A Presentation on

Lasers

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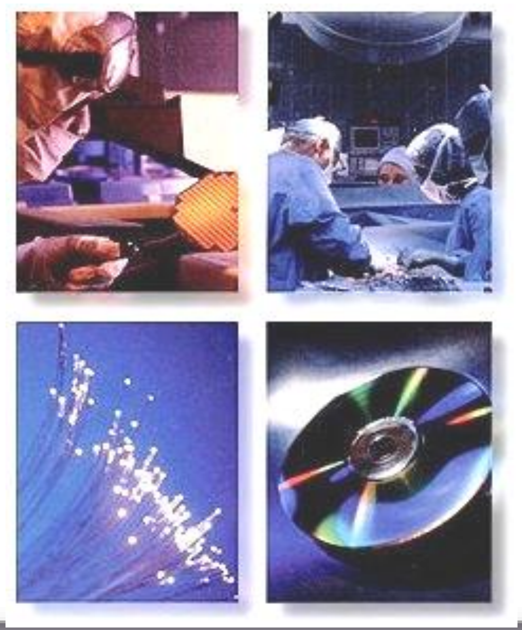
Summary

- What is Laser?
- Classification of light-atom interaction
- Necessary condition for lasing transition
- Properties of Laser
- Types and Use of Laser

What is Laser?

LASER : Light Amplification by Stimulated Emission of Radiation

Laser is a device which can produce a high intense, highly coherent, more directional and highly monochromatic beam



Classification of light-atom interaction

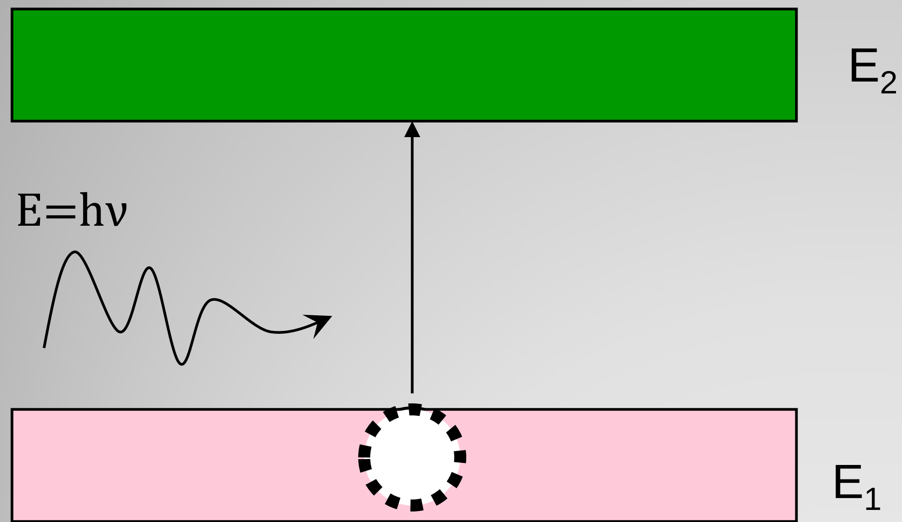
interacts

Radiation → matter

- Absorption
- Spontaneous Emission
- Stimulated Emission

Absorption

Atom be initially in the lower state E_1, If a photon of energy hν is incident on the atom in the lower state, the atom absorbs the incident photon and gets excited to the higher energy state E_2.



$$R_{12} \propto N_1 \rho$$

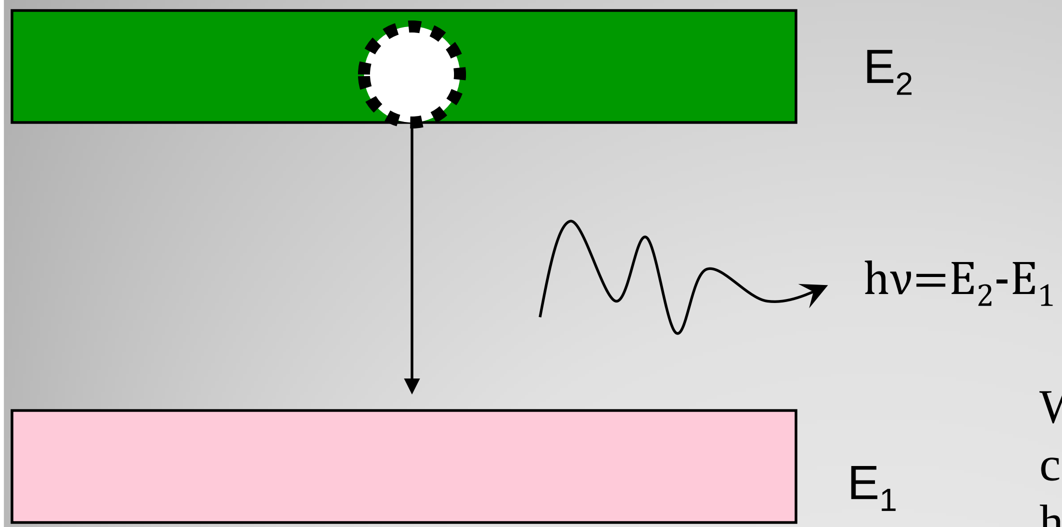
$$R_{12} = B_{12} N_1 \rho$$

Where, B_{12} is the proportionality constant, N_1 is population of the lower energy level, R_{12} is Rate of absorption.

$$\text{Atom} + \text{Photon} \longrightarrow \text{Atom}^*$$

Spontaneous Emission

It is a process in which there is an emission of a photon whenever an atom transits from a higher energy state to a lower energy state without the aid of any external agency.



$$R_{21}(sp) \propto N_2$$

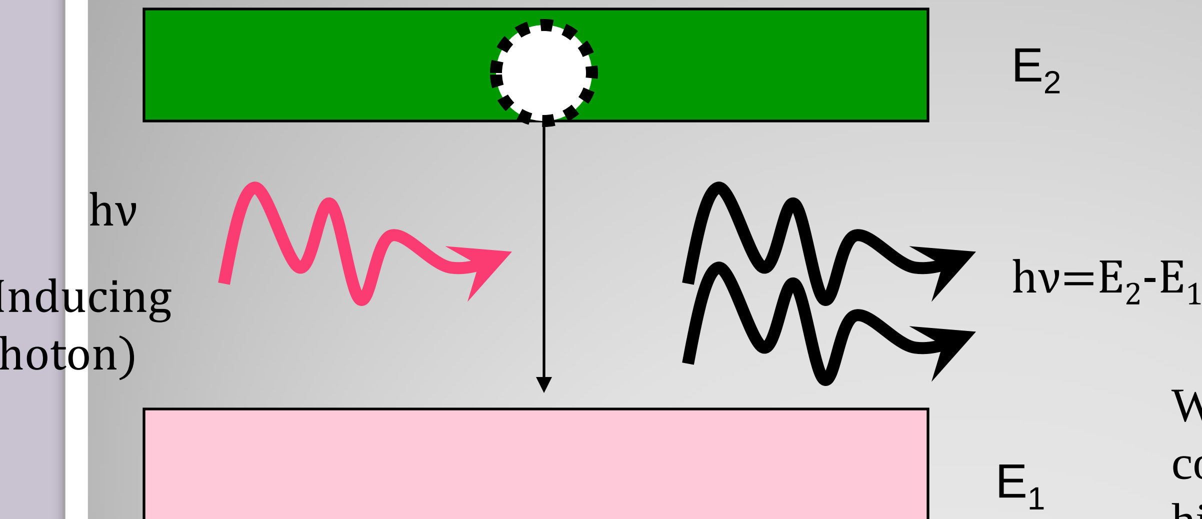
$$R_{21}(sp) = A_{21} N_2$$

Where, A_{21} is the proportionality constant, N_2 is population of the higher energy level, R_{21} is Rate of spontaneous emission.

$$\text{Atom}^* \longrightarrow \text{Atom} + \text{Photon}$$

Stimulated Emission

It is a process in which there is an emission of a photon whenever an atom transits from a higher energy state to a lower energy state under the influence of an external agency. i.e. an inducing photon

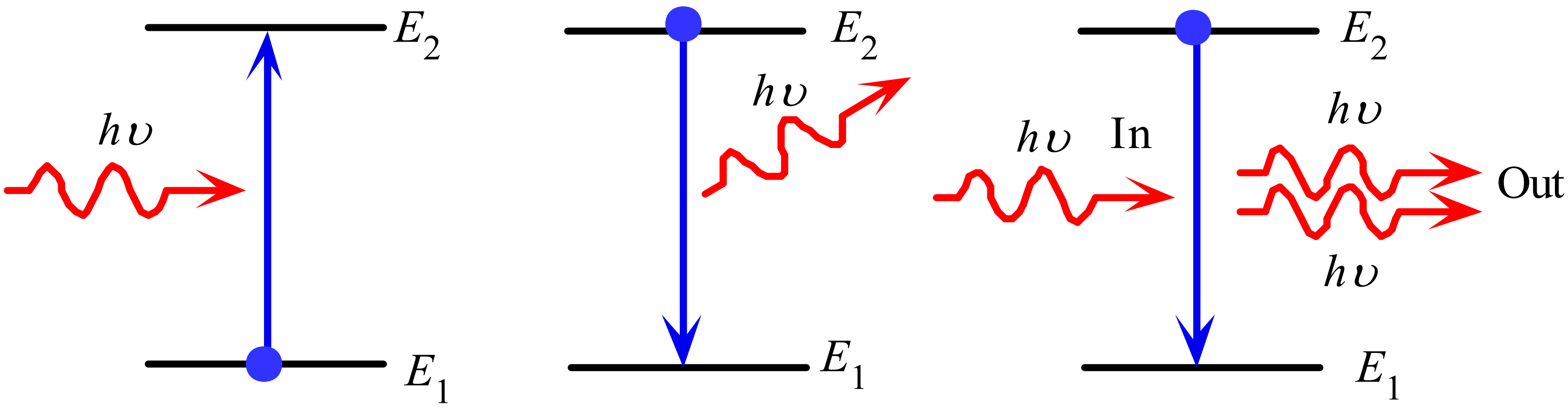


$$R_{21}(st) \propto N_2 \rho$$

$$R_{21}(st) = B_{21} N_2 \rho$$

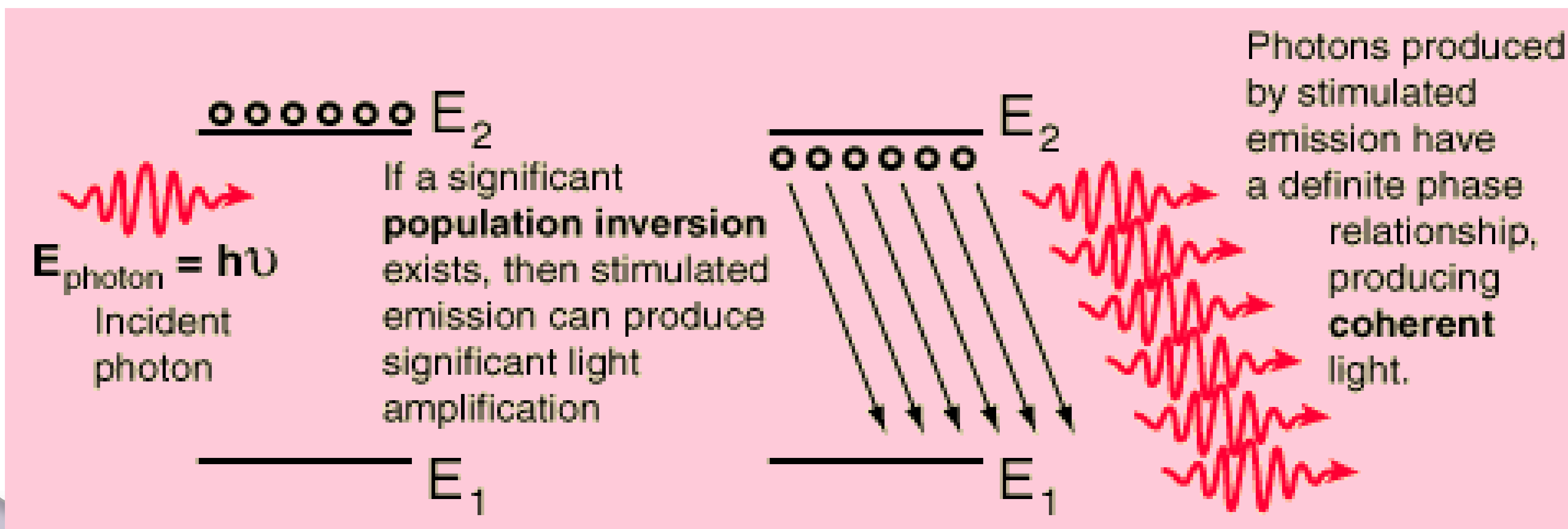
Where, B_{21} is the proportionality constant, N_2 is population of the higher energy level, R_{21} is Rate of stimulated emission.

