frac{1}{1500}$

Area of section, $A = 40 \text{ m}^2$

Chezy's constant, $C = 50$

For the most economical section, using equation (16.11)

$$\frac{b + 2nd}{2} = d\sqrt{n^2 + 1} \quad \text{or} \quad \frac{b + 2 \times \frac{1}{2} \times d}{2} = d\sqrt{\left(\frac{1}{2}\right)^2 + 1}$$

or $$\frac{b + d}{2} = d\sqrt{\frac{1}{4} + 1} = 1.118\, d$$

or $$b = 2 \times 1.118d - d = 1.236\, d$$

But area of trapezoidal section, $A = \frac{b + (b + 2nd)}{2} \times d = (b + nd)^2\, d$

$$= (1.236\, d + \tfrac{1}{2}\, d)\, d \qquad (\because b = 1.236\, d \text{ and } n =$$

$$= 1.736\, d^2$$

$$A = 40 \text{ m}^2$$

∴ $$40 = 1.736\, d^2$$

d : 4.80 m

b : 5.93 m. , m: d/2

: 2.40 m

Discharge, Q : AC√mi

: 40×50 √2.4 × 1/1500

: 80 m³/s

$$\therefore \qquad d = \sqrt{\frac{40}{1.736}} = \mathbf{4.80 \ m. \ Ans.}$$

Substituting the value of d in equation (i), we get

$$b = 1.236 \times 4.80 = \mathbf{5.933 \ m. \ Ans.}$$

Discharge for most economical section. Hydraulic mean depth for most economical section is

$$m = \frac{d}{2} = \frac{4.80}{2} = 2.40 \text{ m}$$

$\therefore$ Discharge

$$Q = AC\sqrt{mi} = 40 \times 50 \times \sqrt{2.40 \times \frac{1}{1500}}$$

$$= \mathbf{80 \ m^3/s. \ Ans.}$$

... canal has side slopes of 3 horizontal to 4 vertical and slope of i ...

A rectangular channel carries water at a rate of 400 lit/s when bed slope is 1 in 2000. Find the most economical dimensions of channel if $C = 50$.

Soln:- $Q = 0.4\ m^3/s$

$i = \frac{1}{2000}$

$C = 50$

For the rectangular channel to be most economical,

i) width :- $b = 2d$

ii) Hydraulic mean depth, $m = \frac{d}{2}$

∴ Area of flow, $A = b \times d$

$= 2d \times d = 2d^2$

Using Eqn for discharge,

$Q = AC\sqrt{mi}$

$0.4 = 2d^2 \times 50 \sqrt{\frac{d}{2} \times \frac{1}{2000}}$

$0.4 = 1.581\, d^{5/2}$

$\therefore d^{5/2} = \frac{0.4}{1.581}$

$d = 0.577\ m.$

$b = 2d$

$= 2 \times .577$

$= 1.154\ m.$

Q.8. A trapezoidal channel has side of 3H to 2V and Q is 10 m³/s at a velocity of 1.5 m/s, so that the amount of Conc. lining for the bed and sides is minimum.

Find i) wetted perimeter

ii) slope of the bed it,

manning's N = 0.014

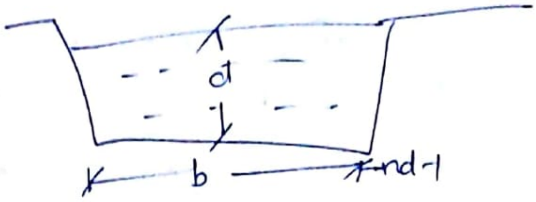
Sol^n

$n = \frac{3}{2} = 1.5$

$Q = 10\ m^3/s$

$V = 1.5\ m/s$

$N = 0.014$



for most economical Trapezoidal channel section,

$$\frac{b+2nd}{2} = d\sqrt{n^2+1}$$

$$\therefore \frac{b+2(1.5\times d)}{2} = d\sqrt{1.5^2+1}$$

$$\frac{b+3d}{2} = 1.8d$$

$$b = 0.6d, \quad n = 1.5$$

Area of trapezoidal Section,

$$A = (b+nd)d$$

$$= (0.6d + 1.5d)d$$

$$= 2.1\,d^2$$

But $A = \frac{Q}{V} = \frac{10}{1.5} = 6.67\ m^2$

$2.1\,d^2 = 6.67 \quad \therefore d = 1.78\ m$

$b = 1.07\ m$

Hence,

ii) Wetted perimeter, $P = b + 2d\sqrt{n^2+1}$

$= 1.07 + (2 \times 1.78)\sqrt{1.5^2+1}$

$P = 7.48\ m.$

ii). Slope of the bed when $N = 0.014$ in manning's formula.

