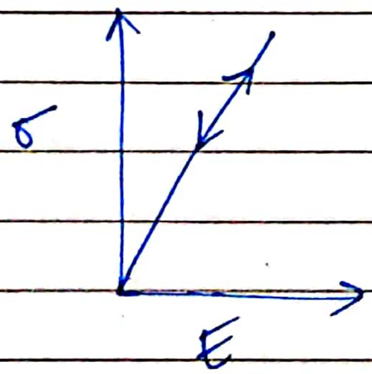
material to return to its original shape & size when forces causing deformatn are removed.

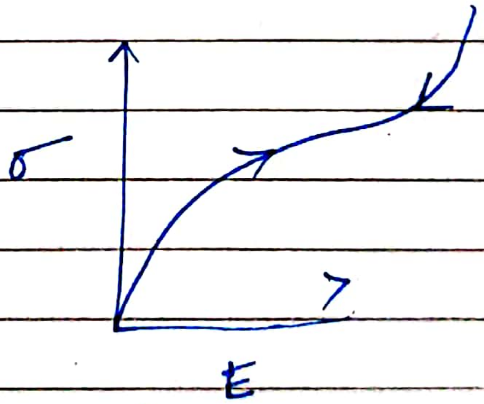
A material is perfectly Elastic if its loading and unloading paths are same

There is no dissipatn of energy

Examples:- Linear Elastic &
Non " " (Hyper elastic)



Linear Elasticity



Non-linear Elasticity

In perfect Elasticity the state of stress at any time is independent of previous history of stresse. Hence the stress is a unique f^n of strain

σ

→ In a non-linear material the stress is a non-" f^n of the strain

→ In both linear & non-linear elasticity σ is obtaind from a energy potential, which a f^n of strain (ε energy)

$$\boxed{\sigma = \frac{\partial \psi(\epsilon)}{\partial \epsilon}}$$

ψ is strain energy.

$E = \frac{\sigma^2}{2E} \cdot V$ → linear

For Non-linear ψ =

3

→ Though, in visco-elastic material, there is no residual strains, there is dissipatn of energy. Different loading & unloading paths.

→ This happens bcz the σ not only depends on ε but also on the rate of strain

$$\sigma = \sigma(\epsilon, \dot{\epsilon})$$

→ Bcz the σ is not a unique f^n of ε, a energy f^n cannot be derived in general.

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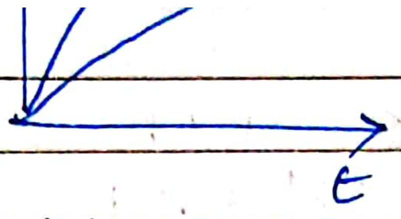


Visco- Elasticity

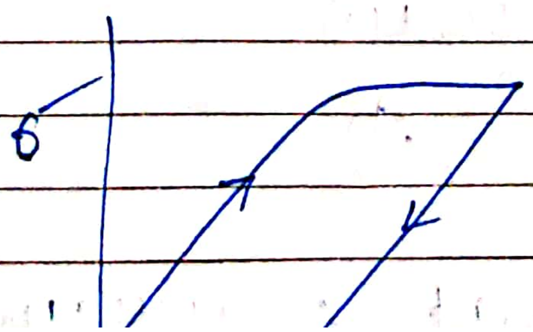
13

In an inelastic material there ~~are~~ permanent strain, when the body is completely unloaded.

There is a dissipatn of energy most of which is converted into heat

In inelasticity, one has to work with increments of ϵ insted of total accumulated ϵ as the σ is not a unique f^n of ϵ

The total ϵ at any time can be obtaind by integrating the ϵ increment



Inelasticity

TOE V/s SOM

1 → In SOM, we make many assumptn to simplify the problem and to arrive at a closed form solution

Flexure Formula:-

$$\frac{M}{I} = \frac{E}{R} = \frac{\sigma}{y}$$

Assumption
- pure bending
- plane sections before Bending remain plane after bending.

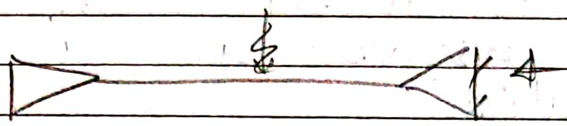
2 → For bending of a cantilever beam where plane sections donot remain plane.