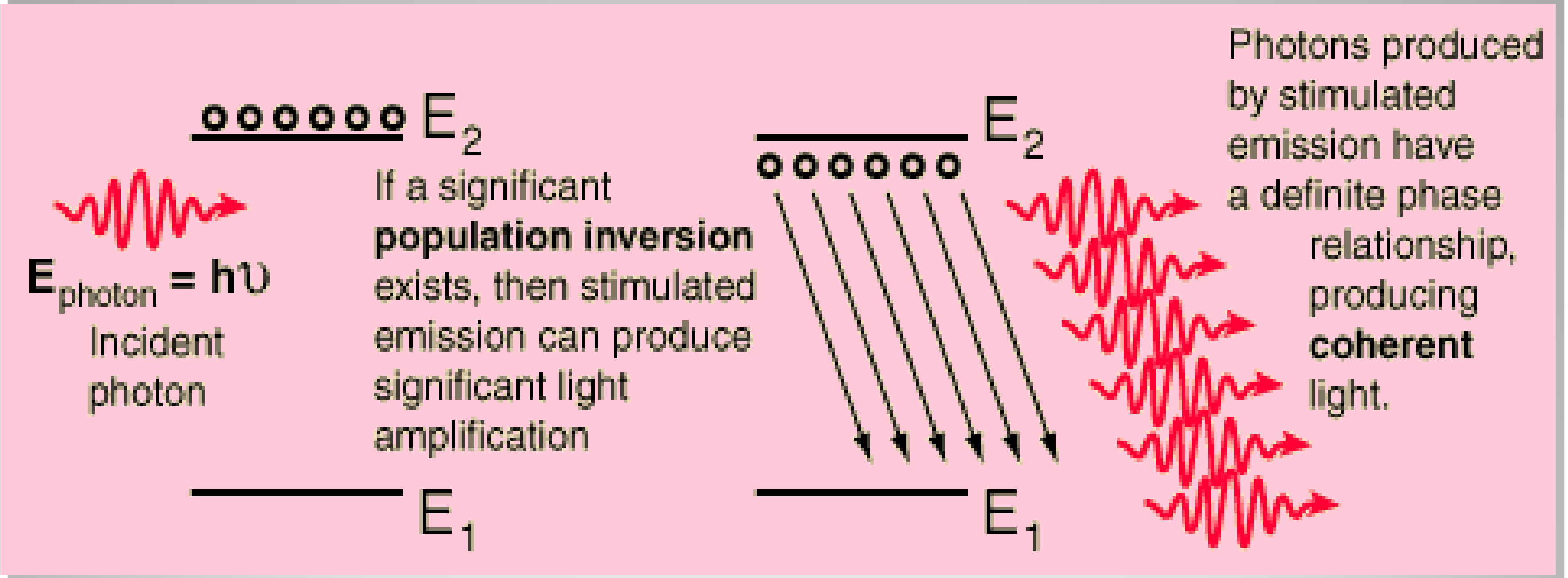
$$\text{Atom}^* + \text{Photon} \longrightarrow \text{Atom} + (\text{Photon} + \text{Photon})$$



(a) Absorption (b) Spontaneous emission (c) Stimulated emission

Absorption, spontaneous (random photon) emission and stimulated emission.

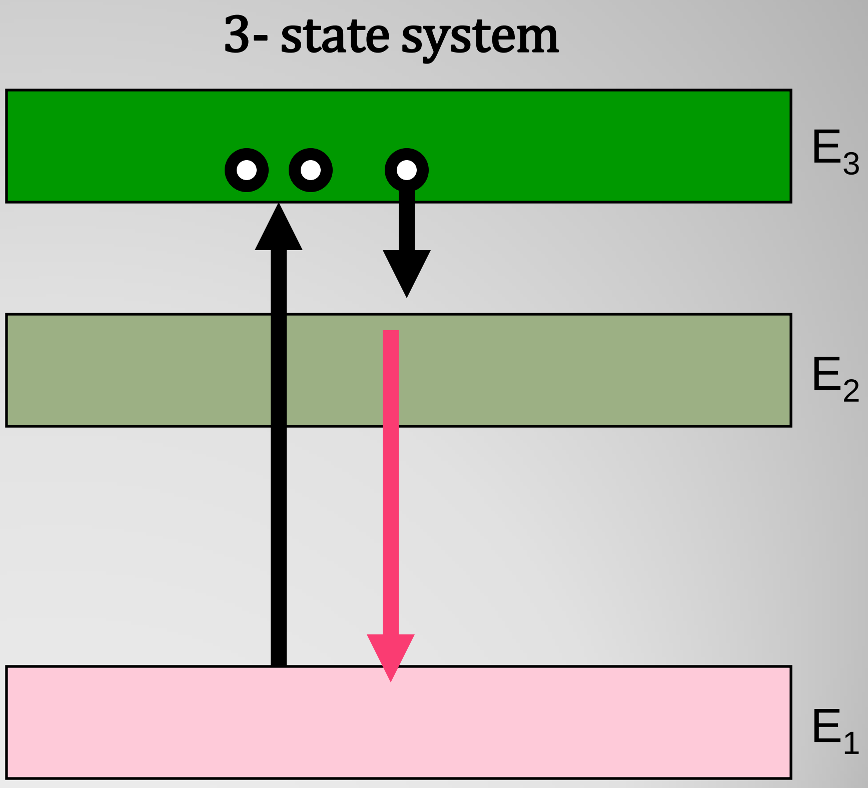




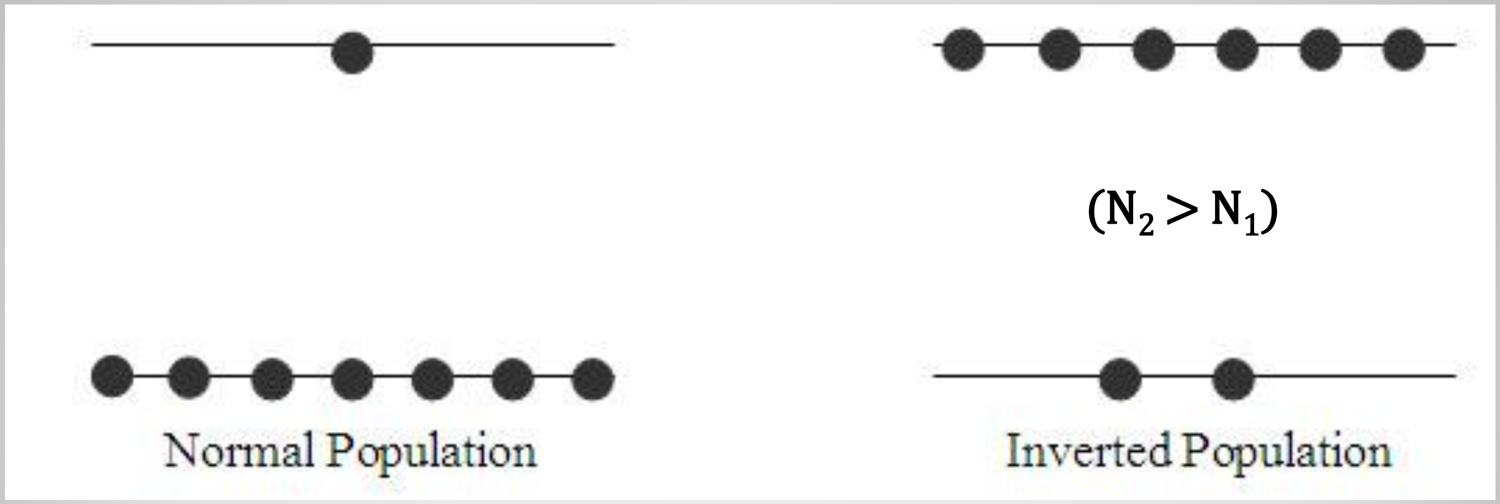
E2
Ephoton = hυ
Incident photon
If a significant
population inversion
exists, then stimulated emission can produce significant light amplification
E1
E2
E1
Photons produced by stimulated emission have a definite phase relationship, producing **coherent** light.

Necessary condition for lasing transition

- Stimulated Emission
- Population Inversion
  ($N_2 > N_1$)
- Metastable State

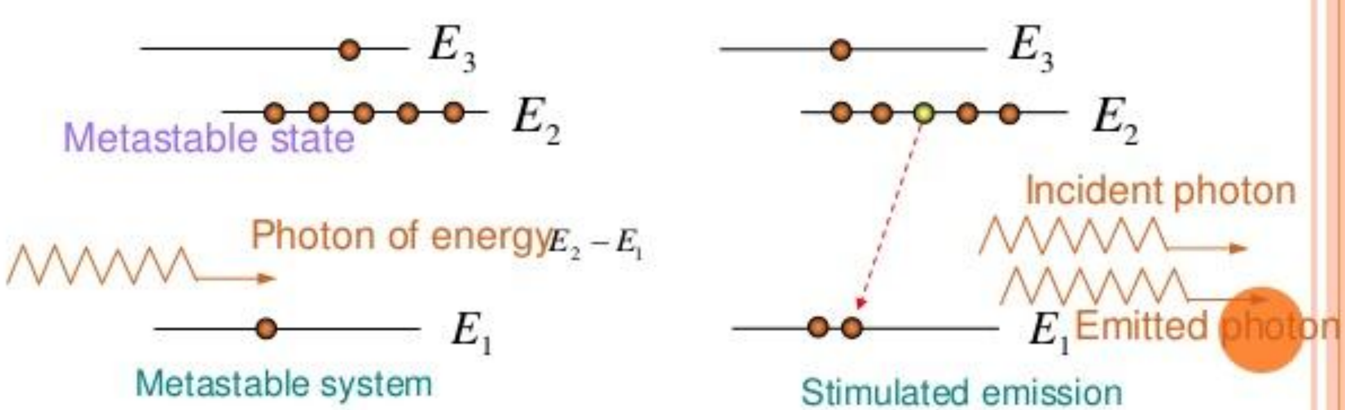


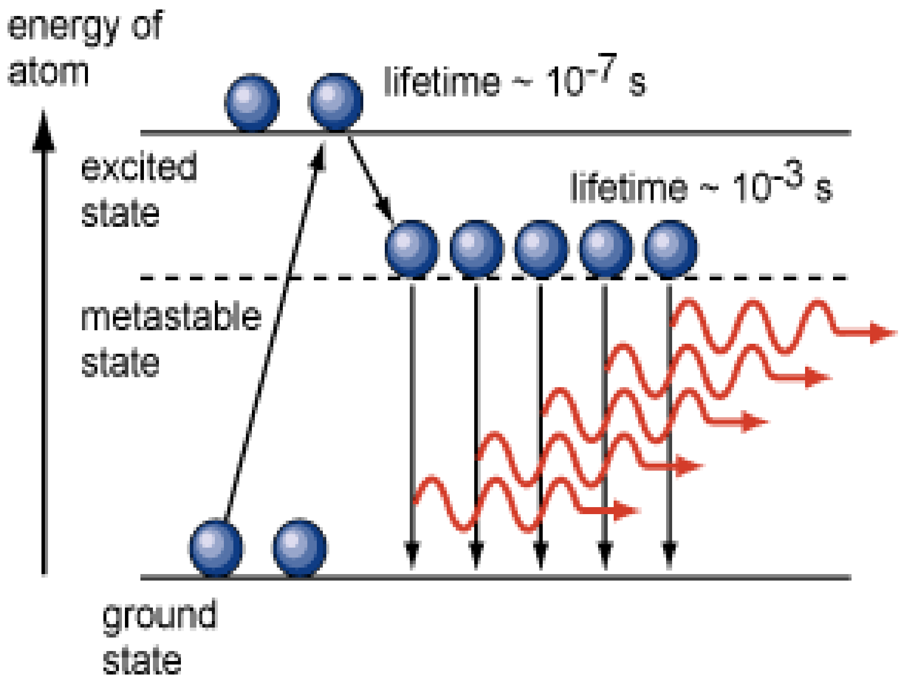
Population inversion: The redistribution of atomic energy levels that takes place in a system so that laser action can occur. Normally, a system of atoms is in temperature equilibrium and there are always more atoms in low energy states than in higher ones. Although absorption and emission of energy is a continuous process, the statistical distribution (population) of atoms in the various energy states is constant. When this distribution is disturbed by pumping energy into the system, a population inversion will take place in which more atoms will exist in the higher energy states than in the lower.

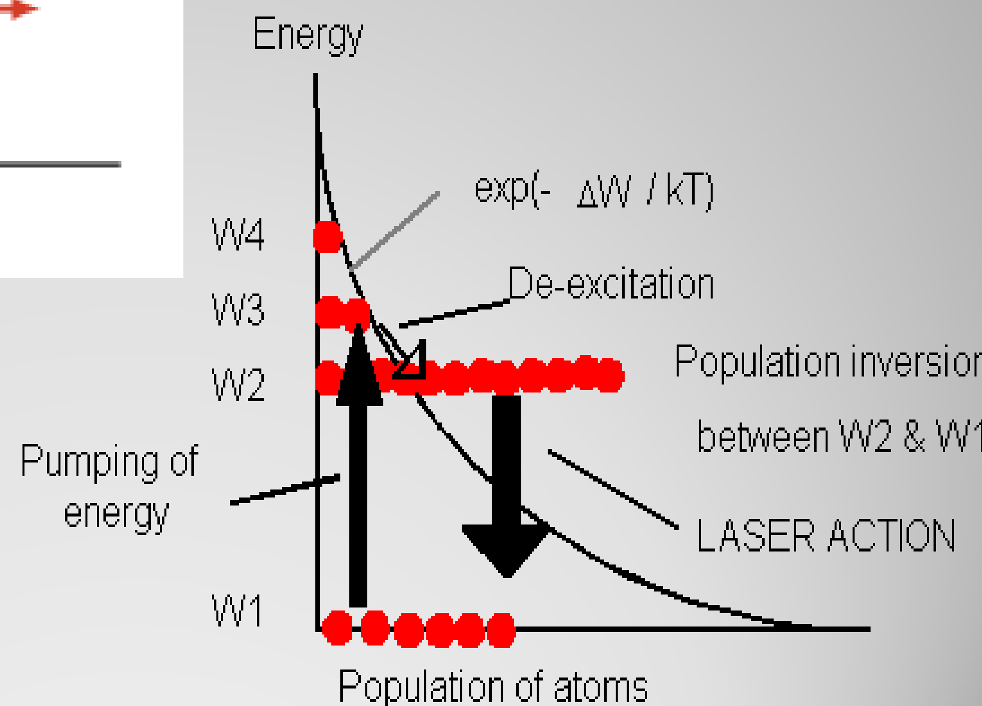


METASTABLE STATE

2. The higher state must be a metastable state – a state in which the electrons remain longer than usual so that the transition to the lower state occurs by stimulated emission rather than spontaneously.




energy of atom
lifetime ~ 10^{-7} s
excited state
lifetime ~ 10^{-3} s
metastable state
ground state


Energy
exp(- ΔW / kT)
W4
De-excitation
W3
Population inversion
W2
between W2 & W1
Pumping of energy
LASER ACTION
W1
Population of atoms

Properties/Characteristic of Laser:

- **High Monochromaticity** - It can emit light of single wavelength. The spread is of the order of 1nm for laser.