$Q = AC\sqrt{mi}$

$m = \frac{d}{2} = \frac{1.78}{2} = 0.89\ m.$

$C = \frac{1}{N} m^{1/6}$

$C = \frac{1}{0.014} \times (0.89)^{1/6}$

$= 66.09$

$Q = AC\sqrt{mi}$

$10 = 6.67 \times 66.09\sqrt{0.89 \times i}$

$i = \frac{1}{1729.4}$

→ Manning's Roughness Co.efficient :

→ it is a coefficient which represent the roughness or friction applied to the Flow by the channel.

→ Roughness Co.efficient represent as a Very imp parameter when Come to the Compution of discharge. Storm water, canal flow in Conduit pipe etc.

★ Factors affecting manning's roughness co.efficient

1. Cross-sectional geometry of channel
2. boundry roughness surface.
3. Vegetation on channel
4. Channel irregularity.
5. channel alignment,
6. Silting & Scouring.
7. Obstruction in channel.
8. Suspended material & bed load.
9. Seasonal changes in channel.
10. Shape & Size of channel.

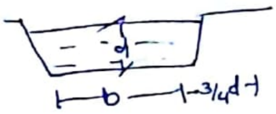
$$C = \frac{1}{N} m^{1/6}$$

Eg A trapezoidal channel has side slopes of 3 H to 4 V and slope of its bed is 1 in 2000. Determine the optimum dimensions of the channel, if it is to carry water at 0.5 m³/s. Take Chezy's const as 80.

Soln

Side slopes, $n = \frac{\text{Horizontal}}{\text{Vertical}} = \frac{3}{4}$

Slope of bed, $i = \frac{1}{2000}$



$Q = 0.5\ m^3/s$

$C = 80$

For the most economical section, the condition is given by eqn as,

$$\frac{b + 2nd}{2} = d\sqrt{n^2+1}$$

$$\therefore \frac{b + 2(3/4)d}{2} = d\sqrt{(3/4)^2 + 1}$$

$$\therefore \frac{b + 1.5d}{2} = 1.25\,d$$

$$b = 2 \times 1.25d - 1.5d$$

$$\therefore b = d \quad \text{——— (i)}$$

For discharge, $Q = AC\sqrt{mi}$ ——— (ii)

But for most economical section,

Hy. mean depth $m = \frac{d}{2}$

$$\therefore 0.50 = A \times 80\sqrt{\frac{d}{2} \times \frac{1}{2000}} \quad \text{——— (iii)}$$

$A = (b + nd)\,d$

$= [d + 3/4 \cdot d]\,d$ $\{b = d,\ n = 3/4\}$

$= 1.75 d^2$

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Substituting the value of A in eqn (iii), we get,

$$0.50 = 1.45d^2 \times 80 \times \sqrt{\frac{d}{2} \times \frac{1}{2000}}$$

$$0.50 = 2.2135\ d^{5/2}$$

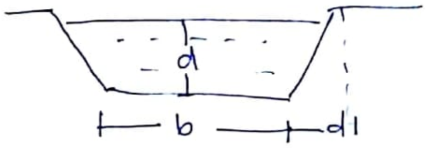
$$\therefore \boxed{d = 0.55m}$$

$$\therefore b = d$$

$$\boxed{d = b = 0.55\ m.}$$

Q.10 A Trapezoidal channel with side slope of 1 to 1 has to be designed to convey 10 m³/s at velocity of 2m/s so that the amount of conc. lining for the bed and sides is the minimum.
Calculate area of lining required for one meter length of Canal.