3 → Another example in which assumptn of planarity is made is the torsion of a member.

4 → This assumption is correct only for a circular section. Any other section will wrap as seen for □ section.

5 → To solve such problems TOE has to be used where governing differential eqn will be solved satisfying Boundary conditions w/o any

simplifying assumptions.



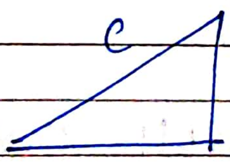
$$\frac{\pi^2 EI}{LE^2}$$

$$\frac{d^4}{dx^2} = \frac{M}{EI} \quad \text{(For pt load)}$$

* INTRODUCTION TO ~~S~~ TENSOR :-

Scalar :- It has only value.
Ex - Time, density

Vector :- It has value as well as direction. Example :- velocity, Displacem



$p = p_1\hat{i} + p_2\hat{j} + p_3\hat{k}$

P_1, P_2, P_3 - components along x, y & z

$c = a + b$

$\hat{i}, \hat{j}, \hat{k}$ - unit vector along those axis.

* Unit Vector :- Have a unit value along the direction of vector

$n = \frac{n}{|n|}$ where $|n|$ = value of vector

→ Second order Tensor :-

A IInd order tensor is a linear transformation which transform a vector (or vector space) to an another vector (ve)

Ex:- $Ax = b$, A is IInd order Tensor

→ A IInd order has 2 direction & magnitude

$$\text{Stress Tensor } [\sigma] = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

$$[\epsilon] = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix}$$

Identity Tensor

Index Notation :-

A vector can be represented by

$$u = u_1 \underset{i}{e_1} + u_2 \underset{j}{e_2} + u_3 \underset{k}{e_3}$$

$$= \sum_{i=1}^{3} u_i e_i$$

Kronecker Delta

$u_{ii} = u_{ij}\,\delta_{ij}$, $\delta_{ij} = 1,\ i = j$

$\delta = 0,\ i \neq j$

$$u_{ii} = u_{11} + u_{22} + u_{33}$$

$$u_i v_j e_j = (u_1 + u_2 + u_3)\, v_j e_j$$

$$= (u_1 + u_2\ u_3)(v_1 e_1 + v_2 e_2 + v_3 e_3)$$

$= (u_{11}\delta_{11} + u_{12}\cancel{\delta_{12}}^{0} + u_{13}\cancel{\delta_{13}}^{0} + u_{21}\cancel{\delta_{21}}^{0} + u_{22}\delta_{22} + u_{23}\cancel{\delta_{23}}^{0}) + u_{31}\delta_{31} + u_{32}\delta_{32} + u_{33}\delta_{33}$

Ex $y_i = a_{ij} x_j = a_{i1} x_1 + a_{i2} x_2 + a_{i3} x_3$

For $i = 1 \Rightarrow j = 1, 2, 3$ $\quad i = 1\ \ j = 1, 2, 3$

$i = 2 \Rightarrow j = 1, 2, 3$ $\quad i = 2,\ j = 1, 2, 3$

$i = 3 \Rightarrow j = 1, 2, 3$ $\quad i = 3,\ j = 1, 2, 3$

$y_1 = a_{11}x_1 + a_{12}x_2 + a_{13}x_3$

$y_2 = a_{21}x_1 + a_{22}x_2 + a_{23}x_3$

$y_3 = a_{31}x \; . \; . \; .$

→ Vector and matrix representatn :-

$$a_i = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}, \quad a_{ij} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{223} \\ \vdots & & \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \end{Bmatrix}$$

→ Vector " " addition :-

$$a_i + b_i = \begin{bmatrix} a_1 + b_1 \\ a_2 + b_2 \\ a_3 + b_3 \end{bmatrix} = a_{ij} + b_{ij} - \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & \cdots \\ a_{31} + b_{31} & \cdots \end{bmatrix}$$

* Outer Product $(a \times b)$:- a dia b
Product of 2 vector with different

$$a_j b_j = \begin{bmatrix} a_1 b_1 & a_1 b_2 & a_1 b_3 \\ a_2 b_1 & a_2 b_2 & a_2 b_3 \\ a_3 b_1 & a_3 b_2 & a_3 b_3 \end{bmatrix}$$

* Decomposition of IInd order tensor into symmetric & skew-symmetric part.

