5)

Define Group and abelian group (Commutative)

If binary operation * defined on a non-empty set G satisfies the following postulates (properties) then G is said to be group.

(1) closureness: $a, b \in G \Rightarrow a * b \in G$

(2) Associative: $\forall a, b, c \in G$
$\Rightarrow (a*b)*c = a*(b*c)$.

(3) Existence of Identity:-
$\forall a \in G \quad \exists e \in G$ such that

$$a * e = a = e * a.$$

here (e) is a identity element

(4) Existence of Inverse:
For $a \in G$, $\exists b \in G$ such that
$a * b = e = b * a$
$a * a^{-1} = 1$

here, b is inverse element of a in G

→ Then G is said to be a group under binary operation *

Notation: $(G, *)$

→ A Group $(G, *)$ is said to be abelian or (commutative) group of $a*b = b*a$; $\forall a, b \in G$

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* Some examples of group and abelian group

(1) $(\mathbb{Z}, +)$ is a group
(i.e $\mathbb{Z}$ is a group under binary operation addition $+$),

moreover $(\mathbb{Z}, +)$ is abelian group.

(2) $(\mathbb{Z}, \cdot)$ → this is not a group.
because for $a \in \mathbb{Z}$ $\not\Rightarrow \frac{1}{a} \in \mathbb{Z}$

i.e Inverse element does not exist.

(3) $(\mathbb{Q}, +)$ is a group as well as abelian.

(4) $(\mathbb{Q}, \cdot)$ is not a group and abelian
→ because Inverse of 0 does not not exist

(5) $(\mathbb{Q}_0, \cdot)$ is a group
because 0 is not include.

(6) $(\mathbb{R}^\circ, \cdot)$ is group but
$(\mathbb{R}, \cdot)$ is not group.

multiplication - m'tion

(EX) * Set $C = \{ a+ib / i^2 = -1, a, b \in R \}$ of all complex numbers form a group under addition as well as multiplication (m'tion)

Ans:

$z_1 = a_1 + ib_1 \in C$

$z_2 = a_2 + ib_2 \in C$

$z_3 = a_3 + ib_3 \in C$

→ binary operation "m'tion".

① $z_1 \cdot z_2 = (a_1 + ib_1) \cdot (a_2 + ib_2)$

$= a_1a_2 + ia_1b_2 + ib_1a_2 + i^2 b_1b_2$

$= (a_1a_2 - b_1b_2) + i(a_1b_2 + b_1a_2) \in C$

∴ closurnes holds

② $(z_1 \cdot z_2) \cdot z_3 = [(a_1a_2 - b_1b_2) + i(a_1b_2 + b_1a_2)] \cdot (a_3 + ib_3)$

$= [a_1a_2 - b_1b_2 + ia_1b_2 + ib_1a_2](a_3 + ib_3)$

$= a_2(a_1 + ib_1)$

$= [a_1 + ib_1] \cdot [(a_2 + ib_2) \cdot (a_3 + ib_3)]$

$= z_1 \cdot (z_2 \cdot z_3)$

∴ Associativity holds

$$\frac{(a_1+ib_1)\cdot(1+i0)}{a_1+ib_1+i0+i^2 0}$$

③ for $z_1 = a_1+ib_1 \in C$ $\exists\ (1,0) \in C$,
such that $z_1 = (a_1,b_1)\cdot(1,0)$
$= (a_1,b_1)$
$= z_1$

Here, $(1,0)$ is identity element in C

④ For $z = (a,b) \in C$ $\exists \left(\frac{a}{a^2+b^2}, -\frac{b}{a^2+b^2}\right) \in C$

such that $(a,b)\cdot\left(\frac{a}{a^2+b^2}, -\frac{b}{a^2+b^2}\right) = (1,0)$

→ Here, $\left(\frac{a}{a^2+b^2}, -\frac{b}{a^2+b^2}\right)$ is an inverse element of $z(a,b)$

→ binary operation "Addition"

① $z_1+z_2 = (a_1+ib_1)+(a_2+ib_2)$
$= (a_1+a_2)+i(b_1+b_2) \in C.$

② $(z_1+z_2)+z_3 = [(a_1+ib_1)+(a_2+ib_2)]+(a_3+ib_3)$
$= (a_1+ib_1)+[(a_2+ib_2)+(a_3+ib_3)]$
$= z_1+(z_2+z_3)$