Sol^n



Side slope, $n = \frac{H}{V} = 1$

$Q = 10\ m^3/s$

$v = 2.0\ m/s$

Area of flow, $A = \frac{Q}{V} = \frac{10}{2} = 5m^2$ — (i)

for, most economical Trape channel section,

$$\frac{b + 2nd}{2} = d\sqrt{n^2+1}$$

$$\frac{b+2d}{2} = 1.414\ d$$

$b = 2 \times 1.414d - 2d$

$= 0.828\ d$

$A = (b + nd)\,d$

$= (0.828d + d)\,d$

$= 1.828\ d^2$

$\therefore A = 5\ m^2$

we will get,

$5 = 1.828\ d^2$

$d = 1.654\ m$

$b = 0.828\ d$

$b = 1.369\,m$

Area of lining rqd for one meter length of Canal = wetted perimeter × length of Canal

$= P \times 1$

where,

$P = b + 2d\sqrt{n^2+1}$

$= 1.369 + 2(1.654)\sqrt{1^2 + 1}$

$= 6.047\ m.$

Area of lining $= 6.047 \times 1$

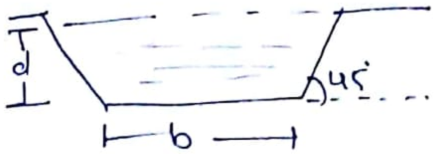
$A = 6.047\ m^2$

Q.11 A trapezoidal channel to carry 142 m^3/minute water is designed to have a min C/s. Find the bottom width and depth if bed slope is 1 in 1200, the side slopes at 45° and chezy's Co-efficient = 55.

Sol^n :-

$Q = 142 \, m^3/min = \frac{142}{60} = 2.367 \, m^3/s$

$i = \frac{1}{1200}$



side slope, $\theta = 45°$

$\tan\theta = \frac{1}{n}$

$\tan 45° = \frac{1}{n}$

$\therefore n = 1$, $C = 55$

For most economical Trapezoidal channel sect,

$\frac{b + 2nd}{2} = d\sqrt{n^2+1}$

$b = 0.828d$ ——————— (i)

Now, $Q = AC\sqrt{mi}$

$2.367 = (b+nd)\,d \times 55\sqrt{\frac{d}{2} \times \frac{1}{1200}}$

$= (1.828d)\,d \times 55\sqrt{\frac{d}{2400}}$

$2.367 = 2.052\, d^{5/2}$

$\boxed{d = 1.06\,m}$

$b = 0.828\, d$

$= 0.828 \times 1.06$

$= \boxed{0.877\,m}$

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Flow through Circular channel :-

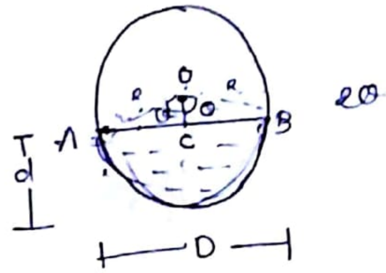
The flow of a liquid through a circular pipe, when the level of liquid in the pipe is below the top of the pipe is classified as an open channel flow. The rate of flow through circular channel is determined from depth of flow and angle subtended by the liquid surface at the centre of circular channel.

- let, d : depth of flow

  2θ : angle subtended by water surface AB at centre in radians.

  R : radius of the channel.

The wetted perimeter & Area is determine as :-



- wetted perimeter, $P = \frac{2\pi R}{2\pi} \times 2\theta$ (length of circle / angle of circle)

$P = \frac{\text{wetted Area}}{\text{wetted length}} = 2R\theta$

| $2\pi r \rightarrow 2\pi$ |
| $2\theta \rightarrow (?)$ |

wetted Area, A : Area of ADBA

= Area of sector OADBO − Area of OAB.

$$= \frac{\pi R^2}{2\pi} \times 2\theta - \frac{AB \times CO}{2}$$

$$= R^2\theta - \frac{2BC \times CO}{2} \quad \therefore (AB = 2BC)$$

$$\therefore R^2\theta - \frac{2 \times R\sin\theta \times R\cos\theta}{2}$$

$$(\because BC = R\sin\theta, \quad CO = R\cos\theta)$$

$$\therefore R^2\theta - \frac{R^2\sin 2\theta}{2} \quad (\because 2\sin\theta \times \cos\theta = \sin 2\theta)$$