→ Symmetric $a_{ij} = a_{ji}$

→ Skew " $a_{ij} = (-a_{ji})$

$a_{ij} = \frac{1}{2}(a_{ij} + a_{ji}) + \frac{1}{2}(a_{ij} - a_{ji})$

$= a_{ij} + a_{ij}$ $\quad \frac{1}{2}[a_{ij} + \cancel{a_{ji}} + a_{ij} - \cancel{a_{ji}}]$

EX $a_{ij} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 4 & 3 \\ 2 & 1 & 2 \end{bmatrix}$ $\quad b_i = \begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix}$

1) $a_{ij} = 1 + 4 + 2 = 7$

2) $b_i b_i = b_1 b_1 + b_2 b_2 + b_3 b_3$

3) $b_i b_j = \begin{bmatrix} b_1 b_1 & b_{12} & \\ b_2 b_1 & & \\ b_3 b_1 & & \end{bmatrix}$

$a_{ij} =$

$a_{ij} b_j = a_{ij} b_1 + a_{ij} b_2 + a_i$

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$$= 2\begin{bmatrix}1\\0\\2\end{bmatrix} + 4\begin{bmatrix}2\\4\\1\end{bmatrix} + 0\begin{bmatrix}0\\3\\2\end{bmatrix}$$

* Kronecker Delta :-

~~Defi~~

$$\delta_{ij} :- \begin{cases}0, & i \neq j \\ 1 & i = j\end{cases} = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

~~Definition :-~~ Property :-

δ_{ji}

$\delta_{ij} = \delta_{ji}$

$\delta_{ii} = \delta_{11} + \delta_{22} + \delta_{33} = 3$

$\delta_{ij}\, x_j = \delta_{ij}\, x_i$

$\delta_{ij}\, a_{jk} = a_{ik}$

$\delta_{ij}\, a_{ij} = a_{ii}$

$\delta_{ij}\, \delta_{jk} = 3$

Permutation symbol:-

$$\epsilon_{ijk} = \begin{cases} +1 \rightarrow \text{for even} \\ -1 \rightarrow \text{Odd} \\ 0 \rightarrow \text{something else} \end{cases}$$

Property:-

$$\epsilon_{123} = \epsilon_{231} = \epsilon_{312} = 1$$

$$\epsilon_{132} = \epsilon_{321} = \epsilon_{213} = (-1)$$

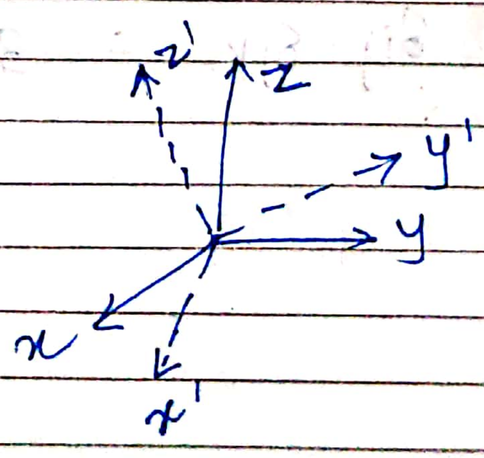
$$\epsilon_{112} = \epsilon_{121} = \epsilon_{222} = 0$$

Co-ordinate Transformation:-

Rotation vector:-

$$e'_i = Q_{ij} e_j \quad , \quad Q_{ij} = \cos(x'_i, x_j)$$

$$e_i = Q_{ji} e'_j$$



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$$\left.\begin{aligned} u' &= l_1 u + m_1 v + n_1 w \\ v' &= l_2 u + m_2 v + n_2 w \\ w' &= l_3 u + m_3 v + n_3 w \end{aligned}\right\} \text{For 3D}$$

$$\boxed{\underset{3\times3}{\epsilon'} = \underset{3\times3}{Q}\, \underset{3\times1}{\epsilon}\, \underset{3\times3}{Q^T}}$$

$Q = \cos(x_i', x_j)$

ϵ = Old strain matrix

$$Q = \begin{bmatrix} \cos(x', x) & \cos(x', y) & \cos(x', z) \\ \cos(y', x) & \cos(y', y) & \cos(y', z) \\ \cos(z', x) & \cos(z', y) & \cos(z', z) \end{bmatrix}$$