$(a_1+ib_1)+(0+i0)$

$a_1+ia_0+ib_1+i^2b_1$

③ for $z_1 = a_1+ib_1 \in c \quad \exists (0,0) \in c$

$\Rightarrow z_1 = (a_1,b_1)+(0,0)$

$= (a_1+0)+i(b_1+0)$

$= (a_1+ib_1)$

$= z_1$

Here, $(0,0)$ is identity element

④ For $z = (a,b) \in c \exists \left(\frac{-a}{a^2+b^2}, \frac{-b}{a^2+b^2}\right) \in c$

$\Rightarrow (a,b)+\left(\frac{-a}{a^2+b^2}, \frac{-b}{a^2+b^2}\right) = (0,0)$

Here, $\left(\frac{-a}{a^2+b^2}, \frac{-b}{a^2+b^2}\right)$ is an inverse element of $z(a,b)$

addition

$z_1 = (a_1+ib_1) \Rightarrow a_1-ib_1$

(a_1-b)

$a \times \bar{a}^1 = e$

$(a_1+ib_1)+(-a_2+ib_2) = 0$ ← Inverse element

$(a_1-a_2)+i(+b_1-b_2) = 0$

$0+i0 = 0$

0

3.(2) (2×2) matrix Group is not Abelian group.

(Ex-2) Let $G = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle/ \begin{matrix} ad-bc \neq 0 \\ a,b,c,d \in R \end{matrix} \right\}$

Then P.T G is a group under binary operation of product in matrices.

Soln :- Let $A = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$, $B = \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$

$$C = \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix} \in G$$

(1) $$AB = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}\begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} \in G$$

$$= \begin{pmatrix} a_1a_2+b_1c_2 & a_1b_1+b_1d_2 \\ c_1a_2+d_1c_2 & c_1b_2+d_1d_2 \end{pmatrix}$$

$\therefore$ closures hold

(2) $(AB).C = A(B.C)$

$$= \left[\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} \cdot \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}\right]\begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix}$$

$$= \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}\left[\begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} \cdot \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix}\right]$$

$$= A(B.C)$$

③ For $A \in G$, $\exists\ I \in G$ such that $A.I = A = I.A$

here, $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is an identity matrix.

④ For $A \in G$, $\exists\ A^{-1} \in G$ such that

$$A^{-1}A = A.A^{-1} = I.$$

Here, $A^{-1} = \frac{adj\,A}{|A|}$ $\qquad A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$= \frac{\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}}{(ad - bc)}$$

is inverse matrix.

$(G, \cdot)$ is Group.

(S.Q) but, $(G, \cdot)$ is not abelian group.

(Ex-2)

(3) Order of the element of a :-

Suppose G is a given group and $a \in G$ If there exist a smallest position integer in such that

$a^m = e$, than m is called the order of element a denoted by $O(a)$, $O(a) = m$

e.g $(-1)^2 = 1 \Rightarrow O(-1) = 2$ order

If there does not exist m such that $(\exists)\ a^m = e$ then

$O(a) = \infty$ or $O(a) = \omega$

I.m.p

(S.Q) Note:- If $a^n = e \Rightarrow O(a) \leq n$ OR $O(a) | n$

(Ex) If $G = \{1, -1, i, -i\}$ is group then find out order of each element in G

Soln.

$(1)^1 = 1 \Rightarrow O(1) = 1$

$(-1)^2 = 1 \Rightarrow O(-1) = 2$

$(i)^4 = 1 \Rightarrow O(i) = 4$

$(-i)^4 = 1 \Rightarrow O(-i) = 4$.

Thm In a group G, $\forall\ a \in G$ $(a^{-1} = a)$ (or $a^2 = e$) then G is abelian.

Proof:
→ Let $a \in G$ and $a^{-1} = a$.

for $a, b \in G \Rightarrow a.b \in G$

$\Rightarrow (a.b)^{-1} = ab$ (∵ Inverse exist)

$\Rightarrow b^{-1}.a^{-1} = ab$

$\Rightarrow ba = ab$ $\left(\because \begin{matrix} a^{-1}=a \\ b^{-1}=b \end{matrix}\right)$

G is abelian. (commutative)

(Ex) If G there are 5 elements then prove that G is abelian.

→ Let $G = \{e, a, b, c, d\}$

.	e	a	b	c	d
e	e	a	b	c	d
a	a	e	d	b	c
b	b	d	a	c	e
c	c	b	e	d	a
d	d	c	c	e	a

from table we can say that G is abelian.

(Ex) In G, There are 4 elements then prove that G is abelian

→ Let $G = \{e, a, b, c\}$

·	e	a	b	c
e	e	a	b	c
a	a	c	e	b
b	b	e	b	a
c	c	b	a	e

$\frac{a^2}{c}, \frac{ab}{e}, \frac{ac}{b}$

→ from table we say that G is abelian

Sub group :-

A non-empty subset H of a group G is called subgroup of G. If H itself a group under the same binary operation as that of G.

