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1 Introduction

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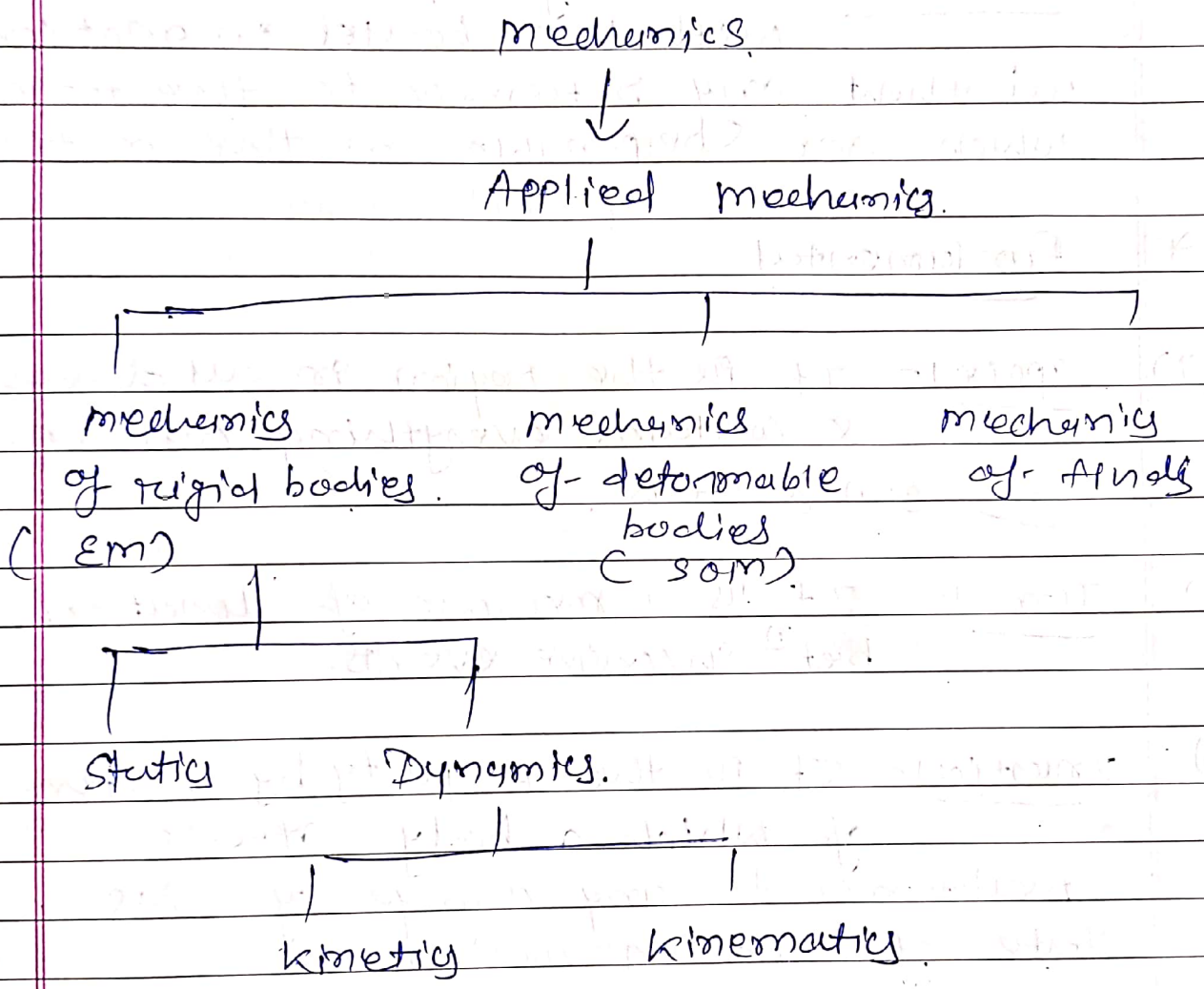
Mechanics :- It is the branch of science which deals with the state of rest or motion of particles and bodies under the action of forces.



Statics :- branch of RM, which deals with the forces & their effects, while acting upon the bodies at rest.

Dynamics :- branch of RM, which deals with the forces & their effects, while acting upon the bodies in motion.

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Kinetics :- branch of dynamics, which deals with the bodies in motion due to the app. of forces, (considering mass & force) torque, Ex. gravity etc.