$i = 1 \Rightarrow j = 1, 2, 3$
$i = 2 \Rightarrow j = 1, 2, 3$
$i = 3 \Rightarrow j = 1, 2, 3$

$- \quad + \quad - \quad +$

$-\lambda^3 + I_1\lambda^2 - I_2\lambda + I_3 = 0$

$\cancel{\epsilon_{ij}}$ = PRINCIPAL DIRECTION :-

* Characteristic Equation :-

$\det(A - \lambda I) = 0$ → Roots of eqn = Eigenvalue

$|a_{ij} - \lambda\delta_{ij}| = 0$

$-\lambda^3 + I_1\lambda^2 - I_2\lambda + I_3 = 0$

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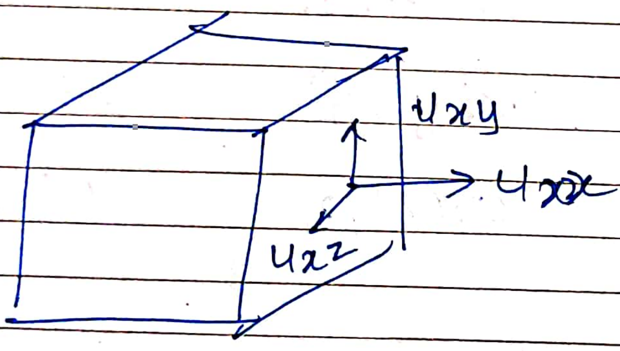
I_1 = Trace of Tensor = trace(A)
= a_{ii}

$$I_2 = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} + \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}$$

$$I_3 = \det(a_{ij}) = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{3} \end{vmatrix}$$



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Note: I_1, I_2, I_3 → Stress invariants

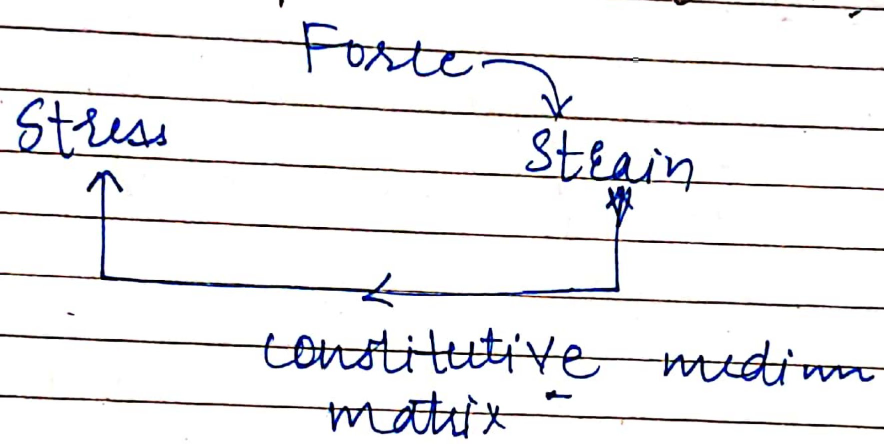
→ The eigenvector n is null space of $A-\lambda I$

→ The number λ is chosen such that $A-\lambda I$ has a null space
i.e. $A-\lambda I$ ~~must~~ must be singular