- **High Directionality** – An ordinary light source emits light in all possible directions. But, laser travels as a parallel beam it can travel over a long distance without spreading

- **High Intensity** - It has a ability to focus over a small area of 10^{-6} cm^2

- **Coherence** - identical in phase and direction. All the constituent photons of laser beam possess the same energy, momentum and propagate in same direction.

Basic concepts:

Population Inversion: It is a state of achieving more number of atoms in the excited state compared to the ground state. i.e., $N_2 > N_1$. It can be achieved by a process called pumping.

Pumping: It is the mechanism of exciting atoms from the lower energy state to a higher energy state by supplying energy from an external source.

Lasing: The process which leads to emission of stimulated photons after establishing the population inversion is referred to as lasing.

Life time: The limited time for which a particle or an atom remains in the excited is known as life time. It is about a nano second.

Metastable State: Metastable states are the energy levels in an atomic system where the life time of atoms is very large (of the order 10-3 to 10-2 second).

Active Medium: A medium in which population inversion is achieved for laser action is called active medium. The medium can be solid, liquid , gas and plasma.

Optical Resonator: It is a pair of reflecting surfaces (mirrors); of which , one is being a perfect reflector and the other being a partial reflector.

DIFFERENT PUMPING MECHANISMS :

i. ***Optical pumping :*** Exposure to electromagnetic radiation obtained from discharge flash tube results in pumping (For solid state lasers)

ii. ***Electrical discharge :*** By inelastic atom-atom collisions, population inversion is established. (For Gas lasers, i.e. CO_2 laser)

iii. ***Chemical pumping :*** By suitable chemical reaction in the active medium, For liquid lasers.

iv. ***Direct Conversions:*** A direct conversion of electric energy into light takes place (for Semiconductor laser).

The Relation between Einstein's Coefficients:

Relation between Einstein's coefficients
Laser action comprises following three processes;
Absorption 2. Spontaneous emission 3. Stimulated emission

In thermal equilibrium, the number of upward transition is equal to the number of downward transition per unit volume per second
In other words;
Rate of absorption = Rate of Spontaneous emission + Rate of stimulated emission

$R_{12} = R_{21}(\mathrm{sp}) + R_{21}(\mathrm{st})$

$B_{12}N_1\rho = A_{21}N_2 + B_{21}N_2\rho$ ………………(1)

Here N_1 **= population density of ground state,**
N_2 **= population density of excited state**
ρ is density of incident radiation

By rearranging equation (1), we get

$(B_{12}N_1$ - $B_{21}N_2$) ρ = $A_{21}N_2$

ρ = $\frac{A_{21}N_2}{(B_{12}N_1 - B_{21}N_2\ )}$

ρ = $\frac{A_{21}}{B_{12}\frac{N_1}{N_2} - B_{21}}$..............................(2)

According to Boltzmann distribution law

N_1= $N_O\ e^{-(E_1/K_BT)}$..............................(3)

N_2= $N_O\ e^{-(E_2/K_BT)}$..............................(4)

On dividing equation (3) by equation (4)

$\frac{N_1}{N_2} = \frac{e^{-(E_1/K_BT)}}{e^{-(E_2/K_BT)}}$

$\frac{N_1}{N_2} = e^{-\left(\frac{E_1}{K_BT}\right)+\left(\frac{E_2}{K_BT}\right)}$ =

$\frac{N_1}{N_2}$= $e^{\frac{(E_2 - E_1)}{K_BT}}$, since $E_2 - E_1 = h\nu$

$\frac{N_1}{N_2}$= $e^{\frac{h\nu}{K_BT}}$ ....(5)

On putting value of $\frac{N_1}{N_2}$ from equation (4) in (2), we get

$$\rho = \frac{A_{21}}{B_{12}e^{\frac{h\nu}{K_BT}} - B_{21}}$$

$$\rho = \frac{A_{21}}{B_{21}} \frac{1}{\frac{B_{12}}{B_{21}} e^{\frac{h\nu}{K_BT}} - 1]} \quad \text{..............(6)}$$

From plank's black body theory of radiation

$$P = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{\frac{h\nu}{K_BT}} - 1} \quad \text{................(7)}$$

By comparing equation (6) and equation (7)

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} \quad \text{......................(8)}$$

And $B_{12} = B_{21}$.....................(9)

Here A_{21} , B_{12} and B_{21} are proportionality constants which are known as Einstein's constant

Equation (9) show that the probability of absorption is equal to the probability of stimulated emission

Type of Laser

- Solid State Lasers (ND-YAG laser, Ruby laser)
- Liquid Lasers
- Gaseous Lasers (He-Ne laser, CO_2 laser)
- Dye Lasers
- Semiconductor Lasers

Essential components of a laser system :

Active medium or Gain medium : It is the system in which population inversion and hence stimulated emission (laser action) is established.

Active Medium → Pumping Mechanism → Optical resonator

Pumping mechanism : To achieve Population Inversion i.e., it is the method for raising the atoms from lower energy state to higher energy state to achieve laser transition.

Optical Resonator: It is a pair of reflecting surfaces (mirrors) of which one is a perfect reflector and the other one is a partial reflector…….used for amplification of photons.

Nd-YAG Laser

Active medium :

- The host medium for this laser is Yttrium Aluminium Garnet (YAG = $Y_3 Al_5 O_{12}$) with 1.5% trivalent neodymium ions (Nd^{3+}) present as impurities.

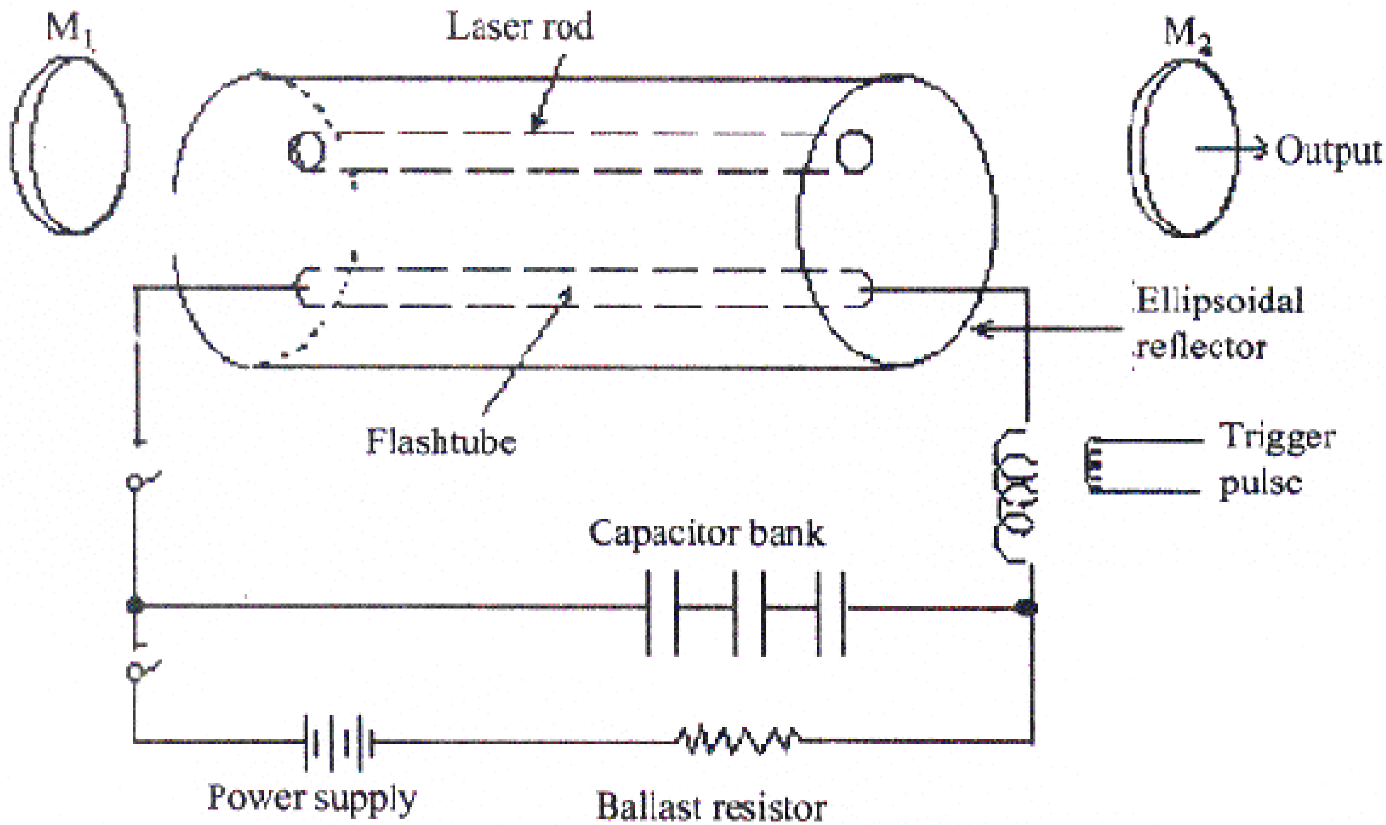
- The (Nd^{3+}) ions occupy the lattice sites of yttrium ions as substitutional impurities and provide the energy levels for both pumping and lasing transitions.

Pumping Source:

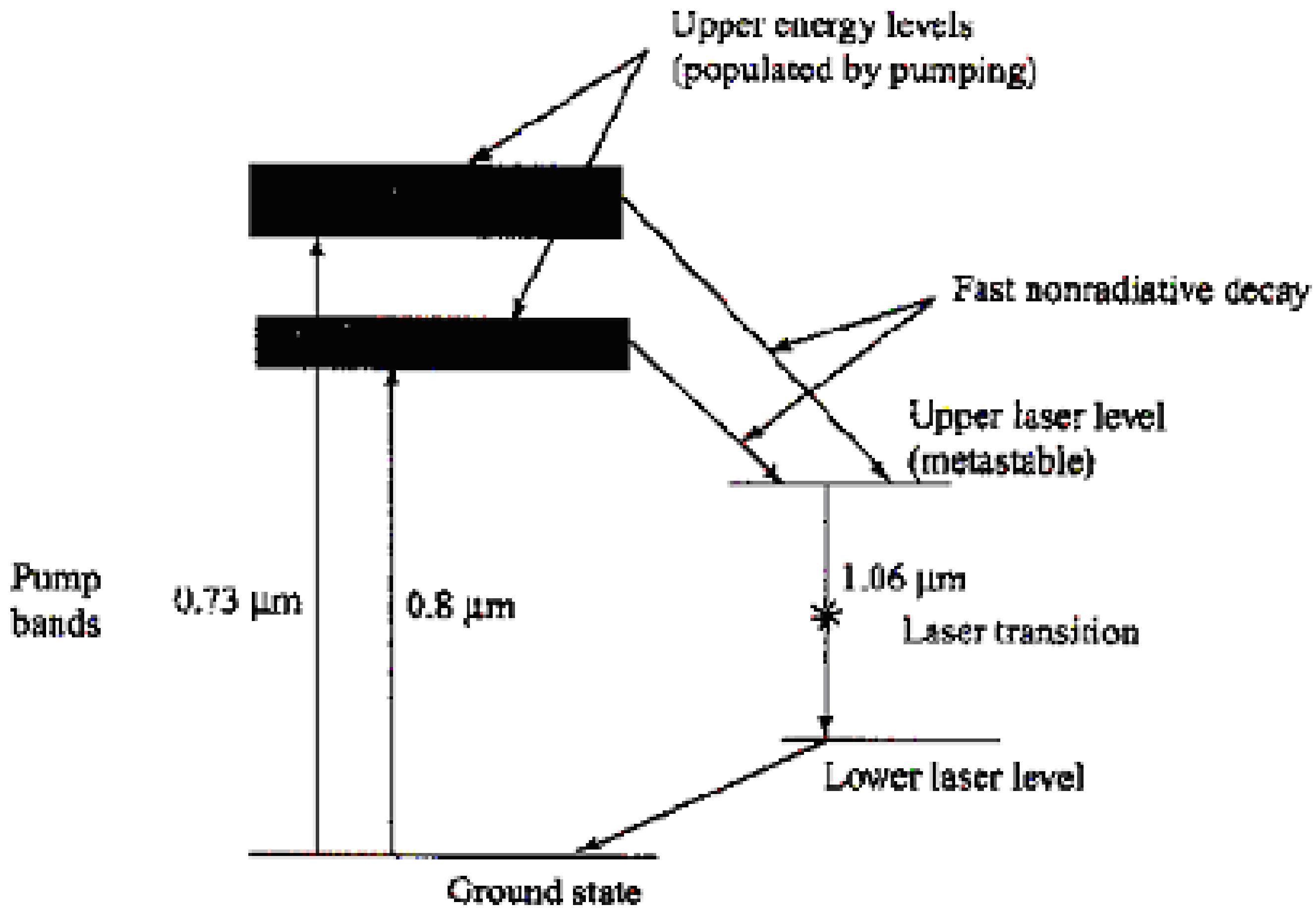
-The principle behind Nd-YAG laser is optical pumping. The population inversion is achieved by a flash light either using xenon or krypton flash tube. As a result, Nd ions are transported into the excited levels.