Kinematics :- branch of dynamics, which deals with the bodies in motion without any reference to the forces which are responsible for the motion (displacement, time, velocity)

how much change?

* Fundamentals

(1) Space :- It is the region in all directions & contains everything within. Ex. sun & stars.

(2) Time :- It is a measure of duration bet^n successive events.

(3) Inertia :- It is the property by virtue of which a body offers resistance to any change of its state of rest or motion.

(4) Mass :- The quantity of matter contained by a body is called mass of the body.

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* Fundamental principles of mechanics:-

(1) parallelogram law of forces

– "If two forces, acting simultaneously on a particle be represented, in magnitude & direction, by the two adjacent sides of a parallelogram, their resultant may be represented in magnitude & dirn by the diagonal of the parallelogram which passes through their point of intersection."

(2) principle of Transmissibility

– "If a force acts at any point on a rigid, it may also be considered to act at any other point on its line of action, provided the point is rigidly connected with the body."

(3) principle of superposition of forces:-

– If two equal, opposite & colinear forces are added to or removed from the system of forces, there will be no change in the position of the body.

(4) Newton's First law of motion

– "Every body continues in a state of rest or uniform motion unless it is compelled to change that state by some external force."

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(5) Newton's second law of motion.

- The acceleration of body is proportional to the impressed force and takes place in the dirn in which the forces acts.

$$F \propto a \quad \therefore F = ma.$$

(6) Newton's Third law of motion.

- For every action there is always an equal and opposite reaction.

* Scalar quantities.

- these quantities which have magnitude only such as length, mass, time, distn, volume, density, temp, speed etc.

* Vector quantities.

- these quantities which have both magnitude & dirn such as force, displacement, velocity, acceleration, momentum etc.

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Coplanar forces.

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* **force** :- It is an agent which produces or tends to produce, destroys or tends to destroy motion of body is called force.

→ unit → (N) → vector.
1 kgf = 9.81 N.

* **characteristics of force.**

(1) **magnitude** :- My of force may be. 10 kN, 50 kN etc.

(2) **Direction** :- towards North etc.

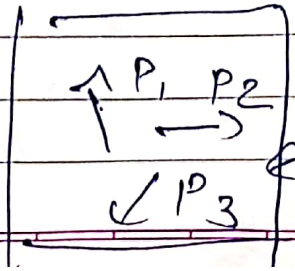
(3) **Nature** :- Tensile or compressive
(pull) (push)

(4) **point of application** :- The point at which the force acts on the body.

* **System of forces** :-

– when two or more forces act on body they are called system of forces.

(1) **Coplanar force** :- forces whose line of action lie on the same plane is called....



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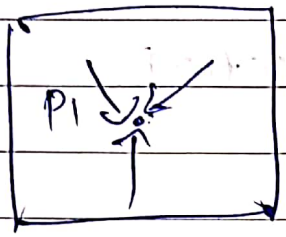
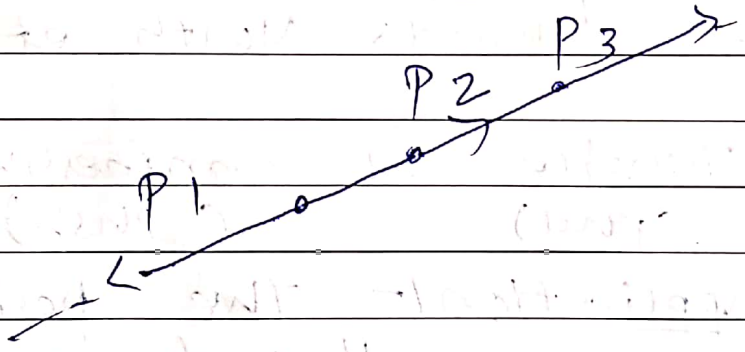
(2) Concurrent forces :-

- forces which meet at one point...



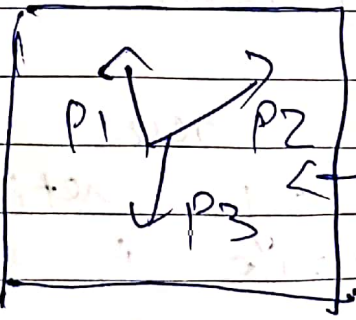
(3) Colinear forces :-

- forces whose lines of action lie on the same line are called...



