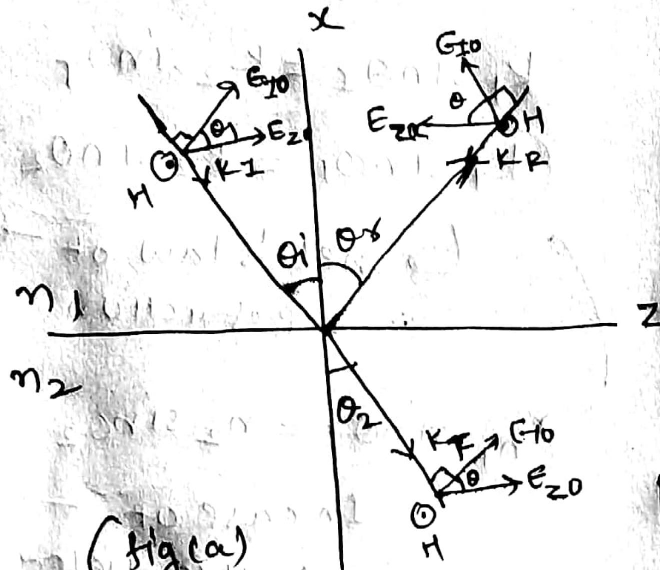
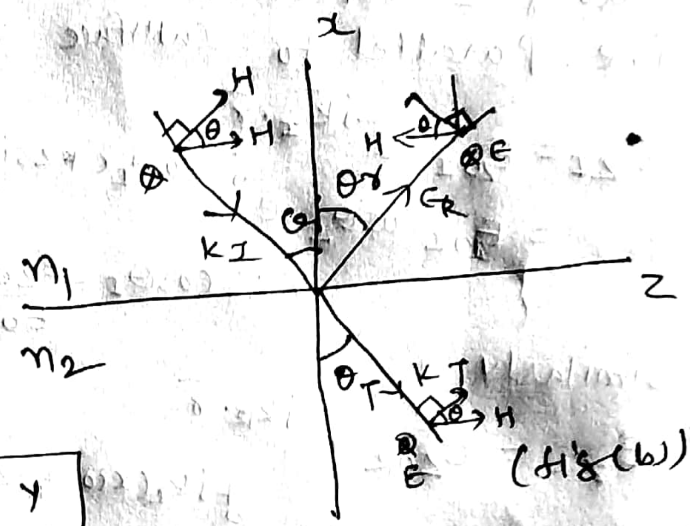
Plane wave Reflection at oblique Incidence:

1. Relation between Incident (θ_i), Reflected (θ_r) and transmitted angle (θ_T)

2. To determine reflection & Transmission Coefficient



(fig(a))



(fig(b))

$\otimes$ y

Parallel Polarization

E in plane of incidence

(Transverse magnetic)

$\boxed{\text{P - Polarization}}$

E ∥ plane of incidence

H ⊥ plane of incidence

$$E_I = E_{0I}\, e^{-j\bar{k}_I\cdot\bar{r}}$$

$$E_R = E_{0R}\, e^{-j\bar{k}_R\cdot\bar{r}}$$

$$E_T = E_{0T}\, e^{-j\bar{k}_T\cdot\bar{r}}$$

where, k is poynting vector

Perpendicular polarization

E perpendicular of plane of incidence

(Transverse Electric)

$\boxed{\text{S - polarisation}}$

E ⊥ plane of Incidence

H ∥ plane of Incidence

$\bar{k}_I = k_I \cos\theta_I \hat{i} + k_I \sin\theta_I \hat{k}$

$\bar{k}_R = -k_R \cos\theta_R \hat{i} + k_I \sin\theta_R \hat{j}$

$\bar{k}_T = k_T \cos\theta_T \hat{i} + k_T \sin\theta_T \hat{j}$

& $\bar{r} = x\hat{i} + z\hat{k}$

Boundary conditions
Projecting E components along z-direction i.e. parallel to Interface

$$E_{zI} = E_{z0I}\, e^{-j\bar{k}_I \cdot \bar{r}}$$
$$= E_{0I} \cos\theta_I\, e^{-jk_I(\cos\theta_I x + z\sin\theta_I)}$$
$$\left(\because \cos\theta_I = \frac{E_{z0I}}{E_{0I}}\right)$$