Resonating Cavity:

- In the Nd-YAG laser, a rod of 5 to 10 cm length and 6 to 9 mm diameter is used. The ends of the rod are polished and made optically flat and parallel. The optical cavity is formed either by silvering the two ends of the rod or by using two external reflecting mirrors. One mirror is made hundred percent reflecting while the other mirror is left slightly transmitting to draw the output



- This laser system has two absorption bands (0.73 μm and 0.8 μm)
- Optical pumping mechanism is employed.
- Laser transition takes place between two laser levels at 1.06 mm.


Upper energy levels
(populated by pumping)
Fast nonradiative decay
Upper laser level
(metastable)
Pump
bands
0.73 µm
0.8 µm
1.06 µm
Laser transition
Lower laser level
Ground state

Working:

-When flash lamp is switched on, neodymium ions acquire energy from the flash light. The Nd ions are excited to energy levels E_3 and E_4 by absorbing energy with wavelengths of respectively 0.73µm and 0.80 µm.

- The Nd ions are not stable in the excited state, it makes a non – radiactive transition from E_3 and E_4 to a metastable state E_2 . Ions can stay for a long time in this state until population inversion is achieved. When population inversion is achieved between E2 and E1 state, a stimulated emission takes place from the energy levels E2 to E1 by emitting radiation of the wavelength 1.064 µm.

- **Nd:YAG applications :**

- The important industrial uses of YAG and glass lasers have been in materials processing such as welding, cutting, drilling.
- Since 1.06 μm wavelength radiation passes through optical fibre without absorption, fibre optic endoscopes with YAG lasers are used to treat gastrointestinal bleeding.
- Remote sensing applications
- Medicines for endoscopic applications, medical surgery, dental surgery etc.

The Use of Lasers

- Science and engineering applications
  - precise measurements and spectroscopy
  - Fiber optic communication

- In Medicine
  - for the treatment of detached retinas
  - eye surgery and in cancers treatments

- In Industry
  - cutting, welding, melting
  - To test the quality of the materials
  - for the heat treatment of metallic and non-metallic materials.

Military applications:

- The laser beam can serve as a war weapon, i.e. a powerful laser beam can be used to destroy in a few seconds, the big size objects like aeroplanes, missiles etc. by pointing the laser beam on to them. For this reason, it can be even called as death ray.

- The laser beam can be used to determine precisely the distance, velocity and direction as well as the size and form of distant objects by means of the reflected signal. It is known as LIDAR.

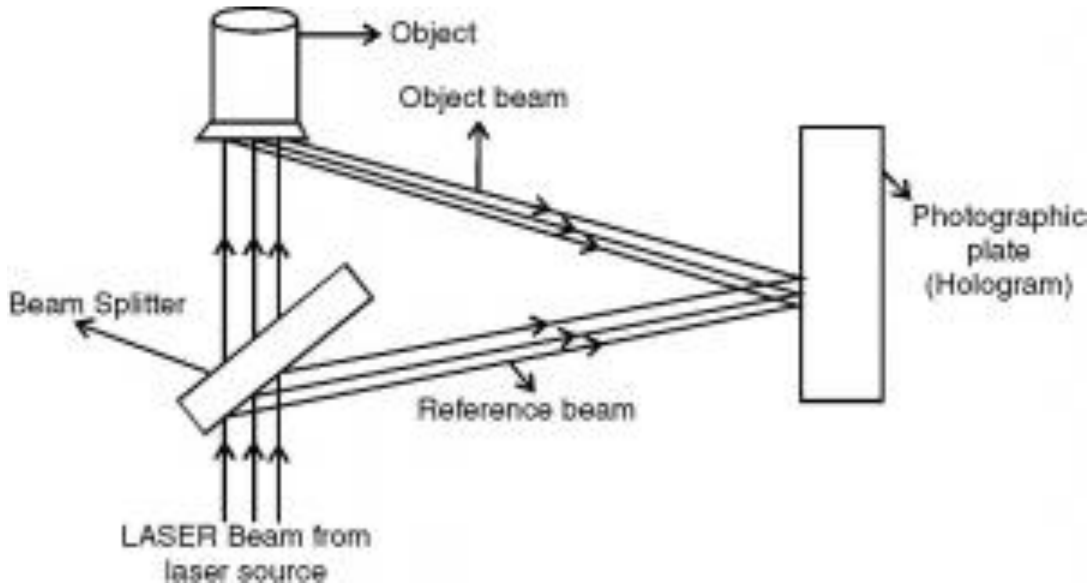
Holography

Holography is a technique of recording the amplitude and phase of the light waves reflected from an object. A three dimensional image of the object can be obtained. The recorded photograph is called a hologram.

Principle:

1. Recording of hologram or construction of hologram (based on interference of coherent light waves).
2. Reconstruction of hologram (based on diffraction of light waves).

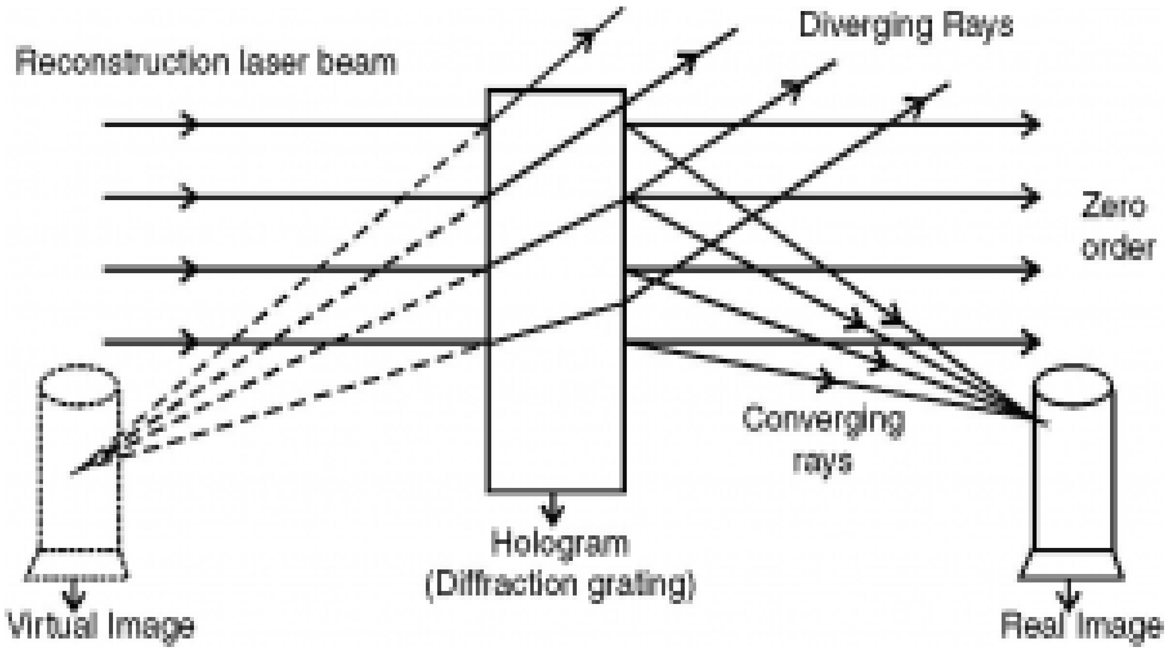
1. Recording of hologram or construction of hologram



The light from a laser source is split into two components – splitter and reference beam. The wave illuminating the object is called the object wave or signal wave and the wave directed towards the photographic plate is called the reference wave.

Thus , the two beams interfere with each other producing interference pattern on the photographic plate. Thus, the record of this interference pattern constitutes a hologram.

2. Reconstruction of hologram



The developed hologram is exposed to a laser beam of the same wavelength. This laser beam interacts with the interference pattern on the hologram and gets diffracted to produced two images of the original object. The virtual image and the real image which is in three dimensional form.

<u>**Optics**</u>

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Syllabus

Introduction to Reflection, Refraction and Total Internal Reflection

Interference: Types of interferences, Thin film interference, wedge shape films;

Diffraction: Huygens principle, Diffraction of light waves, Fraunhofer diffraction at a single slit, Plane diffraction grating, Fresnel diffraction (Introduction)

What is light?

Light is a form of energy that stimulates our vision. Light transports energy from one place to another. Light can be of both nature; corpuscular and/or wave. According to quantum theory, light is emitted in the form of photons, which behave like particles but when their number is large they exhibit the properties of wave.

Velocity of light in vacuum is, $c = 3 \times 10^8$ m/s.

Some Concepts

- A material through which light propagates is called an optical medium.
- Some materials such as air, water and glass readily transmit light are said to be transparent.
- Materials which do not transmit light at all said to be opaque.
- When light travels through any medium, its velocity reduces. The dependence of velocity of light on the medium is characterized by the quantity called optical density.
- The absolute refractive index 'μ' of an optical medium is defined as the ratio of velocity of light in vacuum to the velocity of light in the medium.

$$\mu = \frac{c}{v} \qquad (1)$$

The refractive index is a dimensionless number and greater than unity, since v is always less than c.

1 Reflection, Refraction and Total Internal Reflection

1.1 Reflection

We see most of things with the help of light reflected from them. When a ray of light falls on the boundary separating two optical media, light is partly reflected into the first medium and partly transmitted into the second. If the surface is smooth (polished mirrors, metals, liquid surfaces etc), it reflects light specularly and is known as specular reflection. However, many surfaces have microscopic irregularities, then it reflects light at many angles and the reflection is diffuse. In physics, the term reflection is used to mean specular reflection.

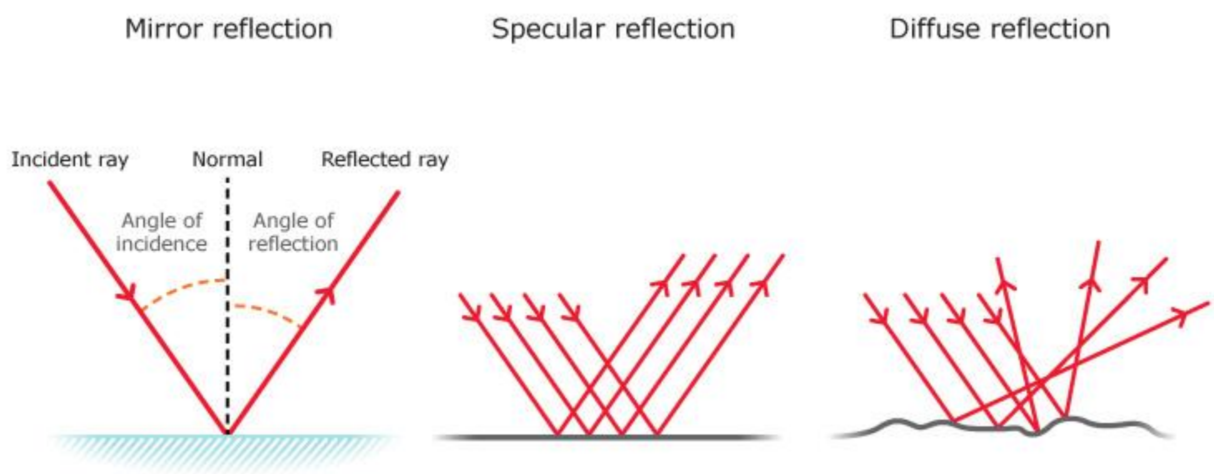


Figure 1: Phenomenon of reflection

Laws of reflection:

- The incident ray, reflected ray and the normal to the surface all lie in the same plane.
- The angle of reflection θ_2 is equal to the angle of incidence θ_1.

1.2 Refraction

The light ray that enters a transparent medium from another transparent medium is bent at the boundary and is said to be refracted. The boundary is known as refracting surface. The angle of refraction 'r' depends on the properties of the two media and on the angle of incidence 'i',

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{\mu_2}{\mu_1} \qquad (2)$$

where v_1 is the velocity of light in medium 1 having refractive index μ_1 and similarly where v_2 is the velocity of light in medium 2 having refractive index μ_2. The relation is more popularly known as Snell's law. It follows from Snell's law that an increase in the angle of incidence causes an increase in the angle of refraction.