(4) Coplanar Concurrent forces :-

- forces which meet at one point & their lines of action also lie on the same plane are known as ---



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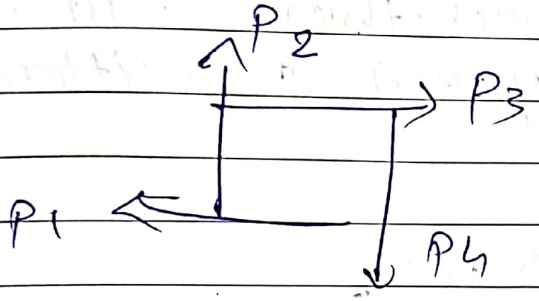
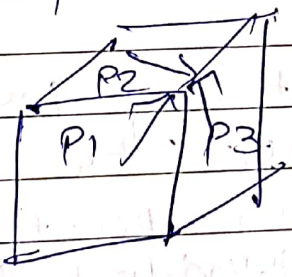
(5) Coplanar Non-concurrent forces:

- forces whose lines of action lie on the same plane but they don't meet at one point are known as....



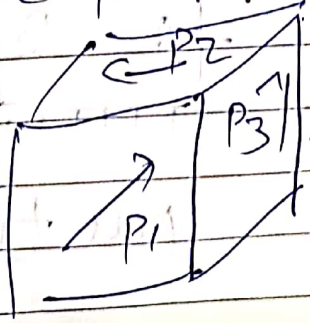
(6) Non-coplanar Concurrent forces

- forces whose lines of action don't lie on the same plane, but they meet at one point are called....



(7) Non-coplanar non concurrent forces.

- forces whose lines of action don't lie on the same plane & they don't meet at one point are known as....



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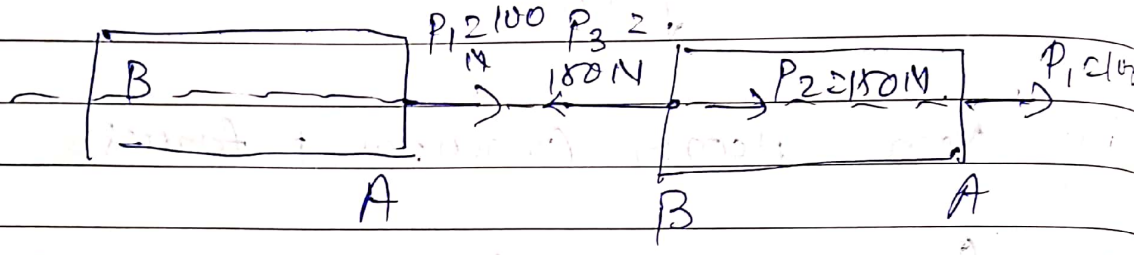
* Principle of superposition of forces.

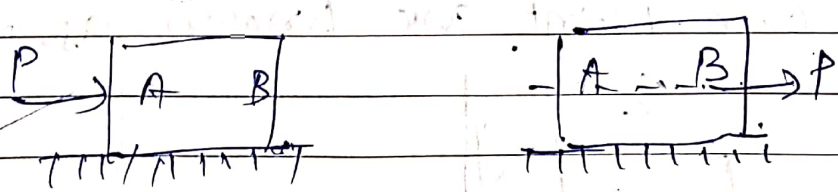
→ "If two equal, opposite & collinear forces are added to or removed from the system of forces, there will be no change in the system & position of the body."



* Principal of Transmissibility of forces.

→ "If a force act at any point on rigid body it may also be considered to act at any other point on its line of action, provided the point is rigidly connected with the body."

Note:- Valid only for rigid body only.



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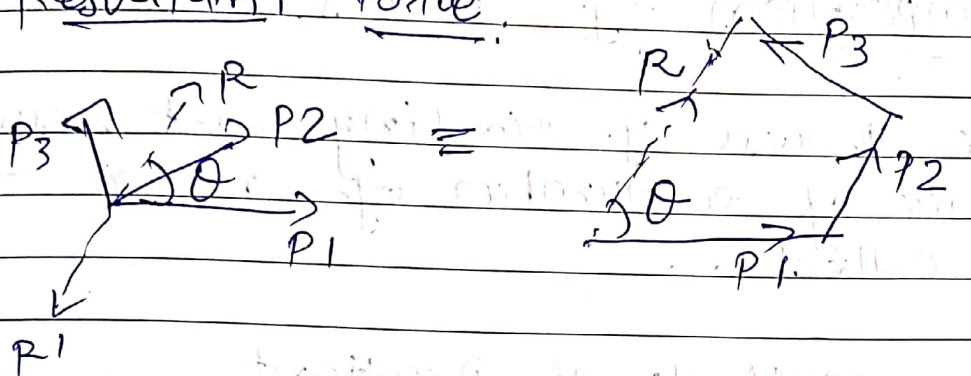
* Resultant force



- If no. of forces P_1, P_2, P_3, P_4 .. etc are acting simultaneously on a particle, it is possible to find out a single force which could replace them. i.e. which would produce the same effect as produced by all the given force. This single force, capable of producing same effect on body is known as resultant forces.