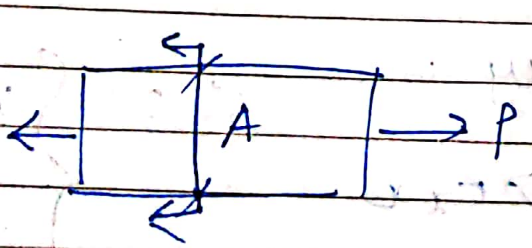
Find Eigen values & Eigen vectors

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 4 & -3 \end{bmatrix}$$

Concept of Stress-Strain:-

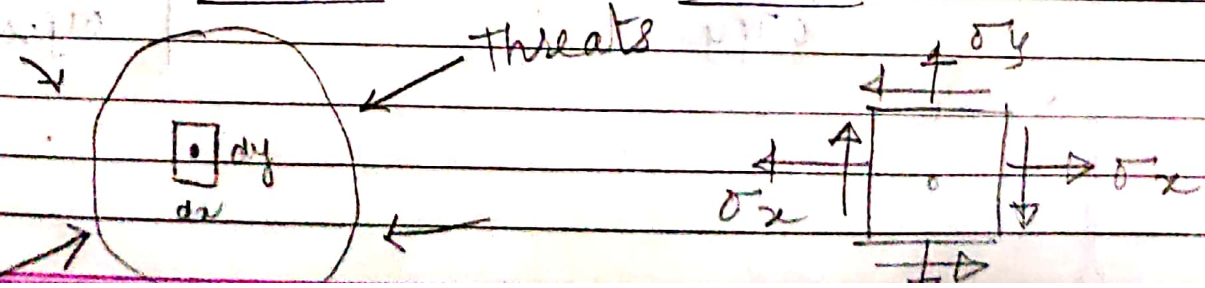


Threat → Material → Response.



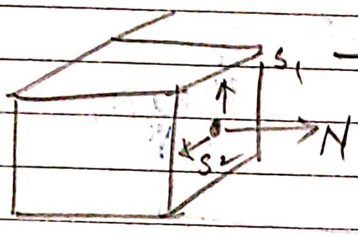
$$\sigma = \frac{P}{A}$$

Infinitesimal area & Volume:-



Normal or Shear Stress $= \lim_{\Delta A \to 0} \frac{\Delta P}{\Delta A}$ → For 2D problems

→ For 3D :-

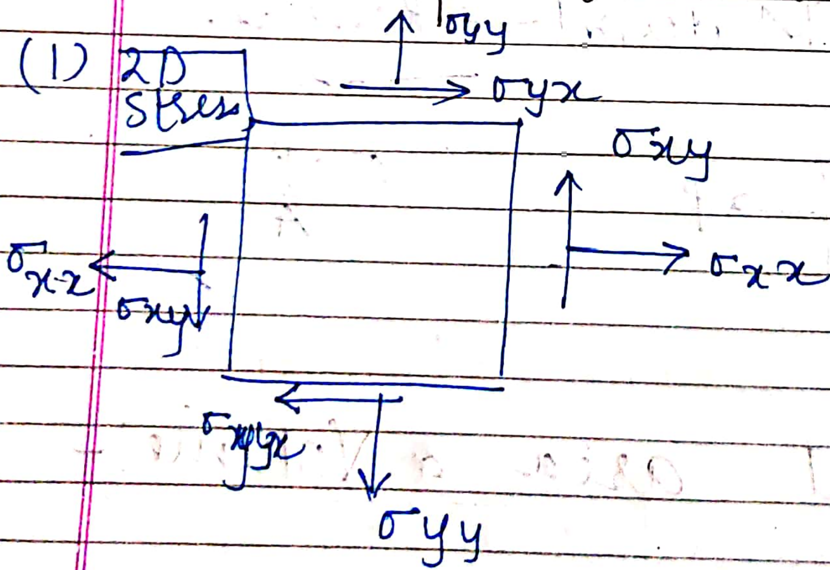


→ point & plane are very imp

→ Here stress is a tensor quantity

→ State of stress at a pt :-

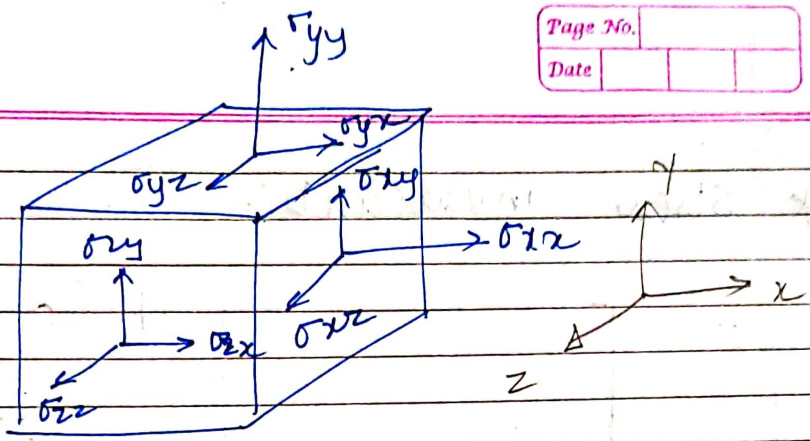
(1) 2D Stress



$\therefore \sigma = \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix}$

$\sigma = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix}$

(2) 3D Stress



$\sigma_{x(x)} \rightarrow$ On the plane in the direction.
$\hookrightarrow$ Normal plane on which it is acting

