Electrostatics & capacitor

* Electrostatics:

Electrostatics is the branch of science which deals with the static charge.

* Electric charge:

The total deficiency or excess of electrons in a body is called its charge.

→ The unit of charge is coulomb

1 coulomb = $6.24 \times 10^{+18}$ electrons

→ Electric charge is denoted by Q.

* Permittivity, Absolute Permittivity and Relative Permittivity:

→ Permittivity is the property of a medium that affects its magnitude of the force between two point charges.

→ While, Absolute or Actual permittivity is nothing but the actual permittivity of a medium.

→ It is denoted by ε.

→ The Absolute permittivity of an air or vaccum is denoted by ε_0 and its value is 8.854×10^{-12} F/m

→ Relative permittivity is the ratio of Absolute permittivity of some material (ε) to the Absolute permittivity of Air or vacuum (ε_0).

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→ It is denoted by ε_r.

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

→ Here, it should be remember that, the relative permittivity for air or vacuum is 1.

Que. Explain the coulomb's laws of electrostatics.

OR

Explain the fundamental laws of electrostatics

Ans: coulomb, a French scientist had given two laws about the electrostatics and which are given below:

* coulomb's first law:

"Like charges of electricity repel each other while unlike charges of electricity attract each other."

* coulomb's second law:

"The force between two charged bodies...
→ is directly proportional to the product of the magnitude of charges.
→ is inversely proportional to the square of the distance between them."

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Mathematically,

$$F \propto \frac{Q_1 Q_2}{d^2} \Rightarrow F = K \frac{Q_1 Q_2}{d^2} \quad \text{---(1)}$$

where,

Q_1, Q_2 = charges at point 1 & 2 respectively

d = distance b/w two point charges

K = constant of proportionality

→ The constant K depends upon the nature of the medium in which charges are placed

→ The value of K is determined by,

$$K = \frac{1}{4\pi\varepsilon_0\varepsilon_r}$$

→ Now, put the value of K into eq^n (1), it becomes,

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0\varepsilon_r d^2} \quad \text{---(2)}$$

Eq^n (2) indicates the equation of force b/w two charges.

* Electric Field:

A region of space around a charge body in which an electric charge experiences a force of attraction or repulsion is called an Electric Field.

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* **Electric Flux:**

The total number of lines of force emanating from a certain charge is called Electric flux.

→ It is denoted by ψ.

→ The unit of it is a coulomb.

* **Electric Flux density:**

It is defined as the electric flux passing through unit area at right angle to the direction of electric field.

→ It is denoted by D.

mathematically, $$D = \frac{\psi}{A}$$

ψ = electric flux in coulomb

A = Area in m^2

→ The unit of it is coulombs/ sq. meter (C/m^2)

Que. What is Electric field intensity? Derive the expression for electric field intensity.

Ans. **Electric field intensity:**

It is defined as the force experienced by a unit positive charge placed at the point.

→ It is denoted by E.

mathematically, $$E = \frac{F}{Q} \text{ or } \frac{V}{d}$$

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Capacitance:

* Some important definitions:

(1) **Capacitance:**
Capacitance is the property of a material which stores the el. charge.
- It is denoted by C.
- Its unit is farade (F)
- It can be found with the help of,

$$C = \frac{Q}{V}$$

(2) **Dielectric:**
Dielectric is nothing but the insulating material by which plates of capacitor are separated.
- Air, paper, mica etc. can be used as a dielectric.

Que. Derive the equation of a capacitance for parallel plate capacitor.

Ans. → For deriving the eq^{n}, lets consider a capacitor which have two parallel plates of area A. m^2 as shown in fig 4(A).

→ Both the plates are separated by a dielectric of thickness 'd' metres having relative permittivity ε_r.