Similarly,

$$E_{zR} = E_{z0R}\, e^{-j\bar{k}_R \cdot \bar{r}}$$
$$= E_{0R} \cos\theta_R\, e^{+jk_R(x\cos\theta_R - z\sin\theta_R)}$$

$$E_{zT} = E_{z0T} \cdot e^{-j\bar{k}_T \cdot \bar{r}}$$
$$= E_{0z} \cos\theta_T\, e^{jk_T(x\cos\theta_T + z\sin\theta_T)}$$

Boundary Condition for Tangential Electric field

$$E_{zI} + E_{zR} = E_{zT} \quad (\text{at } x = 0)$$

$$E_{0I}\cos\theta_I\, e^{-jk_I z \sin\theta_I} + E_{0R}\cos\theta_R\, e^{jk_R z\sin\theta_R} = E_{0T}\cos\theta_T\, e^{-jk_T z\sin\theta_T}$$

Independent of z

To make above eqn hold for all value of all phase terms equa

$$k_I z \sin\theta_I = k_R z \sin\theta_R$$
$$= k_T z \sin\theta_T$$

But $k_I = k_R = k_1$
& $k_T = k_2$

$$k_1 \sin\theta_I = k_2 \sin\theta_T$$

$$\boxed{k_1 \sin\theta_1 = k_2 \sin\theta_2}$$

↳ snell's law of Refraction — (*)

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

↳ In case of non magnetic dielectr — (**)

$$k = \frac{\omega}{v} = \frac{n\omega}{c}$$

$k_1 = n_1\omega/c$ & $k_2 = n_2\omega/c$

NOTE: Material with different permeability & permittivity

$$k_1 = \left(\frac{\omega}{c}\right)\sqrt{\mu_1 \epsilon_1}$$

$$k_2 = \left(\frac{\omega}{c}\right)\sqrt{\mu_2 \epsilon_2}$$

Eqn (**) shows relation between θ_i, θ_r & θ_T

Now, Consider Boundary Condition, (3)

requires Tangential Continuity of $\bar{H}$

at $(x=0)$ ↳ all Negative y-direction

$$H_{0I} + H_{0R} = H_{0T}$$

Using Eqn

$$E_{0I}\cos\theta_I \, e^{-jk_I z \sin\theta_I} + E_{0R}\cos\theta_R \, e^{-jk_R z \sin\theta_R} = E_{0T}\cos\theta_T \, e^{-jk_T z \sin\theta_T}$$

$\because \theta_R = \theta_i$

So, we have

$$E_{0I}\cos\theta_I + E_{0R}\cos\theta_R = E_{0T}\cos\theta_T$$

$(\theta_I \approx \theta_R = \theta_1$, $\theta_T = \theta_2)$

$$E_{0I}\cos\theta_1 + E_{0R}\cos\theta_2 = E_{0T}\cos\theta_2 \quad \text{--- (1)}$$

& $H_{0I} + H_{0R} = H_{0T}$

$$\therefore \frac{E_{0I}}{\eta_1} - \frac{E_{0R}}{\eta_1} = \frac{E_{0T}}{\eta_2} \quad \text{--- (2)}$$

$\because$ Intrinsic Impedance $\eta_1 = \frac{E_{0I_1}}{H_{0I_1}}$, $\eta_2 = \frac{E_{0I_2}}{H_{0I_2}}$

Incase of Reflective wave we use negative sign

Solve Eqn (1) & (2) (1) + (2)

$$\frac{1}{\eta_1}E_{0I}\cos\theta_1 + \frac{1}{\eta_1}\cancel{E_{0R}\cos\theta_1} = \frac{1}{\eta_1}E_{0T}\cos\theta_2$$

$$+ \quad \cos\theta_1\frac{E_{0I}}{\eta_1} - \cancel{\frac{E_{0R}}{\eta_1}\cos\theta_1} = \frac{E_{0T}}{\eta_2}\cos\theta_1$$

$$2\cos\theta_1\frac{E_{0I}}{\eta_1} = E_{0T}\left(\frac{\cos\theta_2}{\eta_1} + \frac{\cos\theta_1}{\eta_2}\right) \quad \text{--- (3)}$$