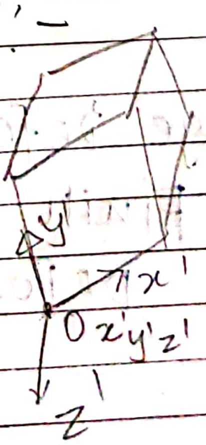
→ Normal stress
& shear stress

$$\therefore \sigma = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zx} & \sigma_{zz} \end{bmatrix} = \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{zx} \end{Bmatrix}$$

* Stress Transformation :-

$\sigma' = Q\sigma . Q^T$

$\sigma'_y =$



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* Stress invariants :-

$\sigma = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$ ← Cauchy Stress Tensor (Augustin - Louis Cauchy) → Small Deformation

$I_\sigma = \sigma_{ii} = tr\,\sigma$

$II_\sigma = \frac{1}{2}(\sigma_{ii}\sigma_{jj} - \sigma_{ij}\sigma_{ji}) = \frac{1}{2}[(tr\,\sigma)^2 - tr(\sigma^2)]$

$III_\sigma = \epsilon_{ijk}\,\sigma_{1i}\,\sigma_{2j}\,\sigma_{3k} = \det\sigma$

→ Statement :- Invariance doesnot change with orientation

Example :- Volume

* Large Deformation (Finite Deformation) → Kirchhoff Stress Tensor

Piola - Kirchhoff Stress tensor

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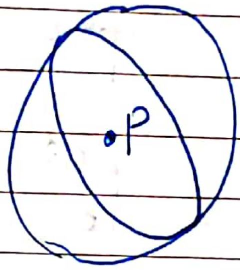
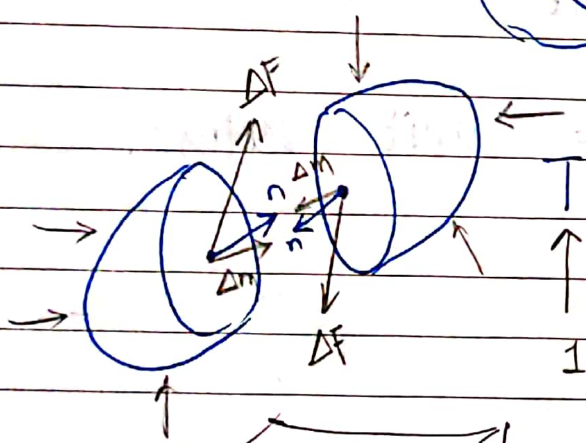
Material