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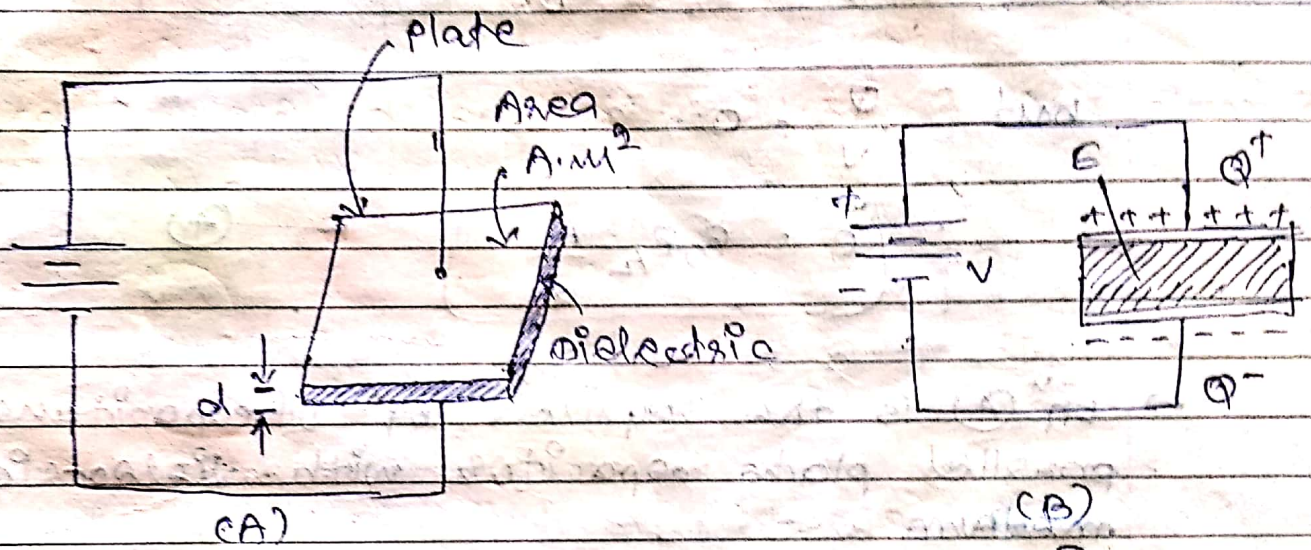
→ Now, when the voltage is applied across the capacitor then +Q and -Q charges are established on plate-A and plate-B respectively shown in fig. Q1(B).



[Fig.1 Parallel-plate capacitor]

→ Now, as we know that, electric flux density in the dielectric is given by,

$$D = \frac{\psi}{A}$$

$$= \frac{Q}{A} \quad \text{---(1)} \quad (Q \propto \psi)$$

and electric field & strength in the dielectric is given by,

$$E = \frac{V}{d} \quad \text{---(2)}$$

→ We also know that,

$$D = \varepsilon_0 \varepsilon_r E$$

from eq⁴ (2), $D = \varepsilon_0 \varepsilon_r \frac{V}{d}$ ---(3)

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→ Now, compare eq^{n} ① & ⑤, we get

$$\frac{Q}{A} = \varepsilon_0 \varepsilon_r \frac{V}{d}$$

$$\therefore \frac{Q}{V} = \varepsilon_0 \varepsilon_r \frac{A}{d}$$

but $\frac{Q}{V} = C$

$$\therefore \boxed{C = \varepsilon_0 \varepsilon_r \frac{A}{d}} \quad \longleftarrow ④$$

→ Eq^{n} ④ is the required eq^{n} of capacitance for parallel plate capacitor with dielectric medium.

→ Now, if the air is the dielectric medium then the eq^{n} ④ becomes,

$$\boxed{C = \varepsilon_0 \frac{A}{d}} \quad (\because \text{for air } \varepsilon_r = 1)$$

Que. Derive the equation of capacitance for a parallel plate capacitor with composite dielectrics.

