Unit-II

(*) Electromagnetic Waves in a Conducting Medium

For Conducting medium (i.e. sea water)

$(\bar{J} \propto \bar{E})$ $\quad J = \sigma E$ $\quad (\sigma =$ conductivity$)$

In Simple Conducting medium, (charge density zero) Maxwell's Equations are in form:

① $\nabla \cdot \bar{E} = \rho/\varepsilon$

② $\nabla \times \bar{E} = -\dfrac{\partial \bar{B}}{\partial t}$

③ $\nabla \cdot \bar{B} = 0$

④ $\nabla \times \bar{B} = \mu\sigma\bar{E} + \mu\varepsilon\dfrac{\partial \bar{E}}{\partial t}$

Equation of Continuity

$$\nabla \cdot \bar{J} = -\frac{\partial \rho}{\partial t}$$

$$\frac{\partial \rho}{\partial t} = -\sigma(\nabla \cdot \bar{E}) = -\sigma\rho/\varepsilon$$

$$\rho = \rho_0\, e^{-(\sigma/\varepsilon)t} \quad \text{—— ①}$$

(general solution)

...y conductor, Conductivity ...decreases Exponentially with time

$\nabla \cdot \bar{E} = 0$

$\nabla \times \bar{E} = -\dfrac{\partial \bar{B}}{\partial t}$

$\nabla \cdot \bar{B} = 0$

$\nabla \times B = \mu\varepsilon\dfrac{\partial \bar{E}}{\partial t} + \mu\sigma\bar{E}$

(Electric fields)

$$\nabla^2 \bar{E} = \mu\varepsilon\frac{\partial^2 \bar{E}}{\partial t^2} + \mu\sigma\frac{\partial \bar{E}}{\partial t}$$

$$\nabla^2 \bar{B} = \mu\varepsilon\frac{\partial^2 \bar{B}}{\partial t^2} + \mu\sigma\frac{\partial \bar{B}}{\partial t}$$

(Magnetic fields)

External E & B,

$$\bar{E}_{Ext} = E_0\, e^{i(kx-\omega t)}$$

$$\bar{B}_{Ext} = B_0\, e^{-i(kx-\omega t)}$$

$$k^2 = \omega^2\mu\varepsilon + i\mu\sigma\omega$$

If $\sigma = 0$, $k = \omega\sqrt{\mu\varepsilon}$

$$k = \gamma + i\delta$$

where, $$\gamma = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2+1\right]$$

$$\& \ \delta = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\varepsilon\mu}\right)^2}-1\right]$$

$$x = 1/\delta$$

Boundary Condition

Two Types : $\sigma << \omega\varepsilon$
$\sigma >> \omega\varepsilon$

Rectangular Waveguides:-

(Transverse Electric)
TE wave

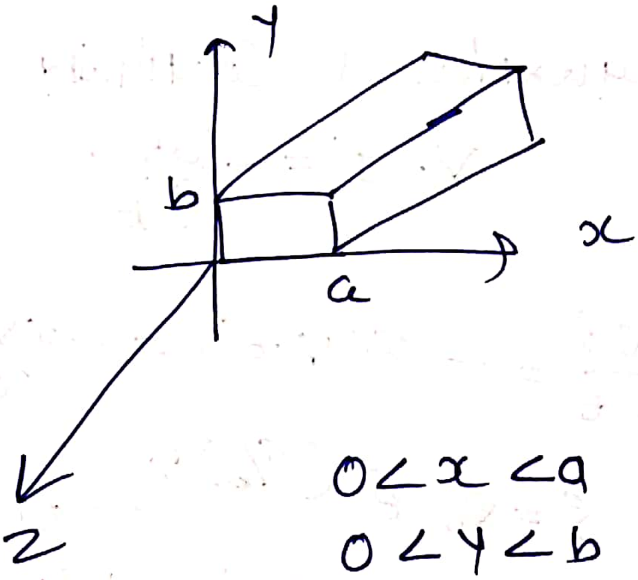
$E = (E_x, E_y, 0)$

$H = (H_x, H_y, H_z)$

(Transverse Magnetic)
TM wave

$E = (E_x, E_y, E_z)$

$H = (H_x, H_y, 0)$



$0 < x < a$
$0 < y < b$
$\& \ a > b$

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TE Wave

$E_z = 0$

$$\nabla^2 H_z + k^2 H_z = 0$$

$$\frac{\partial H_z}{\partial x}(0,y,z) = \frac{\partial H_z}{\partial x}(a,y,z) = \frac{\partial H_z}{\partial y}(x,a,z)$$
$$= \frac{\partial H_z}{\partial y}(x,b,z) = 0$$

where, $k = \omega/c$ = wave number

$$H_z(x,y,z) = h_{mn} \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-jk_z z}$$

when $m = 0, 1, 2, ...$ but $(m,n) \neq (0,0)$
$n = 0, 1, 2, ...$

Spatial components are

$$E_x = \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-jk_z z}$$

$$E_y = \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-jk_z z}$$

$$H_x = \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{jk_z z}$$

$$H_y = \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{jk_z z}$$

Each of this components satisfy Helmholtz equation

k_z is z-component of wave vector, & frequency is given by,

$$k_z = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

This mean m & n values such that

If

$$k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2 > 0$$

or, $$f > \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

when k_z is real & TE_{mn} mode is propagating

NOTE:- TE_{mn}

EM corresponding to (m, n) is called TE_{mn} mode

If

$$k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2 < 0$$

$$f < \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

k_z is Imaginal & TE_{mn} is non-propagal mode

Cut-off frequency for TE_{mn} is given by

$$f_{c\,mn} = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

TM wave

$$\nabla^2 E_z + k^2 E_z = 0$$

$$E_z(0,y,z) = E_z(a,y,z) = E_z(x,0,z) = E_z(x,b,z)$$

$K = \omega/c$ = wave number

Infinitely Many Solution is given by

$$E_{zmn}(x,y,z) = e_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-jk_z z}$$

Spatial Components are same as TE Wave.

given frequency k_z for TE Mode is same as TM Mode

$$k_z = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

Cut-off frequency for TM wave

$$f_{cmn} = \frac{c}{2\pi} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

Example

If $a = 0.3\,m$ & $b = 0.15\,m$, find the lowest cut-off frequency of TM_{11} Mode.

→ Cut-off frequency

$$f_c = \frac{c}{2\pi} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

TM_{11} means $m=1, n=1$

$$= \frac{3 \times 10^8}{2\pi} \sqrt{\left(\frac{1.\pi}{0.3}\right)^2 + \left(\frac{1.\pi}{0.15}\right)^2}$$

$$= 1118\ MHz$$