$$A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right)$$

Then, Hy. mean depth, $m = \frac{A}{P}$

$$m = \frac{R^2\left(\theta - \frac{\sin\theta}{2}\right)}{2R\theta}$$

$$m = \frac{R}{2\theta}\left(\theta - \frac{\sin 2\theta}{2}\right)$$

Discharge is given by,

$$Q = AC\sqrt{mi}$$

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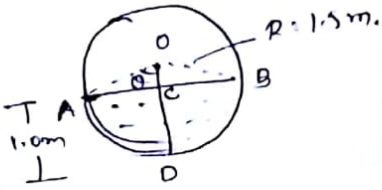
Find the discharge through a Circular pipe of dia 3.0m, if depth of water in pipe is 1.0 and pipe is laid at a slope of 1 in 1000. Take the value of chezy's const as 70.

Soln

Dia of pipe : $D = 3.0$ m.

Radius $R = \frac{D}{2} = 1.5$ m.



depth of flow, $d = 1.0$ m

Bed slope $i = \frac{1}{1000}$

$C = 70.$

Now, $OC = OD - CD$

$= R - 1.0$

$= 1.5 - 1.0 = 0.5$ m.

$AO = 1.5$ m

also, $\cos\theta = \frac{OC}{AO} = \frac{0.5}{1.5} = \frac{1}{3}$

$\theta = 70.53°$

$= 70.53 \times \frac{\pi}{180}$

$= 1.23$ radians.

Now,
wetted perimeter

$P = 2R\theta$

$= 2 \times 1.5 \times 1.23$

$= 3.69$ m.

wetted Area is,

$A = R^2 \left(\theta - \frac{\sin 2\theta}{2}\right)$

$= 1.5^2 \left[1.23 - \frac{\sin(2 \times 70.53°)}{2}\right]$

$= 2.06\ m^2$

Hy. mean depth $m = A/P = 0.5582$

$Q = AC\sqrt{mi} = 2.06 \times 70 \sqrt{0.5582 \times 1/1000} = 3.407\ m^3/s$

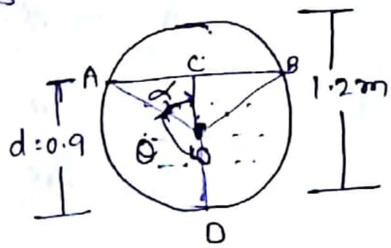
Q.2 Calculate the Qty of water that will be Q at a uniform depth of 0.9m in a 1.2m dia pipe which is laid at a slope 1 in 1000. assume chezy's C = 58.

Soln Dia of pipe = 1.2 m

Radius = 0.6m

depth of flow = d = 0.9m

$i = 1/1000$, $C = 58$



We have, $OC = CD - OD$

$= 0.9 - 0.6 = 0.3m$

$OA = 0.6m$

Now in triangle AOC,

$$\cos\alpha = \frac{OC}{OA} = \frac{0.3}{0.6} = 1/2$$

$\alpha = 60°$

θ = Angle of AOD

$= 180° - \alpha$

$= 180° - 60° = 120° = \frac{120 \times \pi}{180}$

$= 0.667\pi$ Radian

Wetted Perimeter P,

$$P = 2R\theta = 2 \times 0.6 \times 0.667\pi$$

$= 2.528$ m

Area of flow,

$$A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right)$$

$$= 0.6^2\left[0.667\pi - \frac{\sin 2(120°)}{2}\right] \quad \sin 240°$$

$= 0.913\ m^2$

Now, $Q = A \times V$

$= 0.913 \times C\sqrt{mi}$

$$= 0.913 \times 58\sqrt{\frac{0.913}{2.526} \times \frac{1}{1000}}$$

$= 1.007\ m^3/s$.

(G.V.F.) depending upon the change of depth of flow over the length of the channel. If the depth of flow changes abruptly over a small length of the channel, the flow is said as rapidly varied flow. And if the depth of flow in a channel changes gradually over a long length of channel, the flow is said to be gradually varied flow.

16.7 SPECIFIC ENERGY AND SPECIFIC ENERGY CURVE

The total energy of a flowing liquid per unit weight is given by,

$$\text{Total Energy} = z + h + \frac{V^2}{2g}$$

where z = Height of the bottom of channel above datum,

h = Depth of liquid, and V = Mean velocity of flow.