OR

Prove that : $C = \dfrac{\varepsilon_0 A}{\sum \frac{d}{\varepsilon_r}}$

Ans: For the derivation of the eq^{n}, let's consider a capacitor having three dielectric materials shown in below fig. 5

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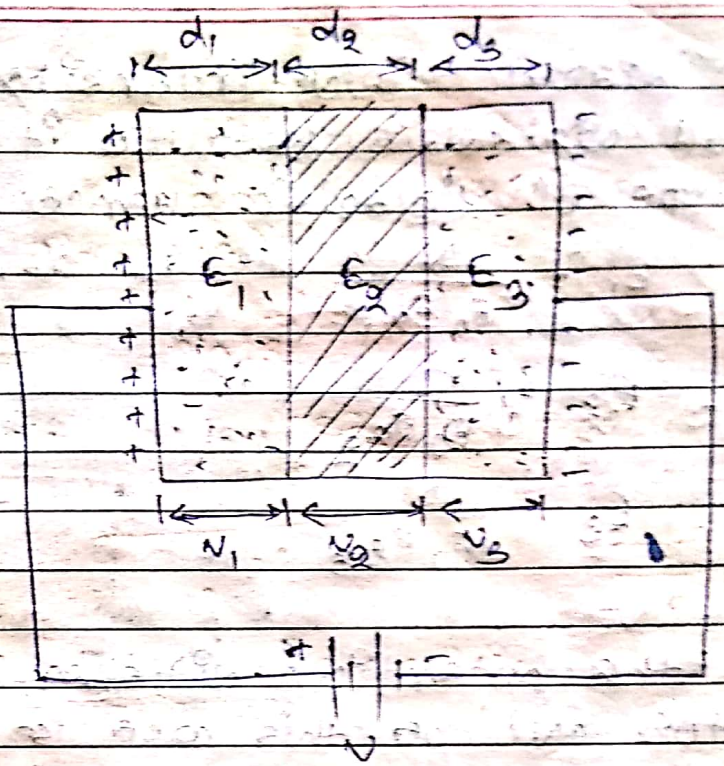

[Fig.5 Parallel plate capacitor with composite dielectrics]

→ Let's here,

d_1, d_2, d_3 = thickness of dielectrics & in m.

$\epsilon_{r1}, \epsilon_{r2}, \epsilon_{r3}$ = Relative permittivities of dielectrics

E_1, E_2, E_3 = Field intensities in dielectrics

V_1, V_2, V_3 = Potential difference across the dielectrics

→ The electric flux density D is same for all dielectric charge because it depends upon the charge and same charge would flow through all dielectrics. Hence,

$$D = \frac{Q}{A}$$

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while electric field intensity E is different for all dielectric because it depends upon the type of dielectric. Hence it is given by,

$$E_1 = \frac{D}{\varepsilon_0 \varepsilon_{r_1}}$$

$$E_2 = \frac{D}{\varepsilon_0 \varepsilon_{r_2}}$$

$$E_3 = \frac{D}{\varepsilon_0 \varepsilon_{r_3}}$$

→ Now, the potential difference across the capacitor (V) for this case is becomes the sum of the potential differences across three dielectrics.

$$V = V_1 + V_2 + V_3$$

but as we know that, $E = V/d$

$$\therefore V = E \cdot d$$

$$V = E_1 d_1 + E_2 d_2 + E_3 d_3$$

$$= \frac{D}{\varepsilon_0 \varepsilon_{r_1}} d_1 + \frac{D}{\varepsilon_0 \varepsilon_{r_2}} d_2 + \frac{D}{\varepsilon_0 \varepsilon_{r_3}} d_3$$

$$= \frac{D}{\varepsilon_0} \left[\frac{d_1}{\varepsilon_{r_1}} + \frac{d_2}{\varepsilon_{r_2}} + \frac{d_3}{\varepsilon_{r_3}} \right]$$

but $D = \frac{Q}{A}$

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$$\therefore V = \frac{Q}{\varepsilon_0 A}\left[\frac{d_1}{\varepsilon_{r1}} + \frac{d_2}{\varepsilon_{r2}} + \frac{d_3}{\varepsilon_{r3}}\right]$$

$$\therefore \frac{Q}{V} = \frac{\varepsilon_0 A}{\frac{d_1}{\varepsilon_{r1}} + \frac{d_2}{\varepsilon_{r2}} + \frac{d_3}{\varepsilon_{r3}}}$$

but $\frac{Q}{V} = C$

$$\therefore C = \frac{\varepsilon_0 A}{\frac{d_1}{\varepsilon_{r1}} + \frac{d_2}{\varepsilon_{r2}} + \frac{d_3}{\varepsilon_{r3}}} \quad \text{---(1)}$$

→ Eq^n (1) is the required eq^n of capacitance for parallel plate capacitor with three dielectrics.

