ASSIGNMENT- 4

Q1. Show that the following boolean expressions A & B are equivalent.

$A(x,y,z) = (x \oplus y) * (x \oplus z)$

$B(x,y,z) = (x * z) \oplus (x' * y)$.

(1) x	(2) y	(3) z	(4) $x \oplus y$	(5) $x' \oplus z$	(6) α	(7) $x * z$	(8) $x' * y$	(9) β
0	0	0	0	1	0	0	0	0
0	0	1	0	1	0	0	0	0
0	1	0	1	1	1	0	1	1
0	1	1	1	1	1	0	1	1
1	0	0	1	0	0	0	0	0
1	0	1	1	0	0	1	0	1
1	1	0	1	1	1	0	0	0
1	1	1	1	1	1	1	0	1

→ From the column (6) and column (9), one can say that $\alpha(x,y,z) = \beta(x,y,z)$, $\forall (x,y,z) \in B^3$ where $B = \{0,1\}$, a two element Boolean algebra.

$\Rightarrow \alpha = \beta$.

Q2. Define the maxterm and find all the max terms of a boolean algebra with two variables x_1, x_2.

→ Let $< B, *, \oplus, ', 0, 1 >$ be any boolean algebra in n variable $x_1, x_2, \ldots, x_n$; then the term $x_1^{a_1} \oplus x_2^{a_2} \oplus x_3^{a_3} \oplus \ldots \oplus x_n^{a_n}$ is called a maxterm of the Boolean algebra, where $a_i = 0$ or 1 $\forall i = 1$ to n and x_i^1 stands for x_i' and x_i^0 for x_i.

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→ Eg.:-

a_1	a_2	Binary Representation	Decimal value	Maxterm
0	0	00	0	$M_0 = x_1^0 \oplus x_2^0 = x_1 \oplus x_2$
0	1	01	1	$M_1 = x_1^0 \oplus x_2^1 = x_1 \oplus x_2'$
1	0	10	2	$M_2 = x_1^1 \oplus x_2^0 = x_1' \oplus x_2$
1	1	11	3	$M_3 = x_1' \oplus x_2' = x_1' \oplus x_2'$

Q3. Define the minterm and find all the minterms of Boolean algebra with three variables x_1, x_2, x_3.

→ A Boolean expression $\alpha(x_1, x_2, \ldots, x_n)$ in n-variables is said to be minterm of variable if

$$\alpha(x_1, x_2, \ldots, x_n) = x_1^{\alpha_1} + x_2^{\alpha_2} + \ldots + x_n^{\alpha_n}$$

$$= \mathop{*}_{i=1}^{n} x_i^{\alpha_i} \quad ; \text{ where } i = 1, 2, \ldots, n$$

in which $\alpha_i \in (0,1)$ x_i^0 stands for x_i' and x_i^1 for x_i.

→ Ex.:- general form of a minterm in variables x_1, x_2, x_3 is $x_1^{a_1} * x_2^{a_2} * x_3^{a_3}$. We write $x_1^{a_1} * x_2^{a_2} * x_3^{a_3}$ as (a_1, a_2, a_3) where $a_i = \{0, 1\}$; $\forall i = 1, 2, 3$.

$$\therefore (a_1, a_2, a_3) = \{0\} \times \{0, 1\} \times \{0, 1\}$$

$$= \left\{ \begin{matrix} (0,0,0), (0,0,1), (0,1,0), (0,1,1), \\ (1,0,0), (1,0,1), (1,1,0), (1,1,0) \end{matrix} \right\}$$

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Symbol	Binary Representation	Decimal value.	Minterm .
(0,0,0)	000	0	$M_0 = x_1' * x_2' * x_3' = x_1^0 * x_2^0 * x_3^0$
(0,0,1)	001	1	$M_1 = x_1' * x_2' * x_3 = x_1^0 * x_2^0 * x_3^1$
(0,1,0)	010	2	$M_2 = x_1' * x_2 * x_3' = x_1^0 * x_2^1 * x_3^0$
(0,1,1)	011	3	$M_3 = x_1' * x_2 * x_3 = x_1^0 * x_2^1 * x_3^1$
(1,0,0)	100	4	$M_4 = x_1 * x_2' * x_3' = x_1^1 * x_2^0 * x_3^0$
(1,0,1)	101	5	$M_5 = x_1 * x_2' * x_3 = x_1^1 * x_2^0 * x_3^1$
(1,1,0)	110	6	$M_6 = x_1 * x_2 * x_3' = x_1^1 * x_2^1 * x_3^0$
(1,1,1)	111	7	$M_7 = x_1 * x_2 * x_3 = x_1^1 * x_2^1 * x_3^1$

Q4. Show any one property of Boolean algebra $< B, *, \oplus, ', 0, 1 >$.