* principle of equilibrium

- To balance the resultant force, a force of same magnitude but of opposite direction is required. This opposite balancing force is called equilibrium force.

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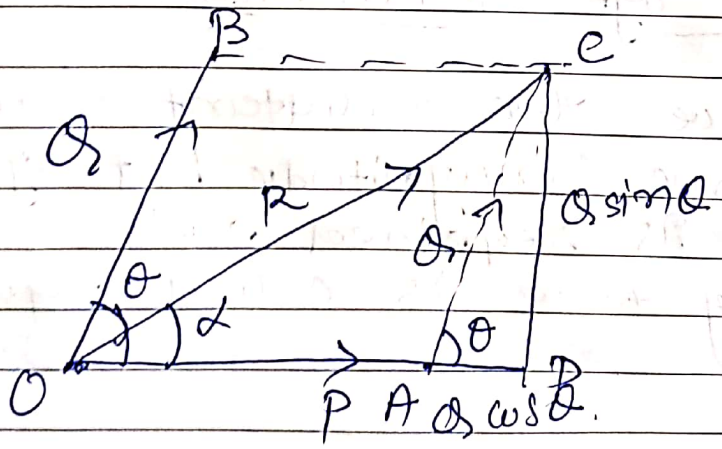
* Composition of forces.

- The process of finding out resultant force of a number of given forces is called ...

Methods of Resultant force

- Analytical
  1. parallelogram
  2. Resolution
  3. Triangle
- Graphical
  1. Triangle law
  2. polygon law.

(1) Parallelogram law of forces.



$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$$

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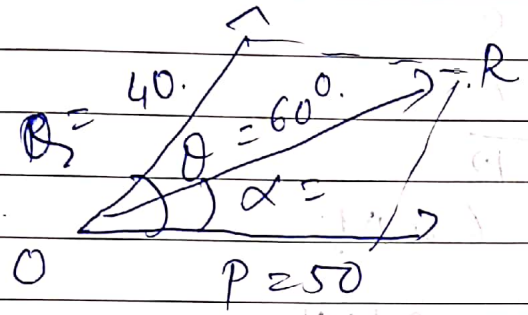
Ex :- Two tensile forces 50 kN & 40 kN acting at a point with angle 60° between them.



$R = 78.10$ kN

$\tan \alpha = 0.495 \quad \alpha = 26.33^\circ$

Ex :- find the magnitude of two forces such that if they act at right angle, their resultant is $\sqrt{40}$ kN. But they act at 60°, their resultant is $\sqrt{52}$ kN.