If the channel bottom is taken as the datum as shown in Fig. 16.25, then the total energy per unit/weight of liquid will be,

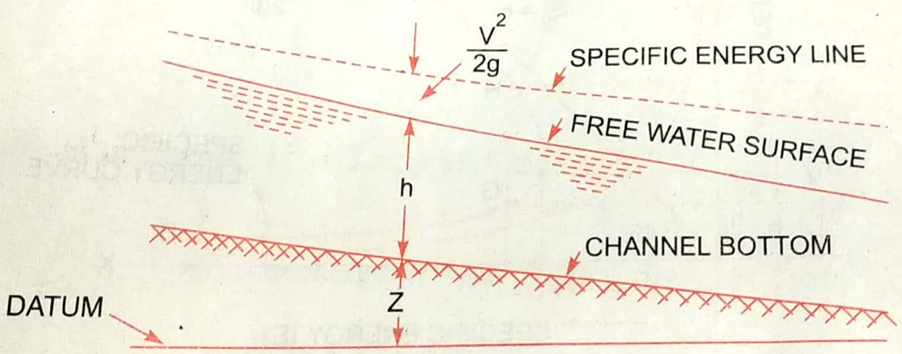


Fig. 16.25 *Specific energy.*

$$E = h + \frac{V^2}{2g} \quad \text{...(16.21)}$$

The energy given by equation (16.21) is known as **specific energy**. Hence specific energy of a flowing liquid is defined as energy per unit weight of the liquid with respect to the bottom of the channel.

Specific Energy Curve. It is defined as the curve which shows the variation of specific energy with depth of flow. It is obtained as :

From equation (16.21), the specific energy of a flowing liquid

$$E = h + \frac{V^2}{2g} = E_p + E_k$$

where E_p = Potential energy of flow = h

E_k = Kinetic energy of flow = $\frac{V^2}{2g}$

Consider a rectangular channel in which a steady but non-uniform flow is taking place.

Let Q = discharge through the channel,

b = width of the channel,

h = depth of flow, and

q = discharge per unit width.

Then $q = \frac{Q}{\text{width}} = \frac{Q}{b}$ = constant $\quad$ ($\because$ Q and b are constant)

Velocity of flow, $V = \frac{\text{Discharg}}{\text{Area}} = \frac{Q}{b \times h} = \frac{q}{h}$ $\quad \left(\because \frac{\theta}{h} = a\right)$

Substituting the values of V in equation (16.21), we get

$$E = h + \frac{q^2}{2gh^2} = E_p + E_k \quad \text{...(16.22)}$$

Equation (16.22), gives the variation of specific energy (E) with the depth of flow (h). Hence for a given discharge Q, for different values of depth of flow, the corresponding values of E may be obtained. Then a graph between specific energy (along X-X axis) and depth of flow, h (along Y-Y axis) may be plotted.

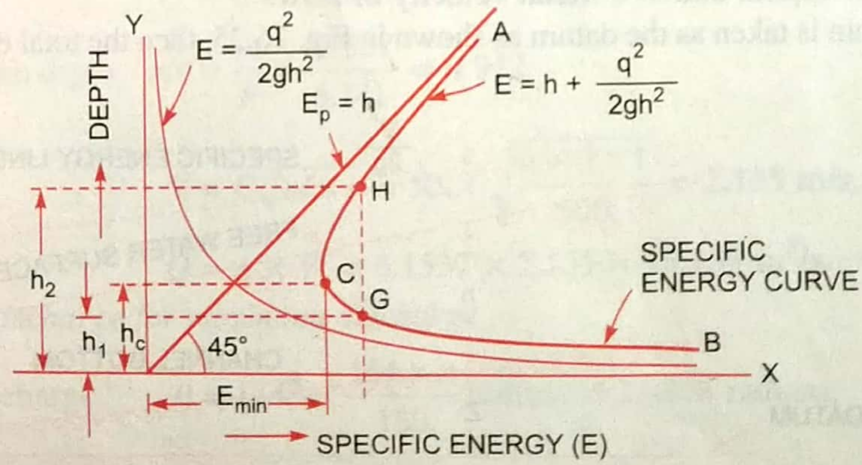


Fig. 16.26 *Specific energy curve.*

The specific energy curve may also be obtained by first drawing a curve for potential energy (*i.e.*, $E_p = h$), which will be a straight line passing through the origin, making an angle of 45° with the X - axis as shown in Fig. 16.22. Then drawing another curve for kinetic energy $\left(i.e., E_k = \frac{q^2}{2gh^2} \text{ or } E_k = \frac{K}{h^2}, \text{ where } K = \frac{q^2}{2g} = \text{constant} \right)$ which will be a parabola as shown in Fig. 16.22. By combining these two curves, we can obtain the specific energy curve. In Fig. 16.22, curve ACB denotes the specific energy curve.