→ But, if the no. of dielectrics are given, then eq^n (1) becomes

$$C = \frac{\varepsilon_0 A}{\sum \frac{d}{\varepsilon_r}}$$

Que. Derive the equation of a equivalent capacitance when capacitors are connected in series.

OR

Prove that : $\frac{1}{C_{eq.}} = \sum_{n=1}^{n} \frac{1}{C_n}$

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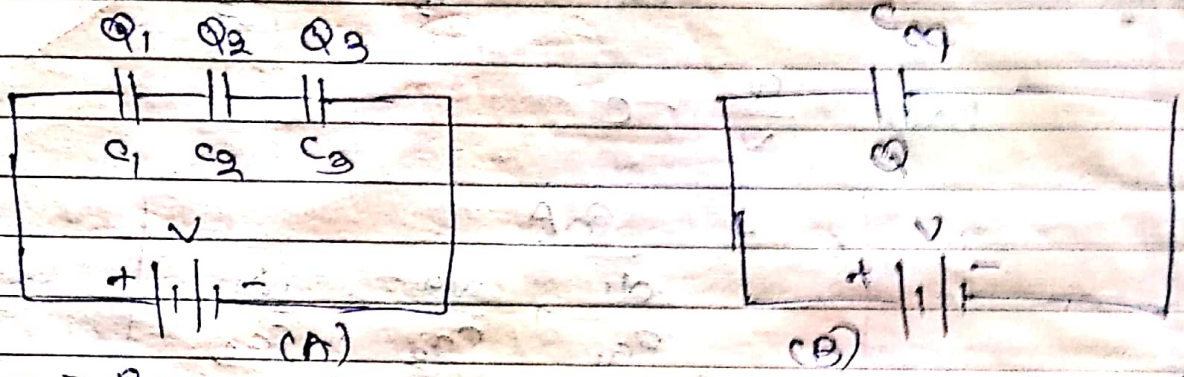
Ans. For deriving the eq^n of a equivalent capacitance, let's consider that three capacitors having its capacitance C_1, C_2 & C_3 are connected in series as shown in fig below:-



[Fig. 6 series connection of capacitors]

→ Here, all three capacitors are connected in series, therefore the current flow through the each capacitor is same at any point.

→ Therefore, let's consider that for time t seconds, the charging current I flows through each capacitor.

→ Hence, due to flow of currents (electrons), the charges are developed on each capacitor. Hence,

$$Q_1 = Q_2 = Q_3 = Q = It \quad \text{---(1)}$$

→ As we know that, when the voltage is applied across the connection then it must

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be equal to the sum of voltages across each capacitors. It means:-

$$V = V_1 + V_2 + V_3 \quad \text{——— (2)}$$

→ But, we know that, $C = Q/V$

$$\therefore V = Q/C$$

therefore, from eq^n (2),

$$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

→ Here, we have to derive the eq^n for a equivalent capacitance. Therefore, let's replace the series connection shown in fig. 6(A). by a equivalent capacitance $C_{eq.}$ shown in fig. 6(B). Therefore,

$$V = \frac{Q}{C_{eq.}}$$

$$\therefore \frac{Q}{C_{eq.}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\therefore \boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} \quad \text{——— (3)}$$

→ Eq^n (3), indicates the eq^n of a equivalent capacitance when three capacitors are connected in series.