Note- ① For any group G,

$H_1 = \{e\}$ and $H_2 = G$ are always subgroup of G, both these subgroup are called improper subgroups of a group G.

A group different from these two subgroup is called proper subgroup

(Ex) If $G = (\mathbb{C}; \cdot)$ and $H = \{a+ib \in G / a^2+b^2 = 1\}$ then P.T. H is subgroup of G.

Proof:- $z_1 = a_1 + ib_1$; $a_1^2 + b_1^2 = 1$

$z_2 = a_2 + ib_2$; $a_2^2 + b_2^2 = 1$

$z_1, z_2 \in H$.

(1) $z_1 \cdot z_2 = (a_1 + ib_1)(a_2 + ib_2)$

$= a_1 a_2 + ia_1 b_2 + ib_1 a_2 + i^2 b_1 b_2$

$= (a_1 a_2 - b_1 b_2) + i(a_1 b_2 + a_2 b_1)$

such that

$= (a_1 a_2 - b_1 b_2)^2 + (a_1 b_2 + b_1 a_2)^2$

$= a_1^2 a_2^2 + b_1^2 b_2^2 + a_1^2 b_2^2 + b_1^2 a_2^2$

$= a_1^2 (a_2^2 + b_2^2) + b_1^2 (b_2^2 + a_2^2)$

$= a_1^2 + b_1^2$

$= 1$ $\quad (\because a_2^2 + b_2^2 = 1, a_1^2 + b_1^2 = 1)$

H is a subgp.

(2) For $z_1 = a_1 + ib_1 = (a_1, b_1) \in H$.

$z_1^{-1} = (a_1 - ib_1)\left(\frac{a}{a^2+b^2}, \frac{-b}{a^2+b^2}\right) \in H$.

such that $z_1 \cdot z^{-1} = e = (1, 0)$
H is a subgroup

Thm A finite subset H of G is a subgroup of G if it is closed under multiplication

Proof: ① Let $a \in H \Rightarrow a \cdot a \in H$
$\Rightarrow a^2 \in H$
$\Rightarrow a^2 \cdot a \in H$
$\Rightarrow a^3 \in H$
$\Rightarrow \ldots \quad a^n \in H.$

but
H is finite,
∴ All those element cannot be distinct

② ∴ ∃ r, s such that $a^r = a^s \ (r > s > 0)$

∴ Taking $a^r \bar{a}^s = a^s \bar{a}^s$

$\therefore a^{r-s} = e \in H$

$\therefore a^{r-s} \in H.$

③ and $a, b, c \in H \Rightarrow a \cdot (b \cdot c) = (a \cdot b) \cdot c.$

④ $a^{r-s-1} \in H \Rightarrow a^{r-s} \cdot \bar{a}^1 \in H.$
$\Rightarrow e \cdot \bar{a}^1 \in H.$
$\Rightarrow \bar{a}^1 \in H.$

→ H is subgroup of G.

(Ex-4) For a commutative group G
$H = \{a \in G / o(a) \text{ is finite}\}$ is a subgroup of G.

Soln:- since, $e \in G$, $e \in H$.
then $H \neq \phi$

→ let $a, b \in H \Rightarrow o(a) = n$; $o(b) = m$.

($\because o(a) \neq o(b)$ is finite)

$\Rightarrow o(a) = n$, $o(b^{-1}) = m$.

Now, $(ab^{-1})^{mn} = a^{mn} \cdot (b^{-1})^{mn}$

$o(a) = n \Rightarrow a^n = e$

$= (a^m)^n \cdot ((b^{-1})^m)^n$

$= e^n \cdot e^n$

$= e$

$\Rightarrow o(ab^{-1}) \leq m.n$ — (II)

$\Rightarrow o(ab^{-1})$ is finite

$\Rightarrow ab^{-1} \in H$.

$\therefore$ H is a subgroup of G.

{e}, H_1 → अपने आप proper subgroup

Identity is subgroup ✓

$G \neq H_1 \cup H_2$

(Ex-5) A group cannot be a union of its two proper subgroup.

sol^n :- suppose H_1 & H_2 are two proper subgroup of G. and $G = H_1 \cup H_2$.

where $H_1 \neq \{e\}$ and $H_1 \neq G$.

$H_2 \neq \{e\}$ and $H_2 \neq G$.

As H_1 is proper subgroup of G,

$\exists$ an element $a \neq e \in G$

$\Rightarrow a \notin H_1$ — (1) $\therefore a \in H_2$.