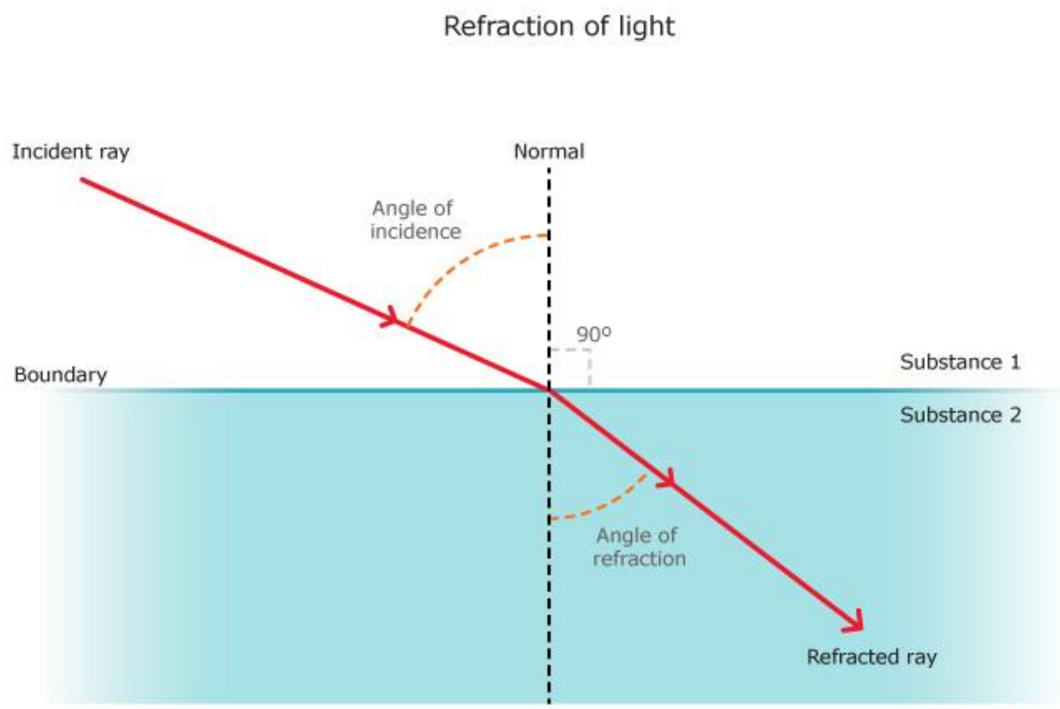


Figure 2: Phenomenon of refraction

When light ray passes from rarer to denser medium, it is bent towards the normal in the denser medium and vice-versa. Further, for a given angle of incidence, the angle of refraction depends on the wavelength of the incident ray and varies from color to color.

1.3 Total Internal Reflection

A medium having a lower refractive index is said to be an optically rarer medium while a medium having a higher refractive index is known as an optically denser medium. When a ray of light passes from a denser medium to a rarer medium, it is bent away from the normal in the rarer medium. Snell's law for this case may be written as,

$$\sin r = \frac{\mu_1}{\mu_2} \sin i \qquad (3)$$

where 'i' is the angle of incidence of light ray in the denser medium and 'r' is the angle of refraction in the rarer medium. Also, $\mu_1 > \mu_2$.

At some particular angle of incidence 'i_c', the refracted ray glides along the boundary surface so that r = 90°, as seen in Fig.3. The angle 'i_c' is known as critical angle. At angles greater than 'i_c',

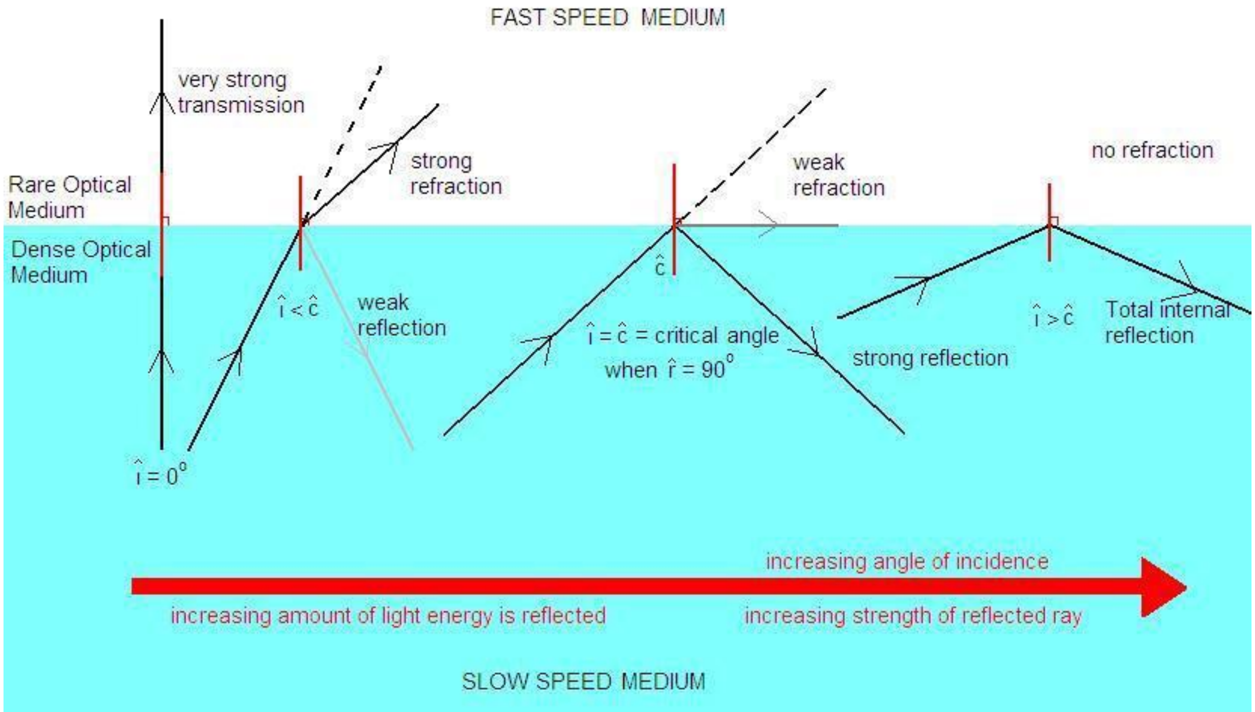


Figure 3: Phenomenon of total internal reflection

There are no refracted rays at all. The rays are reflected back into the denser medium as though they encountered a specular reflecting surface. The phenomenon in which light is totally reflected from a denser-to-rarer medium boundary is known as total internal reflection.

The critical angle can be determined from Snell's law as follows,when i = ic, r = 90°.

Therefore,

$$\sin i_c = \frac{\mu_2}{\mu_1} \qquad (4)$$

Total internal reflection does not take place when light propagates from a rarer to denser medium.

<u>***********</u>**

<u>Assignments:</u>

<u>Definitions:</u>

Reflection, Refraction, Total Internal Reflection, Interference, Constructive Interference, Destructive Interference, Diffraction, Diffraction grating, Refractive Index, wave front

Short Questions:

Q.1What is Wave fronts? Discuss the types of wave fronts.

Q.2 How the secondary wavelets are created? Explain (or Explain Huygen's principal for secondary wavelets.

Q.3 What Interference phenomena of light? Discuss the types of it.

Q.4 Explain Young's double slit experiment for Interference phenomena of light.

Q.5 Differentiate between Interference and Diffraction phenomena of light.

Q.6 What is Diffraction phenomena of light? Discuss types of it.

Q.7 What Plan Diffraction grating? Derive the resultant amplitude formula for the same.

Long Questions:

Q.1 Derive the condition for maxima and minima intensity for the Interference phenomena of light.

Q.2 Derive the formula for fringes width in case of Interference phenomena of light.

Q.3 Prove that Bright fringes width and Dark fringes width will be equal.

Q.4 Derive the condition for principal maxima intensity, minimum intensity and secondary maxima intensity for the diffraction phenomena of light

Examples: (For practice)

1. In Young's double slit experiment the slits are 0.5 mm apart and interference is observed on a screen placed at a distance of 100 cm from the slit. It is found that the 9^{th} bright fringe is at a distance of 8.135 mm from the second dark fringe from the centre pattern. Find the wavelength of light used. (**Ans. 5890 A^0**)
2. Two coherent sources are placed 1mm apart and generate interference fringes on a screen 0.9 m away. The second dark fringe is formed at a distance of 0.9mm from the central fringe. Determine the wavelength of the monochromatic light used.
3. In a Young's double slit experiment , the slits are separated by 0.28 mm and the screen is placed 1.4 m away. The distance between the central bright fringe and the fourth bright fringe has been measured to be 1.2 cm. Determine the wavelength of light.

4. Two straight and narrow parallel slits located 1mm apart are illuminated using a monochromatic light source. A screen placed at a distance of 100 cm is used to obtain fringes. It is found that the distance between consecutive fringes is 0.5mm. Determine the wavelength of light.

Dual Nature of Light

<u>**Properties of Light**</u>

Interference: The phenomenon in which two waves support each other at some points and cancel at others.

Diffraction: The bending or spreading of waves around the edge of an opening or obstacle.

Polarization (of light): The limiting of the vibrations of light, usually to vibrations in one plane.

Photoelectric effect: The emission of electrons by a substance when illuminated by electromagnetic radiation.

Compton effect: The phenomenon in which a photon is scattered by an electron and the scattered photon has a frequency less than its original frequency.

Theories of light:

Several theories have been given to explain the properties of light. We will consider the following four theories.

1. **Newton's Corpuscular theory:**
   According to it light consists of streams of minute particles in motion.
2. **Huygen's wave theory:**
   According to it light travels from one place to another in the form of waves.
3. **Maxwell's Electromagnetic wave theory:**
   According to it light waves are electromagnetic in nature and they consist of an oscillating electric field and an oscillating magnetic field, both are perpendicular to each other and have the same frequency and phase.
4. **Quantum theory of light:**
   According to it light is carried from one place to another in wave packets called 'quanta' or 'photons', each having a definite energy and momentum.

Dual nature of light:

Our present view about the nature of light is that light possesses both wave and particle properties. Sometimes it behaves like waves and sometimes it behave like particles. However, both these behaviors cannot be studied simultaneously.

i) Interference, diffraction and polarization shows its wave nature and can be explained by classical wave theory.

ii) Photoelectric effect and Compton effect exhibit its particle nature and can be explained by quantum theory.

Particles and Waves Reflected by a Mirror

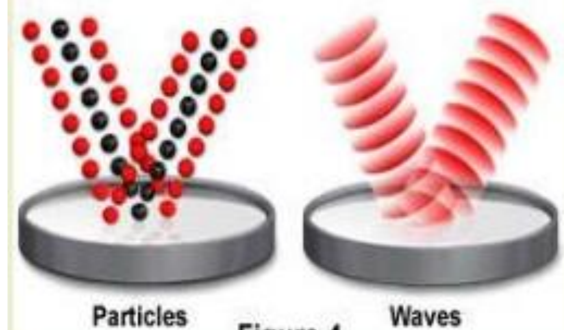


Figure 4

Particle theory is

Diffraction of Particles and Waves

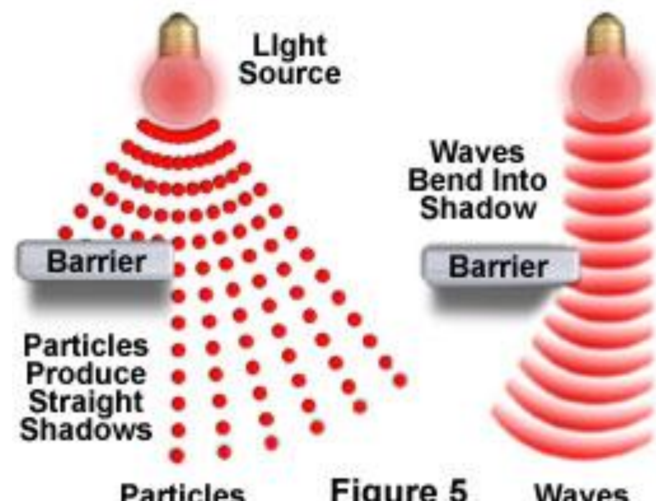


Figure 5

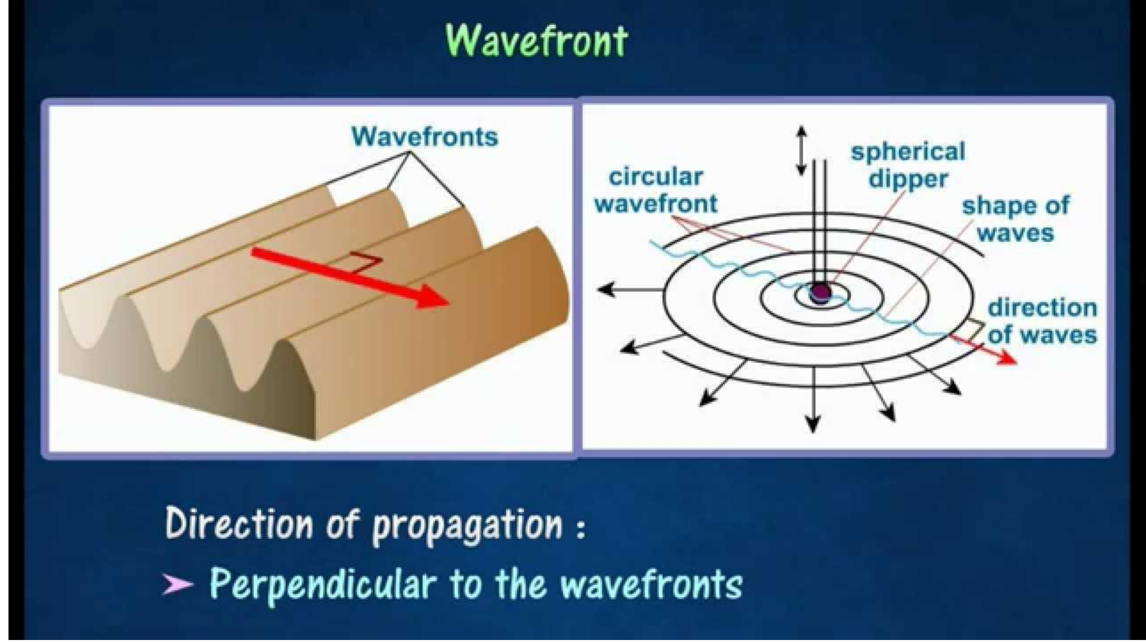
Wave front:

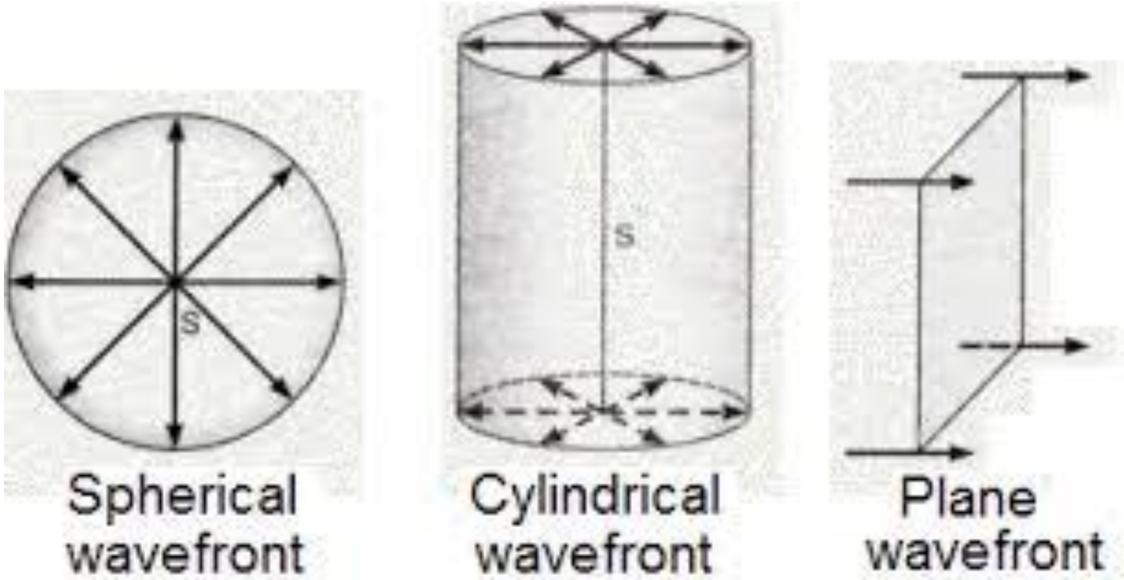
A point of source of light when placed in an isotropic medium emit light waves in all directions.

The disturbances will reach simultaneously to all particles lying on the surface of a sphere with the point source as the centre. Such a sphere is called a wavefront.

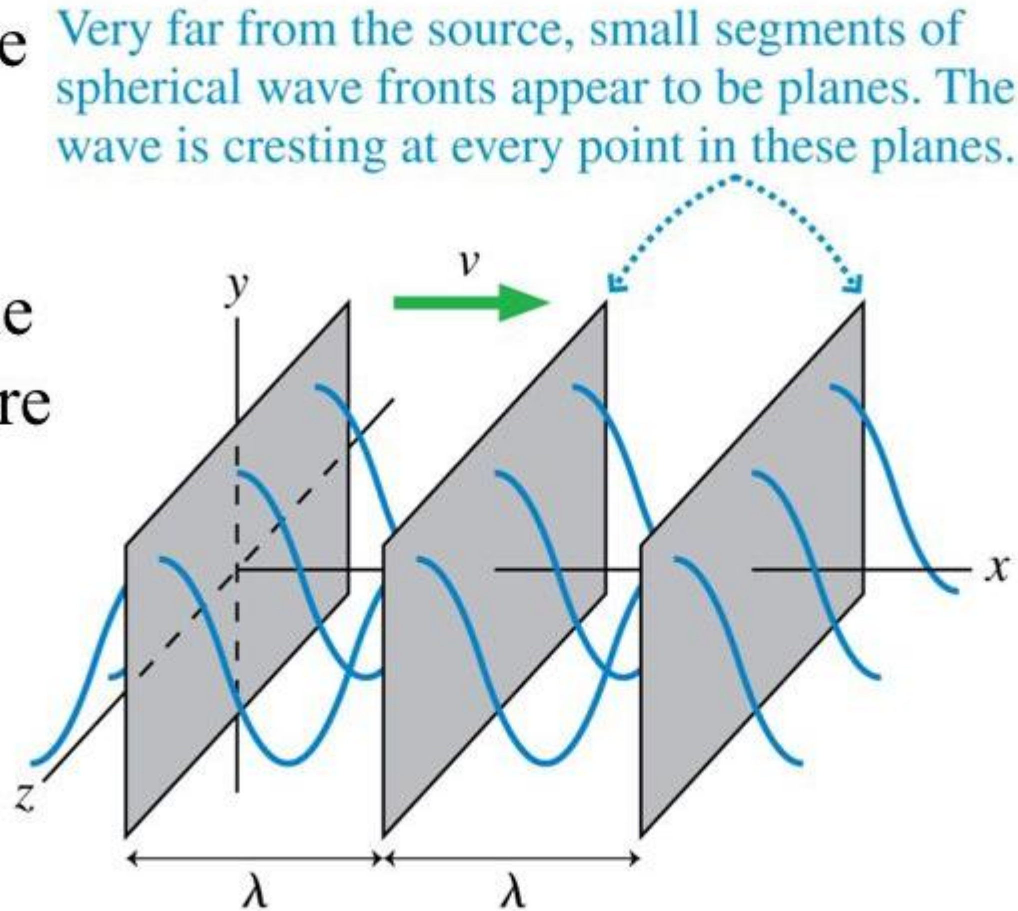
A continuous locus of all the neighbouring particles vibrating in the same phase,.

The shape of wave front depends upon the shape of the light source used.





- When you observe a wave far from its source, the small piece of the wave front is a little patch of the large sphere. The curvature of the sphere will be unnoticed, and the wave front will appear to be a plane.



- A **plane wave** describes observations of waves far from their source. The planes represent the crests of the spherical waves.

Huygens' Construction or Huygens' Principle of Secondary Wavelets:

(Wavelets - Red dots on the wavefront)

1. Each point on a wavefront acts as a fresh source of disturbance of light.
2. The new wavefront at any time later is obtained by taking the forward envelope of all the secondary wavelets at that time.

Note: Backward wavefront is rejected. Why?

Amplitude of secondary wavelet is proportional to ½ $(1+\cos\theta)$. Obviously, for the backward wavelet $\theta = 180°$ and $(1+\cos\theta)$ is 0.

Types of Interference:

It is divided into two class.

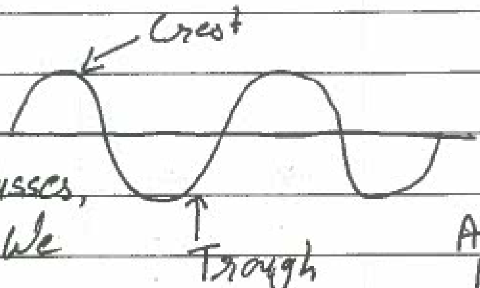
1. **Division of Wavefront** : The incident wavefront is divided into two parts by utilising the phenomenon like reflection, Refraction. Those parts of the same wave front travel unequal distances and reunite at some angle to produce interference bands.

2. **Division of Amplitude:** The amplitude of the incoming beam is divided into two parts either parallel reflection or refraction. These divided parts reunite after travelling different paths and produce interference. In this case it is not essential to employ a point on a narrow line source but a broad line source may be employed to produce bright bands.

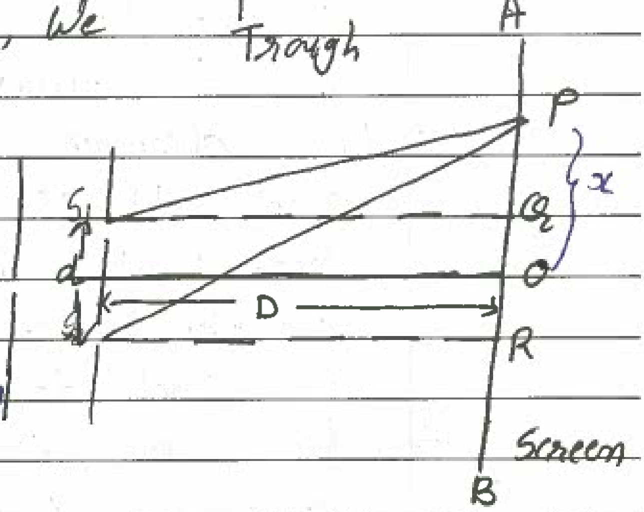
i.e. Newton’s Ring, Michelson Interferometer etc.

Interference:

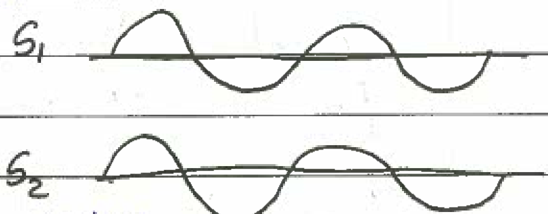
Q. Formation of Fringes:



When Light from the Slit S passes, then from Huygen's Principle, we get wave fronts.



→ The bold lines show the amplitude in positive directions (Crests) and dotted show the amplitude in negative direction (Troughs). These wavelets on entering into Slits S_1 & S_2 will look to be coming from two sources and the radii of these wave fronts increase as they ~~to~~ move away from S_1 & S_2 and so will superimpose on each other.



→ At points where crests fall on crests and Troughs fall on troughs the amplitudes add up and result is intensity increases ($I \propto A^2$) and at points where crests fall on troughs or troughs fall on crests, the amplitudes are reduced (the resultant amplitude is zero.) This is known as destructive interference. Thus, on the screen alternate bright and dark fringes region of equal width, called interference fringes are observed.

Theory of Fringes:

Let S be a monochromatic source of light and let S_1 and S_2 be double slits, equidistant from S. The waves on reaching S_1 and S_2 will have the same phase at all the times.

→ Let distance between S_1 and S_2 be d and D is the distance between ~~S_2~~ Screen AB from S_1 and S_2. The point O on the screen being at equal distance from S_1 and S_2 will have zero path difference.

→ Let us consider a point P at a distance x from the center O. We will consider the conditions for bright or dark fringes at this point.

From the right angled ΔS_1QP

$$S_1P^2 = S_1Q^2 + QP^2 \longrightarrow (1)$$

Similarly from right angled ΔS_2RP

$$S_2P^2 = S_2R^2 + RP^2 \longrightarrow (2)$$

From eq^n (1) & (2)

$$\cancel{S_1P^2 + S_2P^2 = S_1Q^2}$$

$$S_2P^2 - S_1P^2 = S_2R^2 + RP^2 - (S_1Q^2 + QP^2)$$

$$(S_2P+S_1P)(S_2P-S_1P) = \cancel{D^2} + \left(x+\frac{d}{2}\right)^2 - \cancel{D^2} - \left(x-\frac{d}{2}\right)^2$$

If S_1 & S_2 are very close to each other

So, $S_1P + S_2P = 2 \cdot D$

$$\therefore 2D\,(S_2P - S_1P) = \cancel{x^2} + 2x\frac{d}{2} + \frac{d^2}{4} - \cancel{x^2} + 2x\frac{d}{2} - \frac{d^2}{4}$$

$$\therefore 2D\,(S_2P - S_1P) = 4x\frac{d}{2}$$

$$\underbrace{S_2P - S_1P} = \frac{xd}{D}$$

$$\text{Path Difference} = \frac{x \cdot d}{D} \longrightarrow (3)$$

where, D is thousand times greater than x & d.

* Conditions for Maximum Intensity:

The Point P will be the center of maxima if the path difference is an integral multiple of λ because crests of one line will fall on crests of another line and troughs of one light will fall on troughs of other.

i.e. δ Path difference $= \dfrac{x \cdot d}{D} = n\lambda$

$$x = n\lambda \frac{D}{d} \longrightarrow (4)$$

Equation (4) gives the condition of for bright fringes. At O, the path difference is zero hence it will position of maxima. The next bright fringes are formed when n=1,2..

When $n=1$, $x_1 = \frac{D}{d}\cdot\lambda$

When $n=2$, $x_2 = \frac{D}{d}\cdot 2\lambda$

When $n=3$, $x_3 = \frac{D}{d}\cdot 3\lambda$

$\vdots$

When $n=n-1$, $x_{n-1} = \frac{D}{d}(n-1)\lambda$

When $n=n$, $x_n = \frac{D}{d}\, n\lambda$

Now $x_2 - x_1 = \frac{D}{d}\cdot 2\lambda - \frac{D}{d}\cdot\lambda$

$= \frac{D}{d}(2\lambda - \lambda)$

$x_2 - x_1 = \frac{D}{d}\cdot\lambda$

Same for $x_3 - x_2 = \frac{D}{d}\cdot\lambda$

$\vdots$

and

$x_n - x_{n-1} = \frac{D}{d}\cdot\lambda$

i.e. the distance between two consecutive bright fringes is constant.