1 − 2

$$\left(E_{0I}\cos\theta_1 + E_{0R}\cos\theta_2 = E_{0T}\cos\theta_2\right)\frac{1}{n_1}$$

$$\left(\frac{E_{0I}}{n_1} - \frac{E_{0R}}{n_1} = \frac{E_{0T}}{n_2}\right)\cos\theta_1$$

$$-\quad + \quad -$$

$$2\cos\theta_1 \frac{E_{0R}}{n_1} = E_{0T}\left(\frac{\cos\theta_2}{n_1} - \frac{\cos\theta_1}{n_2}\right) \quad \text{--- (4)}$$

Take Ratio of 4 & 3

$$\frac{E_{0R}}{E_{0I}} = \frac{n_2\cos\theta_2 - n_1\cos\theta_1}{n_2\cos\theta_2 + n_1\cos\theta} = R(\bar{\gamma}) = \Gamma_P$$

$\hookrightarrow$ Reflection coefficient

$$= \frac{n_{2P} - n_{1P}}{n_{2P} + n_{1P}}$$

Now from Eq^n 1 & 2

$$E_{0I}\cos\theta_1 + E_{0R}\cos\theta_1 = E_{0T}\cos\theta_2$$

$$\left(\because E_{0R} = E_{0I} - \frac{n_1}{n_2}E_{0T}\right) \text{ from Eq}^n \text{ 2}$$

$$\therefore E_{0I}\cos\theta_1 + \left(E_{0I} - \frac{n_1}{n_2}E_{0T}\right)\cos\theta_1 = E_{0T}\cos\theta_2$$

$$2E_{0I}\cos\theta_1 = E_{T0}\left[\cos\theta_2 + \frac{n_1}{n_2}\cos\theta_1\right]$$

$$= E_{T0}\left[\frac{n_2\cos\theta_2 + n_1\cos\theta_1}{n_2}\right]$$

$$\therefore \frac{E_{T0}}{E_{0I}} = \frac{2n_2\cos\theta_1}{n_1\cos\theta_1 + n_2\cos\theta_2}\left(\frac{\cos\theta_2}{\cos\theta_2}\right)$$

$$\therefore \frac{E_{TO}}{E_{OI}} = \frac{2\eta_{2p}}{\eta_{2p}+\eta_{1p}}\left(\frac{\cos\theta_1}{\cos\theta_2}\right) = \underbrace{\tau_p}_{\text{Transmission Coefficient}}$$

where $\eta_{2p} = \eta_2 \cos\theta_2$

$\eta_{1p} = \eta_1 \cos\theta_1$.

For s-polarization (fig b)

Similarly we can write

$$\Gamma_s = \frac{E_{OR}}{E_{OI}} = \frac{\eta_{2s}-\eta_{1s}}{\eta_{2s}+\eta_{1s}}$$

$$\tau_s = \frac{E_{OT}}{E_{OI}} = \frac{2\eta_{2s}}{\eta_{2s}+\eta_{1s}}$$

Effective impedence for s-polarization are

$\eta_{1s} = \eta_1 \sec\theta_1$

$\eta_{2s} = \eta_2 \sec\theta_2$

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Total Internal Reflection:-

When light strikes an Angle of incidence greater than critical Angle,

- All the light energy will be Reflected back
- None of them will be absorbed.