16.7.1 Critical Depth (h_c). Critical depth is defined as that depth of flow of water at which the specific energy is minimum. This is denoted by 'h_c'. In Fig. 16.22, curve ACB is a specific energy curve and point C corresponds to the minimum specific energy. The depth of flow of water at C is known as critical depth. The mathematical expression for critical depth is obtained by differentiating the specific energy equation (16.22) with respect to depth of flow and equating the same to zero.

or $$\frac{dE}{dh} = 0, \quad \text{where } E = h + \frac{q^2}{2gh^2} \text{ from equation (16.22)}$$

or $$\frac{d}{dh}\left[h + \frac{q^2}{2gh^2}\right] = 0 \quad \text{or} \quad 1 + \frac{q^2}{2g}\left(\frac{-2}{h^3}\right) = 0 \qquad \left(\because \frac{q^2}{2g} \text{ is constant}\right)$$

or $$1 - \frac{q^2}{gh^3} = 0 \quad \text{or} \quad 1 = \frac{q^2}{gh^3} \quad \text{or} \quad h^3 = \frac{q^2}{g}$$

$$\therefore \quad h = \left(\frac{q^2}{g}\right)^{1/3}$$

But when specific energy is minimum, depth is critical and it is denoted by h_c. Hence critical depth is

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} \qquad \text{...(16.23)}$$

16.7.2 Critical Velocity (V_c). The velocity of flow at the critical depth is known as critical velocity. It is denoted by V_c. The mathematical expression for critical velocity is obtained from equation (16.23) as

$$h_c = \left(\frac{q^2}{g}\right)^{1/3}$$

Taking cube to both sides, we get $h_c^3 = \frac{q^2}{g}$ or $gh_c^3 = q^2$ ...(*i*)

But $$q = \text{Discharge per unit width} = \frac{Q}{b}$$

$$= \frac{\text{Area} \times V}{b} = \frac{b \times h \times V}{b} = h \times V = h_c \times V_c$$

Substituting this value of q in (i),

$$\therefore \quad gh_c^3 = (h_c \times V_c)^2$$

or $\quad gh_c^3 = h_c^2 \times V_c^2$ or $gh_c = V_c^2$ [Dividing by h_c^2]

or $\quad V_c = \sqrt{g \times h_c}$ ...(16.24)

16.7.3 Minimum Specific Energy in Items of Critical Depth. Specific energy equation is given by equation (16.22)

$$E = h + \frac{q^2}{2gh^2}$$

When specific energy is minimum, depth of flow is critical depth and hence above equation becomes as

$$E_{\min} = h_c + \frac{q^2}{2gh_c^2} \quad ...(ii)$$

But from equation (16.23), $h_c = \left(\frac{q^2}{g}\right)^{1/3}$ or $h_c^3 = \frac{q^2}{g}$

Substituting the value of $\frac{q^2}{g} = h_c^3$ in equation (ii), we get

$$E_{\min} = h_c + \frac{h_c^3}{2h_c^2} = h_c + \frac{h_c}{2} = \frac{3h_c}{2} \quad ...(16.25)$$

E min

Problem 16.33 *Find the specific energy of flowing water through a rectangular channel of wid*

$$V_c = \sqrt{g \times h_c} = \sqrt{9.81 \times 0.972} = \mathbf{3.088\ m/s.\ Ans.}$$

Problem 16.35 *The discharge of water through a rectangular channel of width 8 m, is 15 m^3/s when depth of flow of water is 1.2 m. Calculate :*

(i) Specific energy of the flowing water, (ii) Critical depth and critical velocity,

(iii) Value of minimum specific energy.