★ **Conditions for dark fringes:**

The point P will be the center of darkness if the path difference is an odd multiple of $\lambda/2$

i.e. Path difference $= \frac{xd}{D} = (2n+1)\lambda/_2$

$$x = \frac{D}{d}(2n+1)\lambda/_2 \longrightarrow (5)$$

Equation (5) gives the distances of the dark fringes from point O.

then $n=1$ $\quad x_1 = \frac{D}{d}\frac{3\lambda}{2}$

" $\quad n=2, \quad x_2 = \frac{D}{d}\frac{5\lambda}{2}$

— — — — —

— — — — —

$n=(n-1) \quad x_{n-1} = \frac{D}{d}(2n-1)\lambda/_2$

$n=n, \quad x_n = \frac{D}{d}(2n+1)\frac{\lambda}{2}$

So We have for dark fringes

$$x_2 - x_1 = \frac{D}{d}\cdot\lambda$$

$$x_3 - x_2 = \frac{D}{d}\cdot\lambda$$

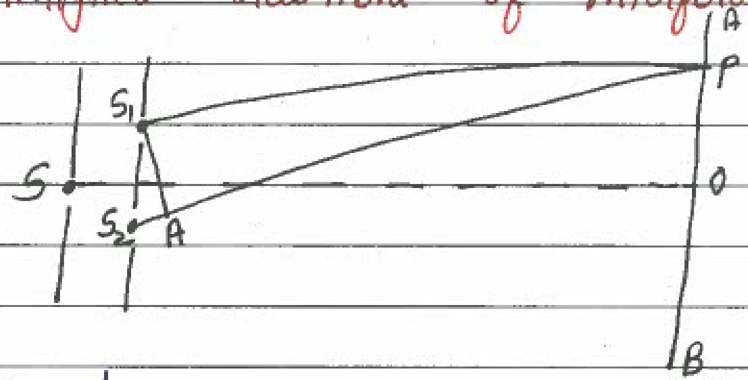
$$\vdots$$

$$x_n - x_{n-1} = \frac{D}{d}\cdot\lambda$$

Hence the distance between any two consecutive dark fringes or bright fringes is the same. This is expressed by $\boxed{\beta = \frac{D}{d}\cdot\lambda}$ is called ~~fringes width. β is directly pro~~

$$\boxed{\beta = \frac{D}{d} \cdot \lambda}$$ which is directly proportional to D and λ and inversely propartional to d.

Analytical treatment of Interference:



→ Consider a narrow slit S illuminated by a monochromatic source of light of w.l. λ. S_1 & S_2 be two parallel slits at equal distance from S. Two Sources S_1 and S_2 will act as Coherent Sources.

→ To find the intensity at any point P on the screen AB. Consider a general wave eqn

$$y_1 = a \sin \frac{2\pi}{\lambda} \cancel{v}t \longrightarrow (1)$$

y_1 is the displacement of a particle from its mean position at any time t, ~~v~~ the velocity of ~~par~~ light a and λ the amplitude and w.l. respectively

→ let y_2 be the displacement of wave reaching P from S_2 and x path difference w.r.to the first wave from S_1.

$$y_2 = a \sin \frac{2\pi}{\lambda} (vt + x) \longrightarrow (2)$$

When the two waves superimpose, the resultant amplitude,

$$y = y_1 + y_2$$

$$= a \sin \frac{2\pi}{\lambda} vt + a \sin \frac{2\pi}{\lambda}(vt + x)$$

$$\left[\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right)\right]$$

$$= 2a \sin \frac{2\pi}{\lambda}\left(vt + \frac{x}{2}\right) \cdot \cos\left(\frac{2\pi}{\lambda} \cdot \frac{x}{2}\right)$$

amplitude $A = 2 \cdot a \cos\left(\frac{2\pi}{\lambda} \cdot \frac{x}{2}\right) = 2a \cos\left(\frac{\pi x}{\lambda}\right) \longrightarrow (3)$

★ For amplitude to be minimum:

$$\cos\left(\frac{\pi x}{\lambda}\right) = 0$$

$$\frac{\pi}{\lambda} \cdot x = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots (2n+1)\frac{\pi}{2}$$

$\therefore \boxed{x = (2n+1)\frac{\lambda}{2}}$ shows Minimum I $\longrightarrow (4)$

When path difference between two waves is equal to an odd multiple of $\lambda/2$.

★ For amplitude to be Maximum:

$$\cos\left(\frac{\pi x}{\lambda}\right) = 1$$

$$\frac{\pi x}{\lambda} = 0, \pi, 2\pi \ldots n\pi$$

$\frac{\pi x}{\lambda} = n\pi \quad \therefore \boxed{x = n\lambda}$ shows Max^m $\longrightarrow (5)$

I if the path difference between two wave is an even multiple of $\lambda/2$.

From Eq^n(3) Resultant amplitude

$A = 2a \cos \delta/2$ where δ is the phase difference between two wave reaching P at any instant.

→ As corresponding to path difference of λ, Phase difference is 2π, so the phase difference for a path difference x is

$$\delta = \frac{2\pi}{\lambda} \cdot x = \frac{2\pi}{\lambda} \times \text{Path difference}$$

Intensity $I \propto A^2 \propto a^2 \cos^2 \delta/2$

$$I \propto \cos^2 \delta/2$$

(i) When phase diff. $= 0, \pm 2\pi, \pm 2(2\pi) \ldots \pm n(2\pi)$
as

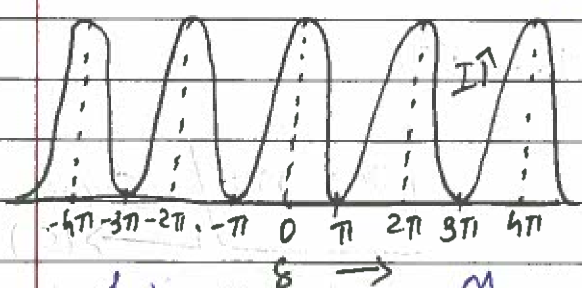
Path diff. $x = 0, \lambda, 2\lambda \ldots n\lambda$

$$\boxed{I = 4a^2}$$

(ii) When Phase diff. $\delta = \pm \pi, \pm 3\pi \ldots \pm (2n+1)\pi$
as

Path diff. $x = \lambda/2, 3\lambda/2 \cdots (2n+1)\lambda/2$

$\therefore A = 0$ $\boxed{\therefore I = 0}$



→ From the principle of Conservation of E, it can't be destroyed. It is transferred from one point of Minimum Intensity to Maximum Intensity. Due to interference it is $4a^2$ at bright point and 0 at dark points.

Interference:

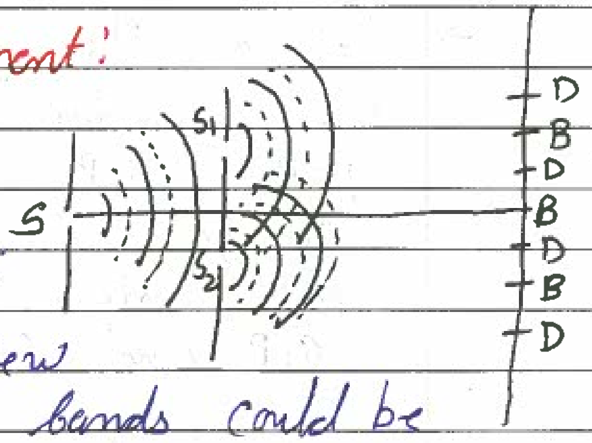
Introduction:

When light from two sources move in a direction then the light wave trains from these sources superimpose upon each other resulting in the modification of distribution of intensity i.e. Position of Maximum Intensity and Minimum Intensity. This modification of light energy due to super position of two or more wave trains are called Interference.

★ When the amplitudes of the two wave trains and there by increasing the intensity of resultant wave It is called Constructive interference.

★ When the amplitude of the two wave trains and there by ~~d~~ decreasing the intensity of resultant wave It is called destructive interference.

★ Young's two slit Experiment:



→ Light was allowed to fall on S and through double slit S_1 & S_2. The Interference pattern was observed on screen. where few coloured bright and dark bands could be seen.

→ If white light is replaced by monochromatic light and double slits are very fine narrow and close to each other so that the dark and bright fringes are observed at equal distance

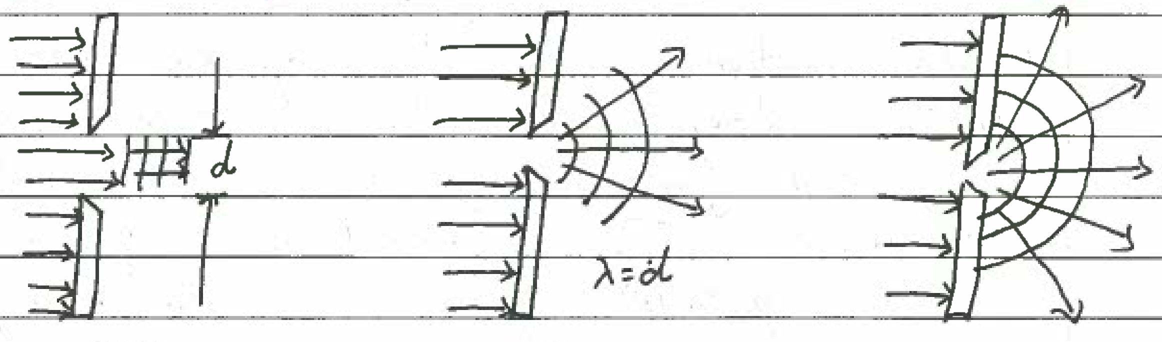
Diffraction:

It is a matter of common experience that the path of light entering a dark room through a hole in the window illuminated by sunlight is straight. But it has been observed that when a beam of light passes through a small opening, it spreads to some extent into the region of the geometrical shadow also.

"The bending of waves around an abstacle and deviation from a recitilinear path is called Diffraction."

When waves pass near an abstacle (barrier), they tend to bend around the edges of the obstacles.

Dependance of the Phenomenon on wavelength.



(a) (b) (c)

→ Figure shows the passage of waves through an opening. When the opening is large compared to the wavelength, the waves do not bend round the edges.

→ When the opening is small, the bending effect round the edges is noticeable.

→ When the opening is very small, the waves spread over the entire surface behind the opening.

→ The opening acts as an independent source of waves, which propagates in all directions. The diffraction effect is observable quite close to the opening when the size of the opening is very small.

→ In general, Diffraction of Light waves become noticeable only when the size of the obstacle is comparable to a wavelength of Light.

★ Two kind of Diffraction:

1) Fresnel's Diffraction

Source and screen are placed at finite distance from the aperture of obstacle having sharp edges. In this case no lenses are used for making the rays parallel or convergent. The incident wave front are either parallel spherical or cylindrical.

2) Fraunhofer's diffraction:

In this case of Diffraction Source and Screen or telescope are placed at infinity or effectively at infinity. The wavefront which is incident on the aperture or obstacle is plane.

★ Difference between Interference and Diffraction:

1) In The Interference Phenomenon, the interaction takes place between two sperate wave fronts originating from two coherent sources while in the phenomenon of Diffraction, the interaction takes place between the secondary wavelets originating from different points of the exposed parts of the same wavefront.

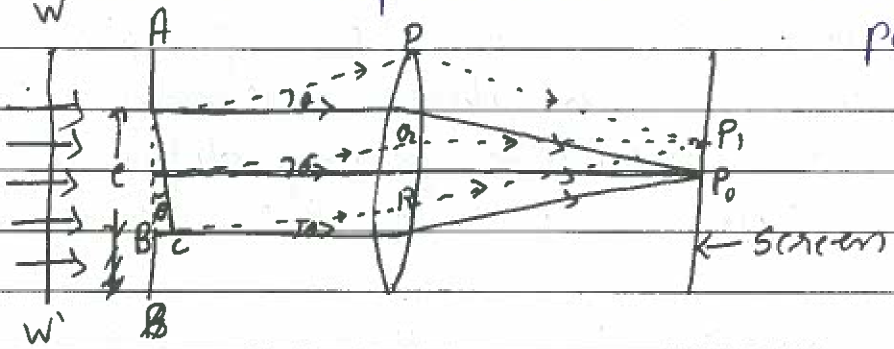
2) In Interference pattern the region of minimum intensity are usually dark perfectly while it is not so in diffraction pattern.

3) The widths of the fringes in interference may or may not be equal or uniform while in diffraction pattern fringes width are never equal.

4) In an Interference pattern all the maxima are of same intensity but in a diffraction pattern they are of varying intensity.

★ Fraunhofer Diffraction at Single Slit:

Figure represents a section AB of a narrow slit of width e perpendicular to the plane of the paper.



→ Let a plane wavefront WW' of monochromatic light of wavelength λ propagating normally to the slit be incident on it. Let the diffracted light be focused by means of a convex lens.

→ According to the Huygen Huygens principle, every point of the wavefront in the plane of the slit is a source of secondary spherical wavelets, which spread out to the right in all directions.