Critical Angle is related to the refractive index of the two media is given by

$$\theta_c = \sin^{-1}(n_2/n_1)$$

From Snell's second law,

$$n_1 \sin\theta_i = n_2 \sin\theta_2$$

θ_i = incident Angle

θ_t = Transmitted angle

$$\sin\theta_1 = \frac{n_2}{n_1}\sin\theta_2$$

Case: I If $\theta_i = \theta_c$, $\theta_t = \pi/2$

If we consider non-magnetic material $(\mu_1 = \mu_2 = 1)$

$$\sin\theta_i = \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}\sin\theta_2$$

$$\sin\theta_c = \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}\sin\pi/2 = \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}$$

$$\theta_c = \sin^{-1}\left(\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\right)$$

$$= \sin^{-1}\left(\sqrt{\frac{\mu_0\mu_{r_2}\varepsilon_0\varepsilon_{r_2}}{\mu_0\mu_{r_1}\varepsilon_0\varepsilon_{r_1}}}\right)$$

$$= \sin^{-1}\left(\sqrt{\frac{\mu_{r_2}\varepsilon_{r_2}}{\mu_r\varepsilon_{r_1}}}\right) = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

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Case: II $\theta_i > \theta_c$

For dielectric medium

$$\mu_{r1} = \mu_{r2} = 1 \quad , \quad \epsilon_1 > \epsilon_2$$

$$\sin\theta_t = \frac{\sqrt{\mu_1 \epsilon_1}}{\sqrt{\mu_2 \epsilon_2}} \sin\theta_i > 1$$

$$\cos\theta_t = \pm\sqrt{1 - \sin^2\theta_t}$$ $\cos\theta_t$ should be Imaginary

Now,

$$\bar{S} = \bar{E} \times \bar{H}$$

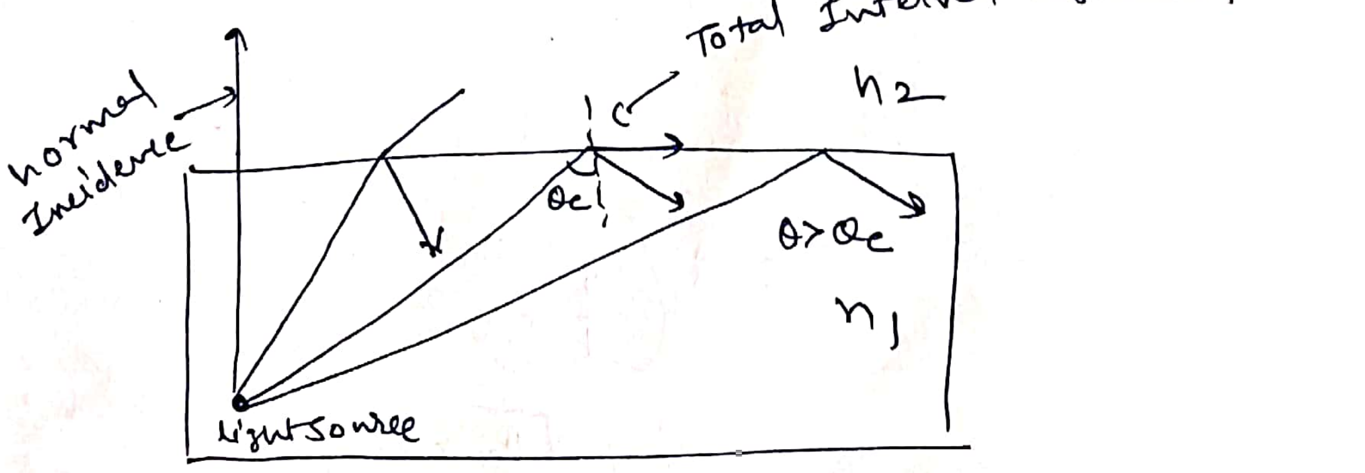
$$= \frac{|E_0|^2}{\eta_2} e^{-jzk_2\cos\theta_t} \sin\theta_t \, \hat{x} + \hat{z} \frac{|E_0|^2}{\eta_2} e^{-jzk_2\cos\theta_t} \cos\theta_t^*$$

For $z > 0$, $\cos\theta_t$ is Imaginary Number
$\cos\theta_t^*$ is purily Imaginary

So, second term is Imaginary, No real power is flowing along z-axis in Region II

- First Term has direction along x-axis & real & is Exponentially decaying

- Hence, when $\theta_i > \theta_c$ from Region I that will be total Internally reflected in Region I



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Example :-

If $\eta_1 = 100\ \Omega$, $\eta_2 = 300\ \Omega$, $E_{in} = 100$ V/m
then Calculate values for the incident, reflected & Transmitted waves