$a \in H_1$ मानेंगे तो a H_2 में होगा.

simi, $\exists$ $b \notin H_2$ — (2); $b \in H_1$

Now $a, b \in G \Rightarrow a.b \in G$,

$\Rightarrow ab \in H_1 \cup H_2$ ($\because G = H_1 \cup H_2$)

if $ab \in H \Rightarrow$ then $(ab).b^{-1} \in H_1$

$\Rightarrow a(b.b^{-1}) \in H_1$,

$\Rightarrow a.e \in H_1$,

which is with eq^n (1)

if $ab \in H_2 \Rightarrow a^{-1}(a.b) \in H_2$

$\Rightarrow b \in H_2$

which $\cdot\dot{x}\cdot$ with e of (2)

Hence $G \neq H_1 \cup H_2$

(Ex) If $G = \{a^0, a^1, a^2, \ldots a^9 / a^{10} = e\}$

and $H = \{a^0, a^5 / a^{10} = e\}$ then find all right coset of G

Soln:- $H = \{a^0, a^5\} = Ha^0 = He$

$Ha^1 = \{a, a^6\}$

$Ha^2 = \{a^2, a^7\}$

$Ha^3 = \{a^3, a^8\}$

$Ha^4 = \{a^4, a^9\}$

$Ha^5 = \{a^5, a^{10}\} = H$

$Ha^6 = \{a^6, a^{11}\}$ here not possible ($\because a^{11} = a^1$)

Thus H, Ha, Ha^2, Ha^3, Ha^4, are distinct right cosets.

ch:-8 - Permutation

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* Notation :-

(1) A(S) = The set of all one-one correspondence on non-empty sets

A(S) is group under composition of mappings (પદાર્થોનું સંયોજન)

(ફંક્શનનો એવો set જે one-one, onto function હોય)

(2) Sn :- Symmetric Group of degree n are permutation group of order n

the elements of Sn are called Permutations.

If S contained n elements then group A(S) is denoted by "Sn"

* Definition :-

Let S be a non-empty set. A one-one and on-to function from S to S is called permutation on S

e.g. $f : S \to S$, $f(S) = S$

(Permutation ફંક્શનનું domain and codomain is equal)

(Ex) Prove that set of all permutation on S is a group under composition.

Soln:- P.T $(A(S), \circ)$ is a group.

$f, g, h \in A(S)$

[1] clouser: $fog : S \to S$

$fog(s) = f(g(s))$

$= f(s)$

$fog(s) = s$

$\therefore fog \in A(S)$

[2] Associative:

$(fog) \circ h(s) = (fog)\, h(s)$

$= f(g(h(s))$

$= f(g \circ h(s))$

$= f \circ (g \circ h)(s)$

$(f \circ g) \circ h = f \circ (g \circ h)$

[3] identity :-

$fog \in A(S)$; $\exists\ I \in A(S)$ ∋

$f \circ I(s) = f(I(s)) = f(s) = s$

& $I \circ f(s) = s$
$f \circ I = I \circ f$.

[4] Inverse:-

since, $f \in A(s)$

f is one-one & onto.

f^{-1} exists

$f \circ f^{-1}(s) = I(s) = s$.

$(A(s), \circ)$ is Group.

Note:- ① Element of s_n is denoted by

$$s_n = \begin{pmatrix} 1 & 2 & 3 & \ldots & n \\ G(1) & G(2) & G(3) & \ldots & G(n) \end{pmatrix}$$

Here, G is permutation

$= \{1, 2, 3 \ldots n\}$.

② s_n is a group of order $n!$

i.e $O(s_n) = n!$

(Ex) If $S = \{1, 2, 3, 4, 5\}$

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 1 & 2 \end{pmatrix}$$

and

$$g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \end{pmatrix}$$

then find fog; gof; f^{-1}, g^{-1}.

Soln:-

$$fog = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 5 & 1 & 2 & 3 \end{pmatrix}$$

$$gof = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 5 & 1 & 2 & 3 \end{pmatrix}$$

$$f^{-1} = \begin{pmatrix} 3 & 4 & 5 & 1 & 2 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix} \times$$

$$\therefore f^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 5 & 1 & 2 & 3 \end{pmatrix}$$

$$g^{-1} = \begin{pmatrix} 2 & 3 & 4 & 5 & 1 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix} \times$$

$\therefore g^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 1 & 2 & 3 & 4 \end{pmatrix}$

(Ex) If $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 4 & 6 & 9 & 8 & 2 & 5 & 7 & 1 & 3 \end{pmatrix}$$

$$g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 1 & 8 & 4 & 3 & 2 & 5 & 6 & 9 \end{pmatrix}$$