Case - 1

$\theta = 90^\circ$

$R^2 = 40.$

$R^2 = P^2 + Q^2 + 2PQ \cos\theta$ ↑ Put $\theta = 90^\circ$

(1) $40 = P^2 + Q^2$

Case - 2

$\theta = 60$

$R^2 = 52.$

$P^2 + Q^2 = 40.$ & $\theta = 60^\circ$

$52 = 40 + PQ$

$PQ = 12$

$\therefore (P^2 + Q^2 + 2PQ) = (P+Q)^2$

$\therefore 40 + 2 \times 12 = (P+Q)^2$

$\therefore P + Q = 8$

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$$(P-Q)^2 = P^2+Q^2-2PQ$$

$$= 16.$$

$$P-Q = 4$$

$$\therefore \quad P+Q = 8$$

$$P-Q = 4$$

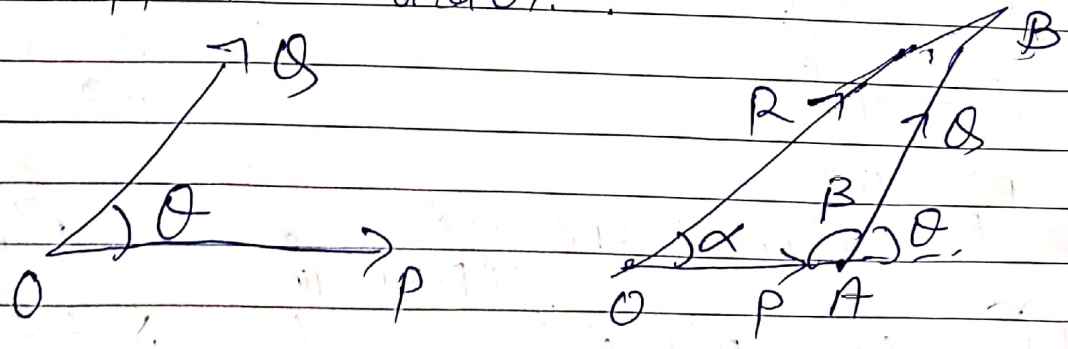
$$2P = 12$$

$$P = 6\,kN.$$

$$\therefore \quad Q = 2\,kN.$$

(2) Triangle law of forces.

→ "If two forces acting at a point be represented in magnitude & direction by two sides of a triangle taken in order, then resultant may be represented in magnitude & dirⁿ by the third side of triangle, taken in opposite order."



$$R = \sqrt{P^2+Q^2-2PQ\cos\beta}$$

$$\therefore \frac{\sin\alpha}{\sin\beta} = \frac{Q}{R} \qquad \alpha = \sin^{-1}\left(\frac{Q}{R}\sin\beta\right)$$

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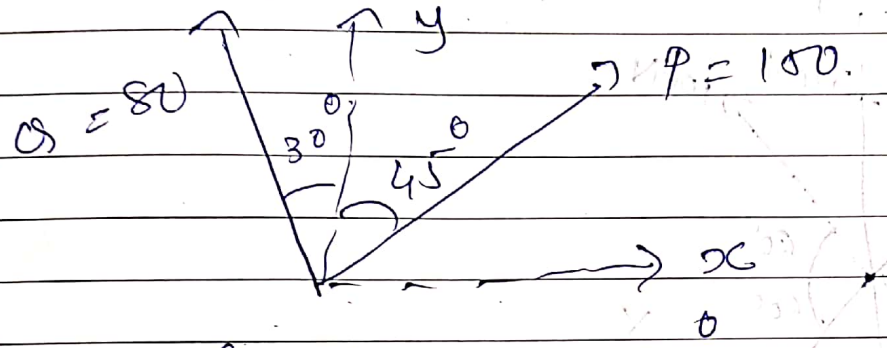
Ex =:

Two forces P & Q are acting at point O as shown in fig. Determine magnitude & dirn of the resultant using triangle law of forces.



$\theta = 30 + 45 = 75^\circ$

$\beta = 180 - \theta = 105^\circ$

$R = 187.38$ kN

$\therefore \frac{R}{\sin\beta} = \frac{Q}{\sin\alpha}$

$\therefore \sin\alpha = 0.412$

$\alpha = 24.33^\circ$

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③ Resolution of a forces.

→ The process of splitting up the given force in to two or more components, in particular dirn, without changing effect on the body is called resolution of a force.

$\Sigma H = 0$, $\Sigma V = 0$, & $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

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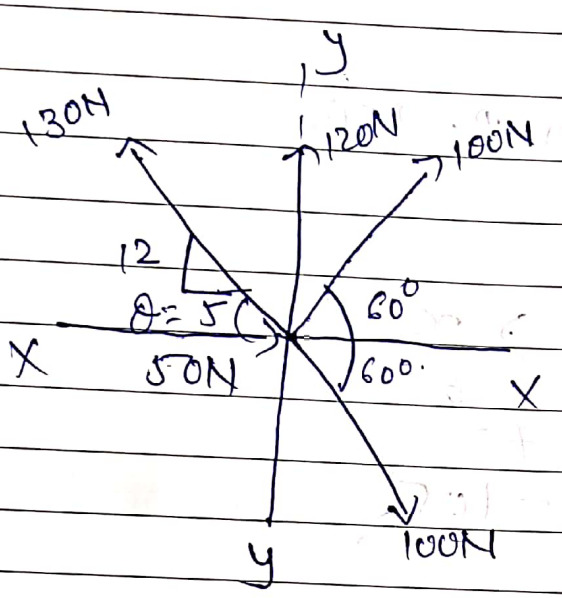
Ex :- Determine magnitude & dirn of resultant force of the force system as shown in fig.