Solution. Given :

Discharge, $Q = 15\ m^3/s$

Width, $b = 8$ m

Depth, $h = 1.2$ m

$\therefore$ Discharge per unit width, $q = \dfrac{Q}{b} = \dfrac{15}{8} = 1.875\ m^2/s$

Velocity of flow, $V = \dfrac{Q}{\text{Area}} = \dfrac{15}{b \times h} = \dfrac{15.0}{8 \times 1.2} = 1.5625$ m/s

(*i*) Specific energy (E) is given by equation (16.21) as

$$E = h + \frac{V^2}{2g} = 1.2 + \frac{1.5625^2}{8 \times 9.81} = 1.20 + 0.124 = \mathbf{1.324\ m.\ Ans.}$$

(*ii*) Critical depth (h_c) is given by equation (16.32) as

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{1.875^2}{9.81}\right)^{1/3} = \mathbf{0.71\ m.\ Ans.}$$

Critical velocity (V_c) is given by equation (16.24) as

$$V_c = \sqrt{g \times h_c} = \sqrt{9.81 \times 0.71} = \mathbf{2.639\ m/s.\ Ans.}$$

(*iii*) Minimum specific energy (E_{min}) is given by equation (16.25)

$$E_{min} = \frac{3h_c}{2} = \frac{3 \times 0.71}{2} = \mathbf{1.065\ m.\ Ans.}$$

16.7.4 Critical Flow. It is defined as that flow at which the specific energy is minimum or

16.7.5 Streaming Flow or Sub-critical Flow or Tranquil Flow. When the depth of flow in a channel is greater than the critical depth (h_c), the flow is said to be sub-critical flow or streaming flow or tranquil flow. For this type of flow the Froude number is less than one *i.e.*, $F_e < 1.0$.

16.7.6 Super-critical Flow or Shooting Flow or Torrential Flow. When the depth of flow in a channel is less than the critical depth (h_c), the flow is said to be super-critical flow or shooting flow or torrential flow. For this type of flow the Froude number is greater than one *i.e.*, $F_e > 1.0$.

16.7.7 Alternate Depths. In the specific energy curve shown in Fig. 16.22, the point C corresponds to the minimum specific energy and the depth of flow at C is called critical depth. For any other value of the specific energy, there are two depths, one greater than the critical depth and other smaller than the critical depth. These two depths for a given specific energy are called the alternate depths. These depths are shown as h_1 and h_2 in Fig. 16.22. Or the depths corresponding to points G and H in Fig. 16.22 are called alternate depths.

16.7.8 Condition for Maximum Discharge for a Given Value of Specific Energy. The specific energy (E) at any section of a channel is given by equation (16.21) as

$$E = h + \frac{V^2}{2g}, \qquad \text{where } V = \frac{Q}{A} = \frac{Q}{b \times h}$$

$$\therefore \quad E = h + \frac{Q^2}{b^2 \times h^2} \times \frac{1}{2g} = h + \frac{Q^2}{2gb^2h^2}$$

or $$Q^2 = (E - h)\, 2gb^2h^2 \quad \text{or} \quad Q = \sqrt{(E-h)2gb^2h^2} = b\sqrt{2g\left(Eh^2 - h^3\right)}$$

For maximum discharge, Q, the expression $(Eh^2 - h^3)$ should be maximum. Or in other words,

$$\frac{d}{dh}(Eh^2 - h^3) = 0 \quad \text{or} \quad 2Eh - 3h^2 = 0 \qquad (\because \ E \text{ is constant})$$

or $$2E - 3h = 0 \qquad \text{(Dividing by } h\text{)}$$

or $$h = \tfrac{2}{3}E \qquad \text{...(16.26)}$$

or $$\boxed{E = \frac{3h}{2}} \qquad \text{...(}i\text{)}$$

But from equation (16.25), specific energy is minimum when it is equal to $\frac{3}{2}$ times the value of depth of critical flow. Here in equation (*i*), the specific energy (*E*) is equal $\frac{3}{2}$ time the depth of flow. Thus equation (*i*) represents the minimum specific energy and *h* is the critical depth. Hence the condition for maximum discharge for given value of specific energy is that the depth of flow should be critical.

Problem 16.36 *The specific energy for a 3 m wide channel is to be 3 kg-m/kg. What would be the maximum possible discharge ?* (A.M.I.E., Winter 1980)