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→ But → If n numbers of capacitors are connected in series, then eq⁴ (5) becomes,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_n}$$

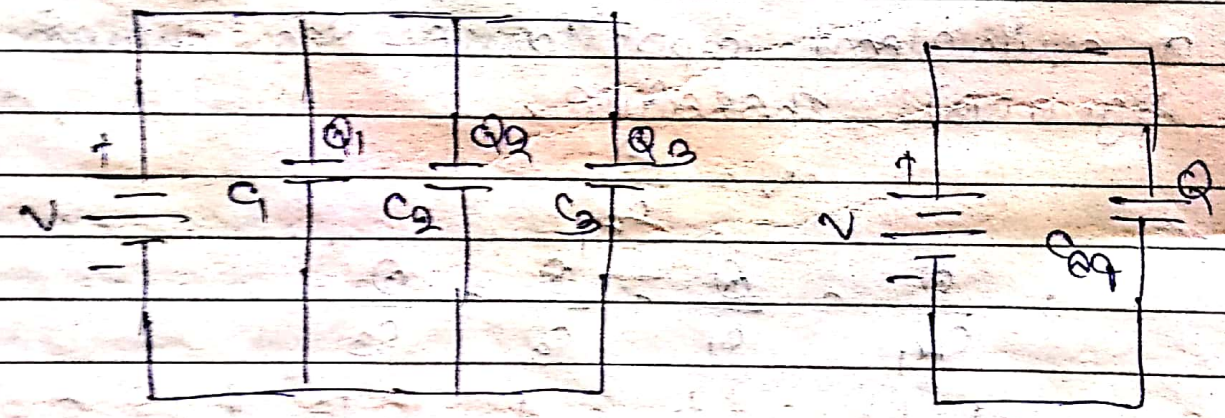
$$\therefore \boxed{\frac{1}{C_{eq}} = \sum_{n=1}^{n} \frac{1}{C_n}}$$

Que. Derive the eq⁴ of a equivalent capacitance when capacitors are connected in parallel.

OR

Prove that: $C_{eq} = \sum_{n=1}^{n} C_n$

Ans.



[Fig. 2(a): parallel connection] [Fig. 2(b) Equ. ckt]

→ For deriving the eq⁴, of a equivalent capacitance, let's consider three capacitors are connected in parallel as shown in fig. 2(a).

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→ When a voltage is applied across the parallel connection of capacitors, then due to closed ckt, current will flow through each capacitor.

→ Here, due to parallel connection, the current flows through the each capacitor is different.

→ Hence, due to different flow of current, the charge produced by each capacitor becomes different, i.e., $Q_1, Q_2, Q_3, \ldots$ etc.

→ For deriving the eq^n for equivalent capacitance, let's consider that the total charge produced by parallel connection must be the sum of charges produced by each capacitor, i.e.:

$$Q = Q_1 + Q_2 + Q_3 \quad \text{---(1)}$$

but, $C = \frac{Q}{V}$

$$\therefore Q = CV$$

→ From eq^n (1),

$$Q = C_1V + C_2V + C_3V$$

→ Here, we need to find out the equivalent capacitance (C_{eq}). Therefore let's replace the parallel connection shown in fig. #(A), by eqv. capacitance shown in fig. #(B).

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→ Eq⁴ (2) indicates the equivalent capacitance when these capacitors are connected in parallel.

→ But $\therefore C_{eq} V = C_1 V + C_2 V + C_3 V$

$$\therefore \boxed{C_{eq} = C_1 + C_2 + C_3} \quad \text{---(2)}$$

→ Eq⁴ (2) shows the eq. capacitance when three capacitors are connected in parallel.

→ But, for n number of capacitors connected in parallel, eq⁴ (2) becomes

$$C_{eq} = C_1 + C_2 + C_3 + \ldots + C_n$$

$$\boxed{C_{eq} = \sum_{n=1}^{n} C_n}$$

Que. What is capacitor? Explain the capacitor action in detail.

Ans. capacitor is a device which stores the electric charge.

→ It consists of two conducting surfaces (plates) separated by an insulating material of dielectric in between.