$$\tan\theta = \frac{12}{5} = 2.4$$

$$\theta = 67.38^\circ$$

$$\Sigma H = 50\cos 0^\circ + 100\cos 60^\circ + 120\cos 90^\circ + 130^\circ \cos 112.62^\circ + 100\cos 300^\circ$$

$$= 100\,N\ (\rightarrow)$$

$$\Sigma V = 50\sin\theta + 100\sin 60^\circ + 120\sin 90 + 130\sin 112.62^\circ + 100\sin 300^\circ$$

$$= 240\,N\ (\uparrow)$$

$$R = \sqrt{(100)^2 + (240)^2}$$

$$= 260\,N$$

$$\tan\theta = \left|\frac{\Sigma V}{\Sigma H}\right| = \frac{240}{100} = 2.40$$

$\theta = 67.38^\circ$ which is in 1st quadrant (+,+)

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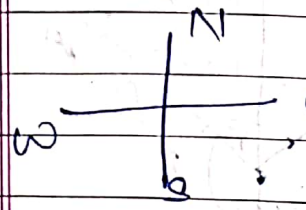
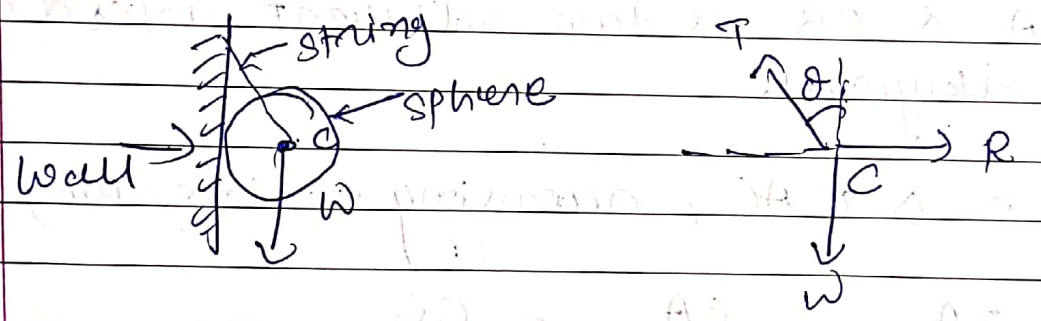
& also, dir^{n} forces may asked in exam.



* Free body diagram

- for a given body, if supports are removed and forces acting on the body & support reactions are shown, the diagram so obtained is known as FBD.



FBD

* Lami's theorem

- "If three coplanar forces acting at point be in equilibrium, then each force is proportional to the sine of angle bet^{n} the other two."

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

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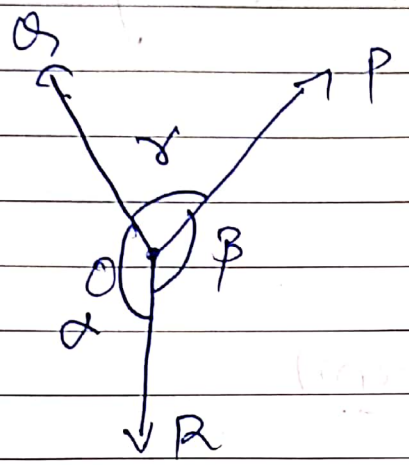

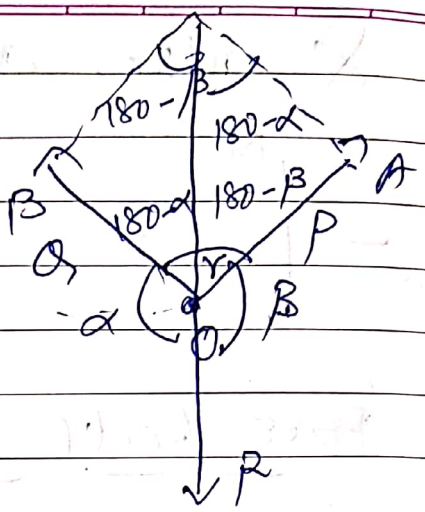

draw two lines OA = force P &
OB = force Q.
complete the parallelogram OACB with
OA & OB as two adjacent sides & ocer
a diagonal.