→ Reflection coefficient at normal Incidence is given by

$$\frac{E_R}{E_I} = \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{300-100}{300+100} = 0.5$$

$$E_{Ref} = \left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right) E_I$$

$$\therefore \quad E_{Ref} = 0.5 \times 100 = 50 \text{ V/m}$$

Transmission coefficient

$$\tau = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{E_T}{E_I}$$

$$\tau = \frac{2 \times 300}{300 + 100} = \frac{6\cancel{00}}{4\cancel{00}} = 3/2$$

$$E_T = \frac{3}{\cancel{2}} \times \overset{50}{\cancel{100}} = \; = 150 \text{ V/m}$$

Electric Intensities are : $E_I = 100$ V/m
$E_R = 50$ V/m
$E_T = 150$ V/m

→ For magnetic Intensity

$$H_I = \frac{E_I}{\eta_1} = \frac{100}{100} = 1 \text{ A/m}$$

$$H_R = \frac{E_R}{\eta_1} = \frac{50}{100} = 0.5 \text{ A/m}$$

$$H_T = \frac{E_T}{\eta_2} = \frac{15\cancel{0}}{30\cancel{0}} = 0.5 \text{ A/m}$$

Example

A uniform plane wave is incident from air onto glass at an angle from the normal of 30°. determine the ~~fraction of the incident~~ coefficient power ~~that is~~ reflected & transmitted for (a) p-polarization (b) s-polarization. Glass has refractive index $n_2 = 1.45$, $n_1 = 1$ (air)

→ Refractive Index

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\sin\theta_2 = \frac{n_2}{n_2}\sin\theta_1 = \cancel{\frac{1.45 \sin 30^\circ}{1}} = \frac{1}{1.45}\sin 30^\circ$$

$$\theta_2 = \sin^{-1}\left(\frac{\sin 30^\circ}{1.45}\right) = 20.2^\circ$$

P-polarization

$$\left(\because \eta_1 = \frac{\eta_0 \rightarrow (377)}{n_1}\right)$$

$$\eta_{1p} = \eta_1 \cos\theta_1 = \cancel{1\cos}\ \frac{(377)}{1}(0.866) = 326\,\Omega$$

$$\eta_{2p} = \eta_2 \cos\theta_2 = \frac{(377)}{1.45}\cos 20.2 = 244\,\Omega$$

Impedance

$$\Gamma_p = \frac{244 - 326}{244 + 326} = -0.144$$

$$\tau_p = \frac{2\eta_{2p}}{\eta_{2p} + \eta_{2p}} = \frac{244}{244 + 326}$$

S-polarization,

$$\eta_{1s} = \eta_1 \sec 30^\circ = \frac{377}{1}(0.886) = 435\,\Omega$$

$$\eta_{2s} = \eta_2 \sec 20.2 = 277\,\Omega$$

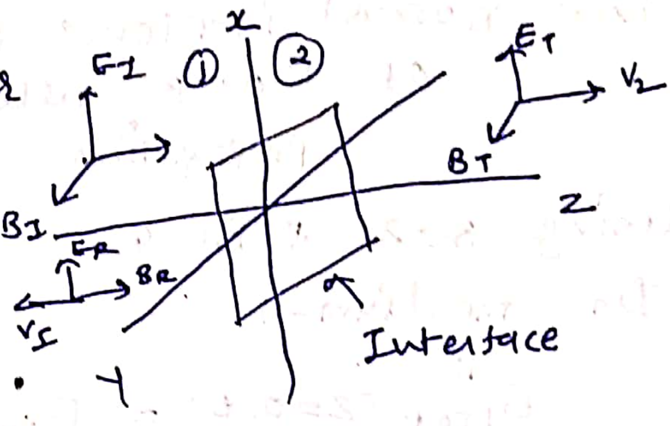
$$\Gamma_s = \frac{\eta_{2s} - \eta_{1s}}{\eta_{2s} + \eta_{1s}} = -0.222$$

$$\tau_s = \frac{2\eta_{2s}}{\eta_{2s} + \eta_{1s}} = \frac{2 \times 277}{277 + 435\,\Omega}$$