→ It uses Air, paper, mica, ceramic etc. as a dielectric b/w plates.

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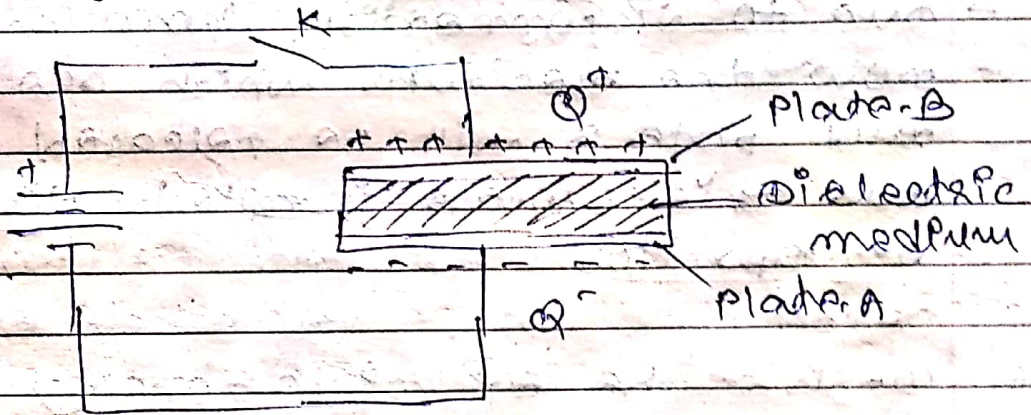
→ A capacitor may be in the form of parallel plates, concentric cylinders, concentric spheres etc. according the shape of the plates.

* Capacitor Action:

→ As we have discussed that the capacitor is used to stored the energy, but how? so, let's understand this concept by considering a below fig. 8



[Fig. 8]

→ When the DC supply is connected across the capacitor then..

→ Due to closed ckt & potential difference, electrons starts flow from negative terminal and positive terminal of the battery.

→ Electrons from negative terminal flows towards the bottom plate A.

→ Due to the presence of dielectric b/w two plates, electrons from plate A does not go further. Therefore, they will remain (collect) at plate A.

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→ Likewise, electrons from positive terminal collects at plate B.
→ Due to collection of charges, plate A & plate B starts to storage of electrons charging and this process is called charging of a capacitor.

→ Now, when the switch K shown in fig. 3) is open, then capacitor disconnects from the supply.
→ Due to disconnection from the supply, now the electrons which are on plate A and plate B will be released into air and this process is called discharging of capacitor.

Que. Derive the eqn of a energy stored in a charged capacitor.

OR

Prove that: $E = \frac{1}{2} \frac{Q^2}{C}$

Ans. → As we know that, when the potential difference is applied across the capacitor, then it starts to charging.
. It means, electrons start to flow from voltage terminals to the plates of capacitor and this transferring of electrons is called workdone.

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→ suppose let's take any instant of charging. At this instant, potential difference across the plates will be V volts.

→ After sometimes, suppose dq amount of charges is transferred then total workdone is given by,

$$dW = V \cdot dq \quad \text{——— (1)}$$

∴ but $C = Q/V$

$$\therefore Q = CV$$

take the differentiation on both sides, we get

$$\therefore dq = C \cdot dV \quad \text{——— (2)}$$

→ put the value of eq^n (2) into eq^n (1), we get

$$dW = V \cdot dq$$
$$= V \cdot C\,dV$$

→ For finding total workdone, take the integration on both sides, we get

$$\therefore \int dW = \int V \cdot C \cdot dV$$

→ As we have seen that, capacitor starts charging when it gets voltage across it and it still charging upto max. volt. V. Hence limit of voltage becomes $0 \to V$

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$$\int dW = \int_0^V V \cdot C \cdot dV$$

$$W = C \int_0^V V \cdot dV$$

$$= C \left[\frac{V^2}{2}\right]_0^V$$

$$= C \left[\frac{V^2}{2} - 0\right]$$

$$\boxed{W = \frac{1}{2} C V^2}$$

→ Due to this workdone, energy will be stored into the capacitor. So, we can take;

Workdone = Energy (E) = $\frac{1}{2} CV^2$

$$E = \frac{1}{2} (CV) V$$

$$= \frac{1}{2} Q \cdot V \quad (\because Q = CV)$$

$$= \frac{1}{2} Q \cdot \frac{Q}{C} \quad (\because V = Q/C)$$

$$\boxed{E = \frac{1}{2} \frac{Q^2}{C} \text{ joules}} \quad \text{---} (3)$$

→ Eq^n (3) indicates the eq^n of energy stored into capacitor.

