Unit-4

Lagrange Interpolation Formula

Lagrange's formula is applicable to problems where the independent variable occurs at equal and unequal intervals, but preferably this formula is applied in a situation where there are unequal intervals for the given independent series. The values of the independent variables $(X)(X)$ are given as a,b,c,d,...a,b,c,d,... etc., and the corresponding values of the function (dependent variable)
are f(a),f(b),f(c),f(d),...f(a),f(b),f(c),f(d),... etc. So the value of the function corresponding to a given value of $X=XoX=Xo$ is given by $f(Xo)f(Xo)$ which is calculated by the formula presented by Lagrange. Thus, if

XX	f(X)f(X)
aa	f(a)f(a)
bb	f(b)f(b)
cc	f(c)f(c)
dd	f(d)f(d)
⋮	⋮

f(Xo) is given as:
$$f(Xo)=(Xo-a)(Xo-b)(Xo-c)/(a-b)(a-c)(a-d)*f(a)+(Xo-a)(Xo-c)(Xo-d)/(b-b)(b-c)(b-d)*f(b)+(Xo-a)(Xo-b)(Xo-d)/(c-a)(c-b)(c-d)*f(c)+(Xo-a)(Xo-b)(Xo-c)/(d-a)(d-b)(d-c)*f(d)+\cdots\cdots\cdots\cdots\cdots\cdots$$

Although this formula looks somewhat lengthy, is in fact very easy to remember and write. The first thing to note is that successive terms are multiplied

by f(a),f(b),f(c),...f(a),f(b),f(c),... etc. respectively, and hence there will be as many terms as the number of items in the data. Thus, for example, if we have four yearly observations on the production of rice we will only need to write four such terms. The numerator in each term contain factors like (Xo−a)(Xo−b)(Xo−c)⋯(Xo−a)(Xo−b)(Xo−c)⋯ etc. The point to note here is that the first term omits (Xo−a)(Xo−a) , the second term omits (Xo−b)(Xo−b), the third term omits (Xo−c)(Xo−c) and so on.

The denominator in each term contains different factors of the independent variable. In the first term the difference is taken from aa, in the second term the difference is taken from bb, in the third term the difference is taken from cc and so on.

When interpolating with this formula it is recommended that you do not look at the formula to write the values of a,b,c,...a,b,c,... and f(a),f(b),f(c),...f(a),f(b),f(c),... etc. but rather directly write down the values as if you are writing the formula with the given values. This is suggested because confusion arises while writing the values of a,b,ca,b,c etc. in the formula

Interpolation refers to the process of creating new data points given within the given set of data.The above MATLAB code computes the desired data point within the given range of discrete data sets using the formula given by Gauss.
The gaussian interpolation comes under the Central Difference Interpolation Formulae which differs from Newton's Forward interpolation formula formula.

Suppose we are given the following value of y=f(x) for a set values of x:

X: x0 x1 x2 ………. xn
Y: y0 y1 y2 ………… yn

Then the process of finding the value of y corresponding to any value of x=xi between x0 and xn is called interpolation. Thus interpolation is the technique of estimating the value of a function for any intermediate value of the independent variable while the process of computing the value of the function outside the given range is called extrapolation. However, extrapolation is beyond the scope of this code.

Suppose that the function y=f(x) is tabulated for the equally spaced values x = x0, x = x0+h , x = x0+2h ,……… x = x0+nh, giving y= y0 , y1, y2,……, yn. . To determine the value of f(x) or f'(x) for some intermediate values of x, the following three types of differences are found useful.
1. Forward Difference
2. Backward difference
3. Central Difference
The common Newton's forward formula belongs to the Forward difference category. However , the Gaussian forward formula formulated in the attached code belongs to the central difference method.
Gauss forward formula is derived from Newton's forward formula which is:

Newton's forward interpretation formula:

Yp=y0+p.Δy0+ p(p-1)Δ2y0/(1.2) + p(p-1)(p-2)Δ3y0/(1.2.3)+….
We have ,
Δ2y0 − Δ2y-1=Δ3y-1

i.e. $\Delta^2 y_0 = \Delta^2 y_{-1} + \Delta^3 y_{-1}$
Similarly,
$\Delta^3 y_0 = \Delta^3 y_{-1} + \Delta^4 y_{-1}$
$\Delta^4 y_0 = \Delta^4 y_{-1} + \Delta^5 y_{-1}$ etc..