(*) Reflection and Transmission of plane waves at normal incidence:

x-y plane Boundary between two linear media

plane wave travelling in z-direction



Incident EM wave

Propagates in +z direction

$$E_{in} = E_{0I}\, e^{i(k_1 z - \omega t)}\, \hat{x}$$

$$B_{in} = \frac{1}{V_1} k_I \times E_I = \frac{1}{V_1} E_{0I}\, e^{i(k_1 z - \omega t)}\, \hat{y}$$

Reflected EM wave

Propagates in -z direction

$$E_R = E_{0R}\, e^{i(-k_1 z - \omega t)}\, \hat{x}$$

$$B_R = \frac{1}{V_1} k_R \times E_R = -\frac{1}{V_1} E_{0R}\, e^{i(-k_1 z - \omega t)}\, \hat{y}$$

Transmitted EM wave

Propagates in +z direction

$$E_T = E_{0T}\, e^{i(k_2 z - \omega t)}\, \hat{x}$$

$$B_T = \frac{1}{V_2} k_T \times E_T = \frac{1}{V_2} E_{0T}\, e^{i(k_2 z - \omega t)}\, \hat{y}$$

Boundary Conditions:

BC1 : Normal $\bar{D}$ continuous $\quad \epsilon_1 E^{\perp}_{1Tot} = \epsilon_2 E^{\perp}_{2Tot}$ (in z direction) (x-y Boundary)

BC2 : Tangential $\bar{E}$ Continuous $\quad E^{\parallel}_{1Tot} = E^{\parallel}_{2Tot}$

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BC 3 Normal $\bar{B}$ continuous : $B_{1Tot}^{\perp} = B_{2Tot}^{\perp}$

BC 4 Tangential H Continuous : $\frac{1}{\mu_1} B_{1Tot}^{\parallel} = \frac{1}{\mu_2} B_{2Tot}^{\parallel}$

For normal incidence (z = 0) – no components of E or B allowed to be along ± z propagation direction,

Using BC-2 & BC-4

In medium-1.

$$E_{1Tot}^{\parallel}(z=0,t) = E_{inc}(z=0,t) + E_r(z=0,t)$$

$$\frac{1}{\mu_1} B_{1Tot}^{\parallel}(z=0,t) = \frac{1}{\mu_1} B_I(z=0,t) + \frac{1}{\mu_1} B_R(z=0,t)$$

In medium-2

$$E_{1Tot}^{\parallel}(z=0,t) = E_T(z=0,t)$$

$$\frac{1}{\mu_2} B_{2Tot}^{\parallel}(z=0,t) = \frac{1}{\mu_2} B_{T}(z=0,t)$$

Using BC2

$$E_{1Tot}^{\parallel} = E_{2Tot}^{\parallel}$$

$$\therefore E_I \pm E_R \mp E_T$$

$$\therefore E_{0I} \cancel{e^{-i\omega t}} \pm E_{0R} \cancel{e^{-i\omega t}} = E_{0T} \cancel{e^{-i\omega t}}$$

$$\therefore E_{0I} + E_{0R} = E_{0T} \quad \text{—— (1)}$$

Using BC-4

$$\frac{1}{\mu_1} B_{1Tot}^{\parallel} = \frac{1}{\mu_2} B_{2Tot}^{\parallel}$$

$$\frac{1}{\mu_1} B_I + \frac{1}{\mu_1} B_R = \frac{1}{\mu_2} B_T$$

$$\frac{1}{\mu_1 v_1} E_{0I} \cancel{e^{-i\omega t}} - \frac{1}{\mu_1 v_1} E_{0R} \cancel{e^{-i\omega t}} = \frac{1}{\mu_2 v_2} E_{0T} \cancel{e^{-i\omega t}}$$

$$\frac{1}{\mu_1 v_1} E_{0I} - \frac{1}{\mu_1 v_1} E_{0R} = \frac{1}{\mu_2 v_2} E_{0T}$$

$$\frac{1}{\mu_1 v_1}(E_{0I} - E_{0R}) = \frac{1}{\mu_2 v_2} E_{0T} \quad (2)$$

Now,

$$E_{0I} - E_{0R} = \frac{\mu_1 V_1}{\mu_2 V_2} E_{0T}$$

$$E_{0I} - E_{0R} = \beta E_{0T} \quad \text{--- (3)} \qquad \left(\because \frac{\mu_1 V_1}{\mu_2 V_2} = \beta\right)$$