→ The secondary wavelets travelling normally to the slit i.e., along OP_0 are brought to ~~focus at~~ focus at P_0 by the lens. Thus P_0 is a bright central image.

→ The point P_1 is the minimum intensity depending upon the path difference between the secondary waves which travells at an angle θ w.r.to normal.

→ To find the Intensity at P_1, draw a perpendicular AC on BR.

The path difference between secondary wavelets from A and B in direction θ

$$= BC = AB\sin\theta = e\sin\theta$$

Corresponding phase difference $= \frac{2\pi}{\lambda} \cdot$ Path diff

$$= \frac{2\pi}{\lambda} e\sin\theta$$

Let us consider that the width of slit is divided into n equal parts and amplitude of the wave from each part is a (Each part is same). The phase difference between any two consecutive waves from these parts would be

$$\frac{1}{n}(\text{total Phase}) = \frac{1}{n}\left(\frac{2\pi}{\lambda} e\sin\theta\right) = d \ (\text{say})$$

Using the method of vector of addition of amplitude ~~as~~ ~~discussed~~ ~~in~~ ~~the~~, the Resultant amplitude R is given by

$$R = a\frac{\sin(nd/2)}{\sin(d/2)}$$

$$= a\frac{\sin(\pi e\sin\theta/\lambda)}{\sin(\pi e\sin\theta/n\lambda)}$$

$$\therefore \alpha = \pi e\sin\theta/\lambda$$

$$\therefore R = a\frac{\sin\alpha}{\sin\alpha/n}$$

If $n \to \infty$, $\alpha/n \to 0$

$$\therefore \sin\alpha/n = \alpha/n$$

$$R = a \frac{\sin\alpha}{\alpha/n}$$

$$R = na \frac{\sin\alpha}{\alpha}$$

Where na will remain constant
$na = A$.

Now Intensity is given by

$$I = R^2 = A^2 \left(\frac{\sin\alpha}{\alpha}\right)^2$$

* Principal Maximum :

R can be written in ascending power of α as

$$R = \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + \dots\right]$$

$$A\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + \dots\right]$$

If R is Max^m Negative term vanish,
$\alpha = 0$.

$$\therefore \alpha = \frac{\pi e \sin\theta}{\lambda} = 0 \qquad \therefore \sin\theta = 0 \text{ or } \theta = 0.$$

Maximum value of R is A and intensity is proportional to A^2. The condition $\theta = 0$ means that this Maximum is formed by those secondary wavelets which travel normally to the slit.

★ Minimum Intensity Positions.

The ~~Mi~~ Intensity will be minimum when $\sin\alpha = 0$

$$\therefore \alpha = \pm\pi, \pm 2\pi, \pm 3\pi \ldots \pm m\pi$$

or $$\frac{e\pi \sin\theta}{\lambda} = \pm m\pi$$

$$e\sin\theta = \pm m\lambda \quad \text{where, } m = 1, 2, 3 \ldots$$

In this way we obtain the points of minimum intensity on either side of the principal maxima.

→ The value of ~~m~~ $m=0$ is not admissible, because for this value $\theta = 0$ and it is corresponds to principal maxima.

† ---- cont. on other page

★ Plane Diffraction Grating:

A Diffraction grating is an extremely useful device and it consits of very Large number of narrow slits placed side by side The slits are separated by opaque surfaces. When a wavefront is incident on a grating surface, light is transmitted by the slits and is obstructed by the opaque portions. Such grating is called Plane transmission grating.

★ Theory of Plane Diffraction grating:

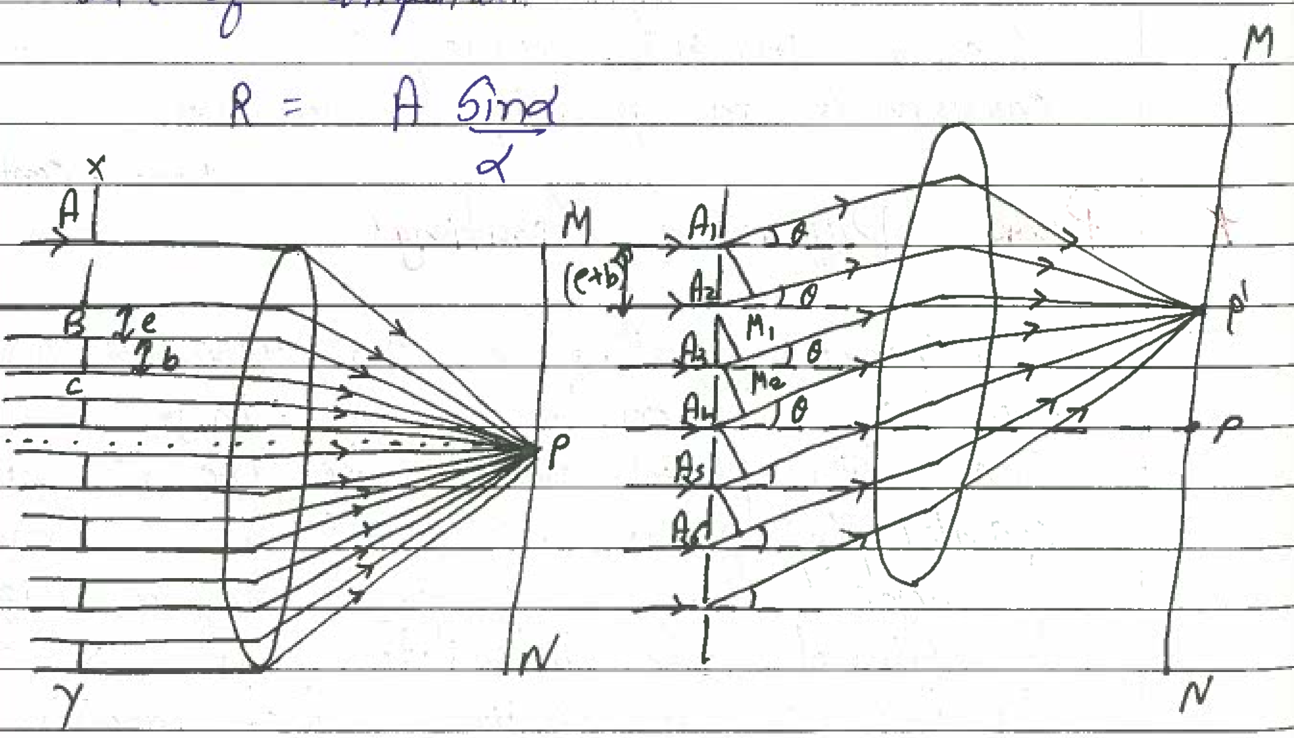
→ It is a N slit arrangement. Plane Wavefront of monochromatic light be incident normally on N-parallel slits of the grating. Each point of the wave sends out secondary wavelengths in all directions.

→ Let 'e' be the width of the slit and 'b' be the separation between any two consecutive slits, then $(e+b)$ is known as the grating element.

→ The diffracted rays from each slit is focussed at a point P' on the screen MN by a convex lens L.

→ Let $A_1, A_2, A_3 \ldots$ be the middle point of each slit and A_1M_1, $A_2M_2 \ldots A_{n-1}M_{n-1}$ be the perpendicular drawn. The wave diffracted from each slit is equivalent to a single wave of amplitude

$$R = \frac{A \sin\alpha}{\alpha}$$



The Path difference between the waves from A_1 & A_2 is

$$A_2M_1 = (e+b)\sin\theta$$

The Path difference between the waves from A_2 and A_3 is

$$A_3M_2 = (e+b)\sin\theta$$

Hence the path difference between the waves from A_{n-1} and A_n is

$$A_nM_{n-1} = (e+b)\sin\theta$$

Thus, the net path difference between consecutive waves is equal to $(e+b)\sin\theta$.

So, the corresponding Phase difference

$$= \frac{2\pi}{\lambda}(e+b)\sin\theta = 2\beta \text{ (say)}$$

The Resultant amplitude at P' is the resultant amplitude of N waves, each amplitude R and common phase difference (2β).

The Resultant amplitude at P' is given by

$$R' = \frac{R\sin\frac{2N\beta}{2}}{\sin\frac{2\beta}{2}} = \frac{R\sin N\beta}{\sin\beta} = \frac{A\sin\alpha}{\alpha}\frac{\sin N\beta}{\sin\beta}$$

Therefore, the resultant intensity at P' is

$$I = R^2 = A^2\frac{\sin^2\alpha}{\alpha^2}\cdot\frac{\sin^2 N\beta}{\sin^2\beta}$$

The factor $\boxed{\frac{A^2\sin^2\alpha}{\alpha^2}}$ gives intensity pattern for single slit and $\boxed{\frac{\sin^2 N\beta}{\sin^2\beta}}$ gives the intensity pattern for N-slit.

Continuation of Fraunhofer diffraction:.......

Secondary Maxima:

There are weak secondary maxima between equally spaced minima. The positions can be obtained with the rule of finding maxima and minima of a given function in calculus.

→ Differentiating the expression of I with respect to α and equating to zero,

$$dI/d\alpha = d/d\alpha \left[A^2 \left(\frac{\sin\alpha}{\alpha}\right)^2 \right] = 0.$$

$$\cancel{A^2 \left[\frac{1}{\alpha^2} 2\sin\alpha\cos\alpha - \sin^2\alpha \frac{2\alpha}{\alpha^3} \right]} = 0.$$

$$A^2 \; \frac{2\sin\alpha}{\alpha} \left[\frac{\alpha\cos\alpha}{\alpha^2} - \sin\alpha \right.$$

$$\cancel{A^3} = A^2 \, d/d\alpha \left[\frac{\sin^2\alpha}{\alpha^2} \right] = 0$$

$$A^2 \left[\frac{1}{\alpha^2} 2\sin\alpha\cos\alpha - \frac{2\sin^2\alpha}{\alpha^3} \right] = 0$$

$$A^2 \frac{2\sin\alpha}{\alpha} \left[\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} \right] = 0.$$

either $\sin\alpha = 0$ $\qquad$ $\alpha\cos\alpha - \sin\alpha = 0$

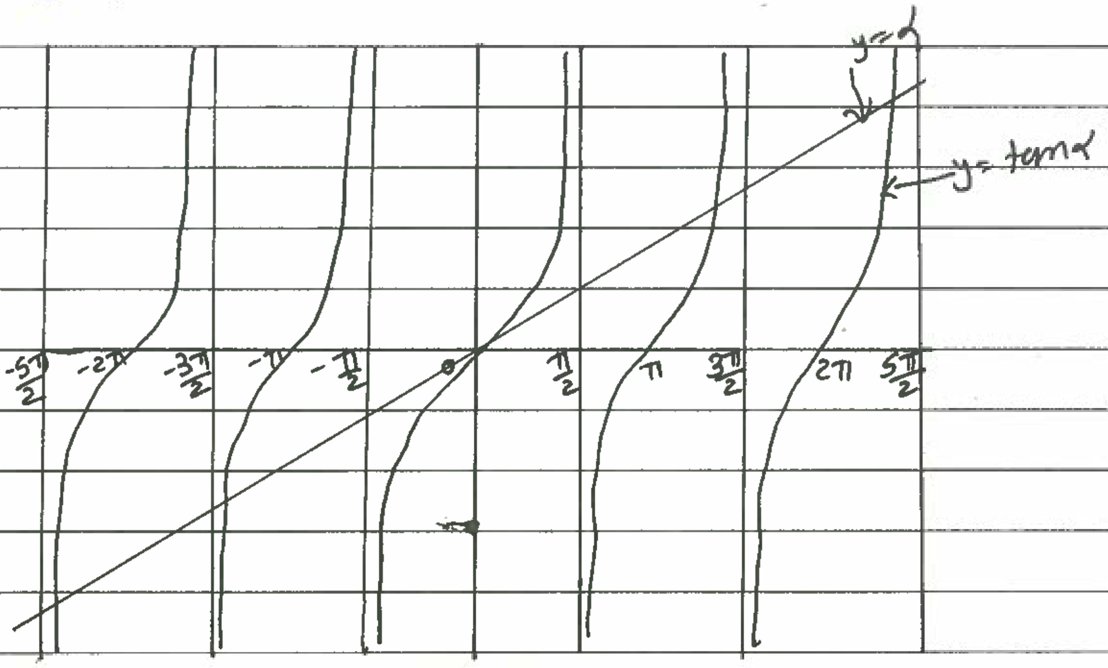
↓

gives minimum I.

$\alpha \cos\alpha = \sin\alpha$

$\tan\alpha = \alpha$

Value of α satisfying the above eq^n are obtained graphically by plotting the curves $y = \alpha$ and $y = \tan\alpha$



The points of intersections are

$$\alpha = 0, \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2}, \text{etc}$$

$\alpha = 0$ gives central maxima $\rightarrow$ Corresponding $I = I_0$

$\alpha = \pm\frac{3\pi}{2}$ gives secondary maxima $\rightarrow$ Intensity I_1

$$I_1 = A^2 \left[\frac{\sin(\frac{3\pi}{2})}{(3\pi/2)}\right]^2$$

$\alpha = \pm\frac{5\pi}{2}$ gives Maxima point $\rightarrow$ Corresponding I_2

$$I_2 = A^2 \left[\frac{\sin(\frac{5\pi}{2})}{5\pi/2}\right]^2$$

$\therefore$ $I_0 > I_1 > I_2 \ldots$ etc