- Body Force
- Surface Force
  - $T^{(n)}$
  - $[\sigma]$

→ Traction:-

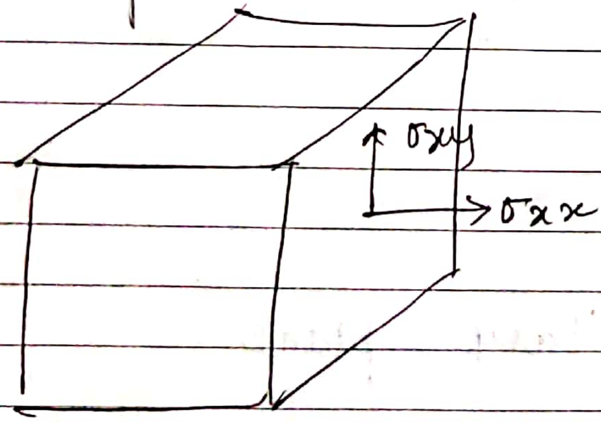


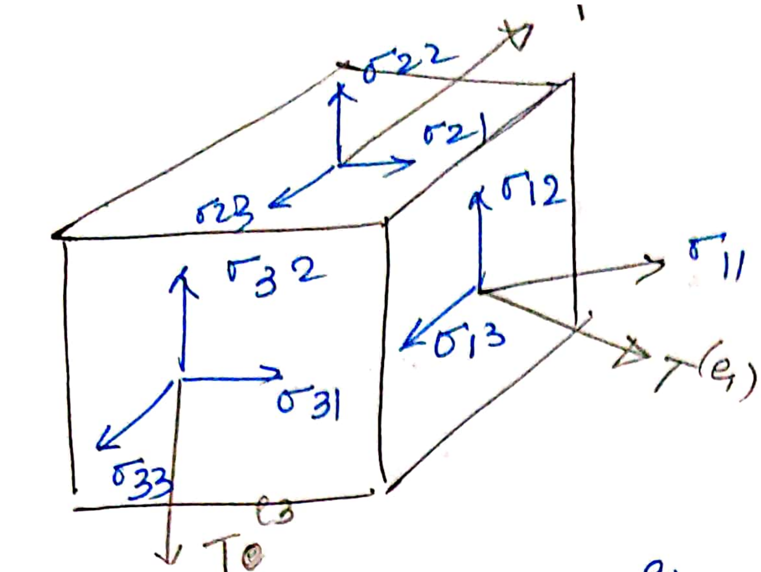


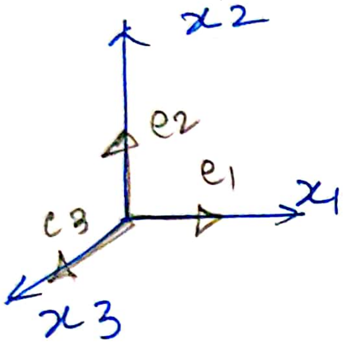
→ If $\Delta M = 0$

$$T = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A}$$

↑ 1^{st} order Tensor







$$\sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots \\ \sigma_{21} & \sigma_{22} & \cdots \\ \sigma_{31} & \sigma_{32} & \cdots \end{bmatrix}$$

$$T^{e_1} = \sigma_{11} e_1 + \sigma_{12} e_2 + \cdots$$

$$T^{e_2} = \sigma_{21} e_1 + \sigma_{22} e_2 + \sigma_{23} \cdots$$

$$T^{e_3} = \sigma_{31} e_1 + \sigma_{32} e_2 + \sigma_{3\cdot} \cdots$$

where e_1, e_2, e_3 = unit vectors.

$e_1 = (1, 0, 0)$.
$e_2 = (0, 1, 0)$.
$e_3 = (0, 0, 1)$

$$\boxed{T^{e_i} = \sigma_{ij} \, e_j}$$

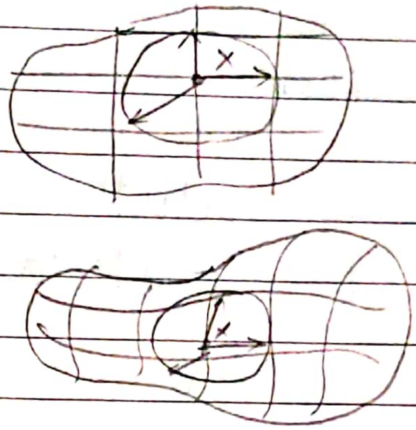
→ For any arbitary plane,

$$\boxed{T^{n} = \sigma_{ij} \, n_i \, e_j} \rightarrow \text{General Formula}$$

* Concept of Strain :-

⇒ Deformation :-

1) Undeformed →
2) Deformed. →



⇒ Strain Tensor :-

$$\epsilon = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix}$$

$$\epsilon_{xx} = \frac{\partial u}{\partial x} \qquad \epsilon_{xy} = \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$$

$$\epsilon_{yy} = \frac{\partial v}{\partial y} \qquad \epsilon_{yz} = \frac{1}{2}\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)$$

$$\epsilon_{zz} = \frac{\partial w}{\partial z} \qquad \epsilon_{zz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)$$

$$\epsilon_{xy} = \frac{\gamma_{xy}}{2}$$

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* CONSTITUTIVE RELATION :-

⇒ Material Behaviour :-

→ Introduction :-

- Constitutive relatn characterize phys[...] property of material.
- It relates stress and strain.
- Elastic solid can recover its config[...] upon removal of loading.
- Elastic " don't include rate / history effect
- If σ is linear f^n of ϵ in elasti[...] solid then linear - elastic solid.