Eq^n (1) + Eq^n (3)

$$E_{0I} + \cancel{E_{0R}} = E_{0T}$$
$$E_{0I} - \cancel{E_{0R}} = \beta E_{0T}$$

$$2E_{0I} = (1+\beta) E_{0T}$$

$$E_{0T} = \left(\frac{2}{1+\beta}\right) E_{0I} \quad \text{--- (4)}$$

Eq^n (1) − Eq^n (3)

$$\cancel{E_{0I}} + E_{0R} = E_{0T}$$
$$\cancel{E_{0I}} - E_{0R} = \beta E_{0T}$$
$$- \quad + \quad -$$

$$2E_{0R} = (1-\beta) E_{0T}$$

$$E_{0R} = \left(\frac{1-\beta}{2}\right) E_{0T} \quad \text{--- (5)}$$

Put Eq^n (4) → Eq^n (5)

$$E_{0R} = \left(\frac{1-\beta}{\cancel{2}}\right)\left(\frac{\cancel{2}}{1+\beta}\right) E_{0I}$$

$$E_{0R} = \left(\frac{1-\beta}{1+\beta}\right) E_{0I}$$

As we know,

$$\beta = \frac{\mu_1 V_1}{\mu_2 V_2}$$

$$v_1 = \frac{c}{n_1}, \quad V_2 = c/n_2$$

$$n_1 = \sqrt{\frac{\epsilon_1 \mu_1}{\epsilon_0 \mu_0}} \qquad n_2 = \sqrt{\frac{\epsilon_2 \mu_2}{\epsilon_0 \mu_0}}$$

$$\beta = \frac{\mu_1 (c/n_1)}{\mu_2 (c/n_2)} = \frac{\mu_1 n_2}{\mu_2 n_1}$$

$$\beta = \frac{\mu_1 \sqrt{\epsilon_2 \mu_2 / \epsilon_0 \mu_0}}{\mu_2 \sqrt{\epsilon_1 \mu_1 / \epsilon_0 \mu_0}}$$

$$= \sqrt{\frac{\epsilon_2 \mu_1}{\epsilon_1 \mu_2}}$$

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If two Media are both paramagnetic or diamagnetic

$$|\chi_{m_{1,2}}| \ll 1$$

$$\mu_1 = \mu_0(1+\chi_{m_1}) \approx \mu_0 \quad , \quad \mu_2 = \mu_0(1+\chi_{m_2}) \approx \mu_0$$

So, $\beta = \dfrac{\mu_1 v_1}{\mu_2 v_2} = \dfrac{v_1}{v_2} = \dfrac{n_2}{n_1}$

So, now we have

$$E_{0R} = \left(\frac{1-\beta}{1+\beta}\right) E_{0I} = \left(\frac{1-(v_1/v_2)}{1+(v_1/v_2)}\right) E_{0I}$$

$$E_{0R} = \left(\frac{v_2 - v_1}{v_2 + v_1}\right) E_{0I}$$

$$E_{0T} = \left(\frac{2}{1+\beta}\right) E_{0I} = \left(\frac{2}{1+(v_1/v_2)}\right) E_{0I}$$

$$E_{0T} = \left(\frac{2v_2}{v_1+v_2}\right) E_{0I}$$

Reflection Coefficient

$$R = \frac{E_{0R}}{E_{0I}}$$

$$= \frac{v_2 - v_1}{v_2 + v_1}$$

$$= \frac{n_1 - n_2}{n_1 + n_2}$$

Transmission Coefficient

$$T = \frac{E_{0T}}{E_{0I}}$$

$$= \frac{2v_2}{v_1+v_2}$$

$$= \frac{2n_1}{n_1+n_2}$$