Also, $\Delta^3 y_{-1} - \Delta^3 y_{-2} = \Delta^4 y_{-2}$
I.e. $\Delta^3 y_{-1} = \Delta^3 y_{-2} + \Delta^4 y_{-2}$

Similarly $\Delta^4 y_{-1} = \Delta^4 y_{-2} + \Delta^5 y_{-2}$ etc..

Substituting for $\Delta^2 y_0$, $\Delta^3 y_0$, $\Delta^4 y_0$….. from (2),(3),(4)….. in (1), we get
$Y_p = y_0 + p\Delta y_0 + (p(p-1).(\Delta^2 y_{-1} + \Delta^3 y_{-1}))/(1.2) + (p.(p-1).(p-2)(\Delta^3 y_{-1} + {}^4 y_{-1}))/(1.2.3)$
$+ (p.(p-1)(p-2)(p-3)(\Delta^4 y_{-1} + \Delta^5 y_{-1}))/(1.2.3.4) + ……..$

Hence $y_p = y_0 + p\Delta y_0 + (p(p-1)\Delta^2 y_{-1})/2! + ((p+1)p(p-1)\Delta^3 y_{-1})/3! +$
$((p+1)p(p-1)(p-2)\Delta^4 y_{-2})/4! + ………$ using (5)……….

Which is then called gauss's forward interpolation formula.

One more representation of this formula is in the central difference notation , this formula will be

$y_p = y_0 + p\delta y_{1/2} + (p(p-1)\delta^2 y_0)/2! + ((p+1)p(p-1)\delta^3 y_{1/2})/3! +$
$((p+1)p(p-1)(p-2)\delta^4 y_0)/4! + ……$

These interpolation formulae are applicable for interpretation near the beginning and end of tabulated values.
However, the gaussian forward formula are best suited for interpolation near the middle of the table.

The coefficients in the central difference formula such as that of gauss are smaller and converge faster than those in Newton's formulae. Therefore, as much, whenever possible, central difference formulae should be used in preference to Newton's formulae.
The above code starts with asking the no. of data. The input data is denoted by x.
The difference between the two consecutive values of x is denoted by h.
One more parameter used in this formula is p, which is computed by the following formula

$p = (x - x_0)/h$
The corresponding value of y is also asked from the user.
After entering this values, the code computes the parameters h and p automatically and displays the result on calling.

The final desired answer is stored in a variable named ans and is displayed on calling in MATLAB window.
An example :-
No. of variables =5
The x values are :- 21 , 25 , 29 , 33 ,37 and the corresponding values of y are given by

18.478 , 17.8144 ,
17.8144 , 16.3432 and 15.5154.

x p	yp	Δyp	^{2}yp	^{3}yp	^{4}yp	^{5}yp
21	-2	18.478	-0.6564	-0.0510	-0.0074	-0.0022
25	-1	17.8144	-0.7074	-0.0564	-0.0076	
29	0	17.8144	-0.7638	-0.0640		
33	1	16.3432	-0.8278			
37	2	15.5154				

On running the code for the desired value of y30, we get the following values :-
h=4
p=0.25
Ans=16.9216

A formula in which the nodes (cf. Node) nearest to the interpolation point x are used as interpolation nodes. If $x = x_0 + th$, the formula

$$G_{2n+1}(x_0 + th) = f_0 + f^1_{1/2}t + f^2_0 \frac{t(t-1)}{2!} + \ldots +$$

$$+ f^{2n}_0 \frac{t(t^2-1)\ldots[t^2-(n-1)^2](t-n)}{(2n)!},$$

written with respect to the nodes $x_0, x_0 + h$, $h_0 - h, \ldots, x_0 + nh$, $x_0 - nh$ is called the Gauss forward interpolation formula, while the formula

$$G_{2n+1}(x_0 + th) = f_0 + f^1_{-1/2}t + f^2_0 \frac{t(t+1)}{2!} + \ldots +$$

$$+ f^{2n}_0 \frac{t(t^2-1)\ldots[t^2-(n-1)^2](t+n)}{(2n)!},$$

written with respect to the nodes $x_0, x - h$, $x_0 + h, \ldots, x_0 - nh$, $x_0 + nh$ is called the Gauss backward interpolation formula, [1], [2]. Formulas (1) and (2) employ finite differences, defined as follows:

$$f^1_{i+1/2} = f_{i+1} - f_i, \qquad f^m_i = f^{m-1}_{i+1/2} - f^{m-1}_{i-1/2}.$$

The advantage of Gauss' interpolation formulas consists in the fact that this selection of interpolation nodes ensures the best approximation of the residual term of all possible choices, while the ordering of the nodes by their distances from the interpolation point reduces the numerical error in the interpolation.