In △ OAC, according to sine rule,

$$\frac{OA}{\sin\angle ACO} = \frac{CA}{\sin\angle COA} = \frac{OC}{\sin\angle OAC} \quad \text{——(1)}$$

$\angle ACO = 180^\circ - \alpha$,
$\angle COA = 180^\circ - \beta$,
$\angle OAC = 180^\circ - \angle ACO - \angle COA$
$= 180^\circ - (180-\alpha) - (180^\circ - \beta)$
$= \alpha + \beta - 180^\circ$
$= (360^\circ - \gamma) - 180^\circ$
$= 180^\circ - \gamma$

$$\frac{OA}{\sin(180^\circ-\alpha)} = \frac{CA}{\sin(180^\circ-\beta)} = \frac{OC}{\sin(180^\circ-\gamma)}$$

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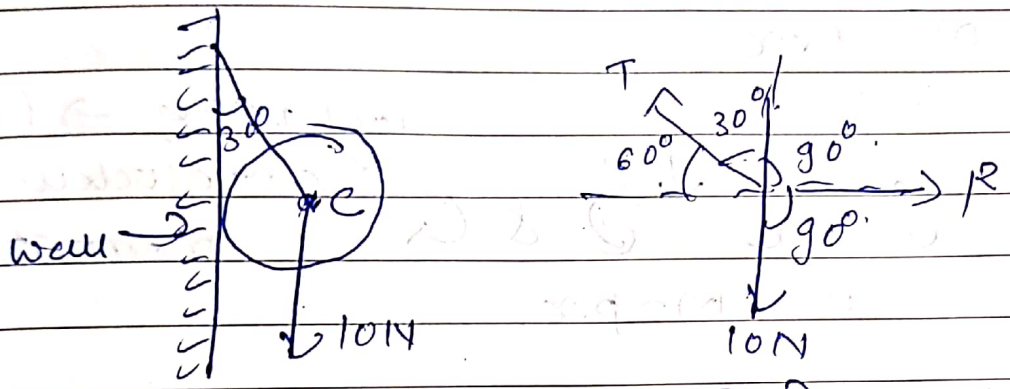
$$\therefore \frac{OA}{\sin\alpha} = \frac{CA}{\sin\beta} = \frac{OC}{\sin\gamma}$$

$$\therefore \boxed{\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}}$$

Ex.



FBD.

$$\frac{T}{\sin 90} = \frac{R}{\sin 150} = \frac{10}{\sin 120}.$$

$$\therefore \frac{T}{1} = \frac{R}{0.5} = \frac{10}{0.866}$$

$$\therefore T = \frac{1 \times 10}{0.866} = 11.55\,N.$$

$$R = \frac{0.5 \times 10}{0.866} = 5.77\,N.$$

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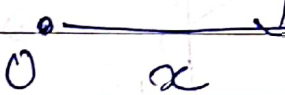
Non-Coplanar forces.

- Moment of force :- product of force & perpendicular dist^{n} of line of action of force from the point about which the moment is req^{d}.

$$m = p \times x.$$

clockwise ⇒ (+) & anticlockwise moment (−)



$\therefore m = p.x.$

- Couple.

Two equal & opposite forces whose lines of action are diff^{n} form a couple.

- Varignon's principle of moments.

or Law of moments.

- "If a no. of coplanar forces are acting simultaneously on a particle, the algebric sum of the moments of all the forces about any point is equal to the moment of their resultant force about the same point."

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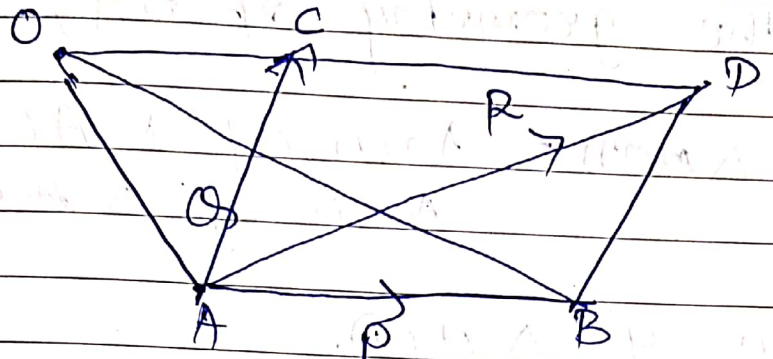

- Consider two forces P & Q acting at a point A represented in magnitude & dirn by AB & AC shown in fig. below.
- let O be the point, about which the moments are taken. Through O, draw a line CD parallel to the dirn of force P, to meet the line of action of Q at C. Now with AB & AC as two adjacent sides, complete parallelogram ABDC. Join the diagonal AD of the parallelogram and OA and OB. Diagonal AD represents in magnitude & dirn the resultant of two forces P and Q.