→ A complemented distributive lattice is called boolean algebra. It is denoted by $< B, *, \oplus, ', 0, 1 >$.

- Property:

(i) Idempotent:

$a * a = a$; $a \oplus a = a$ $\quad \forall a \in B$.

(ii) Commutative:

$a * b = b * a$; $a \oplus b = b \oplus a$ $\quad \forall a, b \in B$.

(iii) Associative:

$a * (b * c) = (a * b) * c$

$a \oplus (b \oplus c) = (a \oplus b) \oplus c$ $\quad \forall a, b, c \in B$

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(iv) Absorption:

$a * (a \oplus b) = a$

$a \oplus (a * b) = a \quad \forall\, a, b \in B$

(v) Distributive:

$a * (b \oplus c) = (a * b) \oplus (a * c)$

$a \oplus (b * c) = (a \oplus b) * (a \oplus c) \quad \forall\, a, b, c \in B$.

(vi) There exist 0-elements, 1-element in B such that $0 \le a \le 1$; $\forall\, a \in B$.

(vii) $a * 0 = 0 * a = 0$, $0 \oplus a = a = a \oplus 0$

$1 * a = a * 1 = a$, $a \oplus 1 = 1 = 1 \oplus a \quad \forall\, a \in B$.

(viii) $a * a' = 0$, $a \oplus a' = 1$

$0' = 1$, $1' = 0 \quad \forall\, a \in B$

(ix) De Morgan's law:

$(a \oplus b)' = a' * b'$

$(a * b)' = a' \oplus b' \quad \forall\, a, b \in B$

(x) $< B, \le >$ is a lattice as a poset where $\le$ is defined as $a \le b$ if $a * b = a$ then $a * b = GLB\{a, b\}$, $a \oplus b = LUB\{a, b\}$ $a \le b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$.

Q5. Define the equivalent Boolean expression. Show that α and β are equivalent Boolean expression.

$\alpha(x_1, x_2, x_3) = x_1 * (x_2 * x_3)'$ and $\beta(x_1, x_2, x_3) = (x_1 * x_2') \oplus (x_2 * x_3')'$.

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→ Two Boolean expressions α_1 and α_2 (or α and β) in n-Boolean variables are said to be equivalent or equal boolean expression if one can be obtained from the other say, a finite number of applications of the identities of boolean algebra.

$$\alpha(x_1, x_2, \ldots x_n) = \beta(x_1, x_2, \ldots x_n)$$

→ Ex.: $\alpha(x_1, x_2, x_3) = x_1 * (x_2 * x_3)'$

$$= x_1 * (x_2' \oplus x_3')$$

$$= (x_1 * x_2') \oplus (x_1 * x_3')$$

$$\therefore \alpha(x_1, x_2, x_3) = \beta(x_1, x_2, x_3).$$

Q6. Define Boolean variables and boolean form.

→ (i) Boolean variables:

Let $< B, *, \oplus, ', 0, 1 >$ be a boolean algebra, let $x_1, x_2, \ldots, x_n$ be n symbols then we will consider $x_1, x_2, \ldots, x_n$ as n-boolean variables.

→ (ii) Boolean Form / Expression:

A boolean expression, form in n-variable $x_1, x_2, \ldots, x_n$ is any finite string of symbols formed in following ways:

① "0" and "1" are boolean exp.

② $x_1, x_2, \ldots, x_n$ are boolean exp.

③ If α_1 and α_2 are boolean exp. then $\alpha_1 * \alpha_2$ & $\alpha_1 \oplus \alpha_2$ are also boolean expressions.

④ If α is boolean expression then (α') is also boolean exp.

Ex.:- $x_1, x_2 * x_3, (x_3 \oplus x_1) * x_1'$.

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Q7. State the stone's representation theorem.

→ Every finite boolean algebra is isomorphic to the power set Boolean algebra of the set of all atoms.

→ In other words, if $< B, *, \oplus, ', 0, 1 >$ is a finite boolean algebra and A is the set of all atoms of B then

$$< B, *, \oplus, ', 0, 1 > \cong < P(A), \cap, \cup, ^c, \phi, A >.$$

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