* Constitutive relatn of uniaxial solid

- Simple tensile Test is carried out in UTM
  ↳ to find σ, ϵ curve → to find [...]

→ ϵ = is found by dividing Stress.

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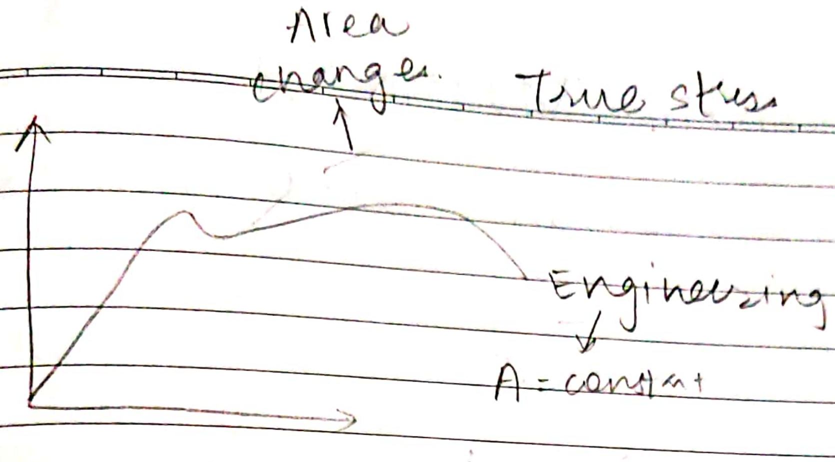


→ Steel, Aluminium → Ductile material

* Elasticity Tensor :-

$\sigma_{ij} = C_{ijkl} \, \epsilon_{kl}$ = Tensorial form

→ C_{ijkl} = 4th Order Elasticity tensor

→ No. of components = $3^4 = 81$

→ Monoclinic material - have single plane of symmetry which reduces the components to 13

→ Orthotropic " have 3 plane
9 independent components

(E, ν, G)

→ Transversely isotropic have 5 in"

→ Isotropic Material - 2 independent "
(E, ν)

* Homogeneous Material :-
Material property dont vary pt

* Isotropic :-
M " " is same along any directⁿ at a pt

* Homogeneous Isotropic
– " " dont change any dir & remain same throughout its bo

* Isotropic Tensor :- (Not imp)
→ Elastic $C_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu (\delta_{ik} \delta$
→ λ = Lame's constant

$$\lambda = \frac{-E\mu}{(1+\mu)(1-2\mu)}$$ poisons ratio.

→ μ = Shear modulus

λ can be written with help of E & μ (poisons ratio)

* Generalize Hook's law:-

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \mu(\sigma_y + \sigma_z)\right]$$

$$\epsilon_y = \frac{1}{E}\left[\sigma_y - \mu(\sigma_x + \sigma_z)\right]$$

$$\epsilon_z = \frac{1}{E}\left[\sigma_z - \mu(\sigma_x + \sigma_y)\right]$$

$$\gamma_{xy} = \frac{G}{\tau_{xy}}$$

$$\gamma_{yz} = \frac{G}{\tau_{yz}}$$

$$\gamma_{zx} = \frac{G}{\tau_{xz}}$$

* Physical meaning of Elastic Moduli

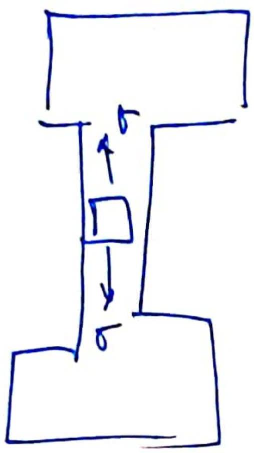
Simple Tension test:-

→ Test is carried to find E & μ.

→ Load Data, lateral & longitudinal ϵ is measured

→ loading is along x direction so we will have only σ_x

Stress Tensor $= \begin{bmatrix} \sigma_x & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$



Strain $= \begin{bmatrix} \frac{\sigma_x}{E} & 0 & 0 \\ 0 & -\frac{\sigma_x \mu}{E} & 0 \\ 0 & 0 & -\frac{\sigma_x \mu}{E} \end{bmatrix}$