Now, taking moment of forces P, Q & R about O.

moment of force P, about O,
= 2 x Area of triangle AOB —(1)

Similarly, moment of force Q, about O,
= 2 x Area of triangle AOC —(2)

moment of Resultant R, about O,
= 2 x Area of triangle AOD —(3)

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From, the geometry of Fig,

Area of ΔAOD = Area of ΔAOC + Area of ΔAeD.

but area of ΔAeD.
= Area of ΔAOB
= Area of ΔAOB.

$\therefore$ (∵ two triangles AOB & AOB one on the same base AB and between the same parallel lines).

$\therefore$ Area of ΔAOD = Area of ΔAOC + Area of ΔAOB.

multiplying both sides by 2,
2 × Area of ΔAOD = 2 × Area of ΔAOC + 2 × Area of ΔAOB.

$\therefore$ moment of force R about O
= moment of force P about O
+ moment of force Q about O.

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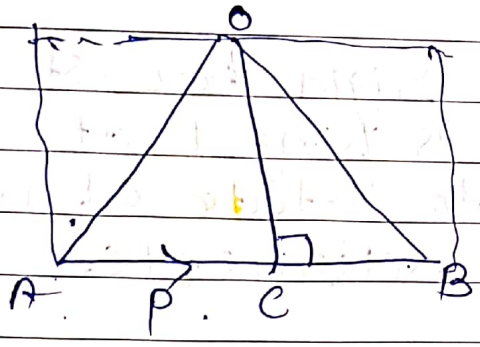
* Graphical Representation of moment.



$(\Delta ex\ OG) \times 2$

Consider a force P represented, in magnitude & dirn, by the line AB, let O be point, about which the moment of this force is req^{d} to be found out, as shown in fig. From O, draw OC perpendicular to AB, Join OA & OB.

Now, moment of force P about O,

$= P \times OC = AB \times OC$.

but $AB \times OC$ = twice the area of triangle AOB,

moment of force, about any point = twice the area of triangle, whose base is line to some scale representing the force & whose vertex is the point about which the moment is taken.

* Applications of moments

(1) position of the resultant force.

(2) Levers.

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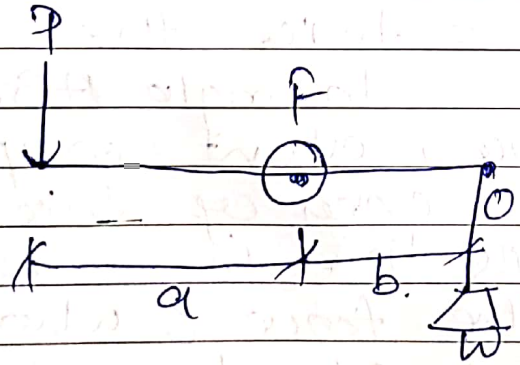
* Levers

- A lever is a rigid bar (straight, curved or bent) & is hinged at one point. It is free to rotate about the hinge. Ex pair of scissors, etc.

* Types of levers

(1) simple levers
(2) Compound levers.

* Simple levers



A lever which is consists of bar having one hinge is known as simple levers

P = load applied
W = weight lifted
a = length bet^n hinge & effort
b = " " & weight

Now taking moment of the effort & F & equating the same.

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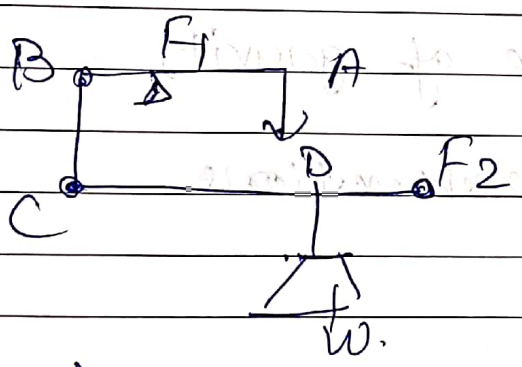
$P.a = W.b$

$\therefore \frac{W}{P} = \frac{a}{b}$ → to lift a heavier load with the app of the same force.

- Term W/P & a/b known as mechanical advantantage & leverage.
- If increase the mechanical advantage either length of the lever arm (a) is to be increased or length of the load arm (b) is to reduced.

* Compound levers

- A lever which consists of a no. of simple lever is called Compound lever.



Ex (9) by R.S Khurmi