Then verify $(f \circ g)^{-1} = (g^{-1} \circ f^{-1})$

and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$

Soln:-

$$(f \circ g) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 4 & 1 & 8 & 9 & 6 & 2 & 5 & 3 \end{pmatrix}$$

$$(f \circ g)^{-1} = \begin{pmatrix} 7 & 4 & 1 & 8 & 9 & 6 & 2 & 5 & 3 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{pmatrix} \times$$

$$(f \circ g)^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 7 & 9 & 2 & 8 & 6 & 1 & 4 & 5 \end{pmatrix} \quad \text{--- (1)}$$

$$f^{-1} = \begin{pmatrix} 4 & 6 & 9 & 8 & 2 & 5 & 7 & 1 & 3 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{pmatrix} \times$$

$$f^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8 & 5 & 9 & 1 & 6 & 2 & 7 & 4 & 3 \end{pmatrix}$$

$$g^{-1} = \begin{pmatrix} 7 & 1 & 8 & 4 & 3 & 2 & 5 & 6 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{pmatrix} \times$$

$$g^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 6 & 5 & 4 & 7 & 8 & 1 & 3 & 9 \end{pmatrix}$$

$$g^{-1} \circ f^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 7 & 9 & 2 & 8 & 6 & 1 & 4 & 5 \end{pmatrix} \quad \text{---(2)}$$

$\Rightarrow$ by eqn (1) & (2) $\underline{(f \circ g)^{-1} = g^{-1} \circ f^{-1}}$

$$\rightarrow (g \circ f) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 4 & 2 & 9 & 6 & 1 & 3 & 5 & 7 & 8 \end{pmatrix}$$

$$(g \circ f)^{-1} = \begin{pmatrix} 4 & 2 & 9 & 6 & 1 & 3 & 5 & 7 & 8 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{pmatrix}$$

$$(g \circ f)^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 5 & 2 & 6 & 1 & 7 & 4 & 8 & 9 & 3 \end{pmatrix} \quad \text{---(3)}$$

$$f^{-1} \circ g^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 5 & 2 & 6 & 1 & 7 & 4 & 8 & 9 & 3 \end{pmatrix} \quad \text{---(4)}$$

$\Rightarrow$ by eqn (3) & (4)

$\underline{(g \circ f)^{-1} = f^{-1} \circ g^{-1}}$

Transposition: A cycle:

A function $f: S \to S$ defined by $f(p) = q$, $f(q) = p$ and $f(i) = i$ for $i \neq p, q$, then permutation is called transposition.

Notation: (p, q)

Also, $f^{-1}: S \to S$ is also transposition

e.g $S = \{1, 2, 3, 4, 5\}$

and $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 5 & 3 & 4 & 2 \end{pmatrix}$

$\sigma = (2, 5)$ is transposition.

⇒ A Permutation $g: S \to S$ is said to be a cycle of length k if $\exists$ elements $p_1, p_2, p_3, \ldots p_k \in S$ such that

$g(p_1) = p_2$, $g(p_2) = p_3 \ldots g(p_{k-1}) = p_1$ and $g(m) = m$.

(where $m \neq p_i$ $(i = 1, 2, 3, \ldots k)$)

which is denoted by

$g = (p_1, p_2, \ldots p_k)$

e.g

① $G_1 : S \to S$; $S = \{1, 2, 3\}$

$$G_1 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$$

$G_1 = (1\ 2\ 3)$ is cycle

② $G_2 : S \to S$; $S = \{1, 2, 3, 4, 5, 6\}$

$$G_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 4 & 5 & 6 & 1 \end{pmatrix}$$

$G_2 = (1\ 2\ 3\ 4\ 5\ 6)$ is cycle

③ $G_3 : S \to S$, $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$$G_3 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 2 & 5 & 6 & 7 & 8 & 1 & 4 \end{pmatrix}$$

$= (1\ 3\ 5\ 7)(4\ 6\ 8)$ is cycle.

(Ex) If $f = (1\ 4\ 5\ 6) \in S_6$ & $g = (2\ 1\ 5) \in S_6$
$g\ (2\ 1\ 5) \in S_6$
then find fog & gof

$$fog = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 2 & 3 & 5 & 6 & 1 \end{pmatrix}$$

$$g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 1 & 3 & 4 & 2 & 6 \end{pmatrix}$$

$$fog = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 4 & 3 & 5 & 2 & 1 \end{pmatrix}$$

$$= (1, 6)(2, 4, 5)$$

$$gof = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 1 & 3 & 2 & 6 & 5 \end{pmatrix}$$

$$= (1, 4, 2)(5, 6)$$

hence composition of two will not to be again cycle