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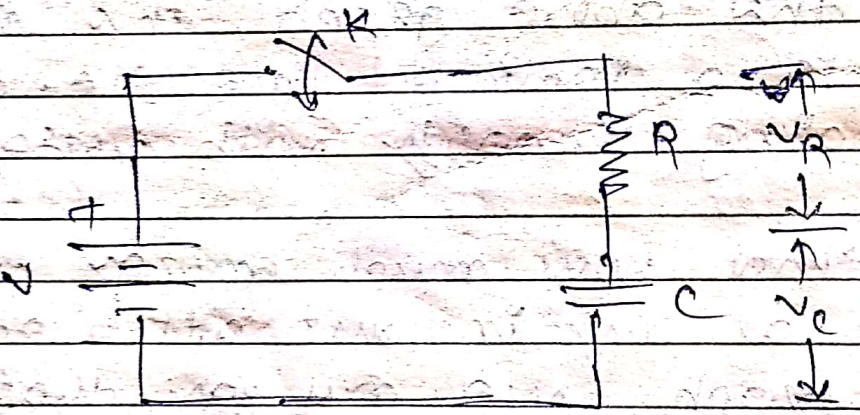
Que. Explain the charging of a capacitor through resistor.

OR

Derive the eq^n, $V_c = V(1 - e^{-t/RC})$

Ans. For understanding the concept of a charging capacitor through resistor, let's consider Fig. 10:



[Fig. 10 charging of a capacitor through resistor]

→ Fig. 10 shows that one resistor R is connected in series with capacitor.

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→ When the supply is connected across capacitor then it starts charging through resistor i.e. charging of capacitor can be controlled by the resistor.

→ Due to potential difference, same current would flow through capacitor as well as resistor.

$$i_C = i_R \quad \text{——(1)}$$

→ As we know that, $C = Q/V$

$$\therefore Q = CV$$

$$Q = CV_C$$

Q = charge on capacitor
C = capacitance of a capacitor
V_C = voltage across the capacitor

→ Take differentiation w.r. to time on both sides, we get

$$\therefore \frac{dq}{dt} = C \cdot \frac{dV_C}{dt}$$

but $q = it \Rightarrow i_C = \frac{q}{t} = \frac{dq}{dt}$

→ so, $i_C = C \cdot \frac{dV_C}{dt}$ ——(2)

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→ While the current through resistor is given by,

$$i_R = \frac{V_R}{R}$$

but from fig. 10, $V_R = V - V_C$

$$\therefore i_R = \frac{V - V_C}{R} \quad \text{---(3)}$$

→ Put eq^n (2) & (3) in eq^n (1), we get

$$C \frac{dV_C}{dt} = \frac{V - V_C}{R}$$

$$\therefore RC \cdot \frac{dV_C}{dt} = V - V_C$$

$$\therefore \frac{1}{V - V_C} dV_C = \frac{1}{RC} dt$$

→ Now, take the integration on both sides, we get,

$$\int \frac{dV_C}{V - V_C} = \int \frac{dt}{RC}$$

$$\therefore -\ln(V - V_C) + K = t/RC$$

where K is constant of integration. The value of K is obtained by considering initial conditions i.e. $V_C = 0$, $t = 0$

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$$\therefore \frac{0}{RC} = -\ln(V-0) + K$$

$$\therefore 0 = -\ln V + K$$

$$\therefore K = \ln V$$

→ Put the value of K into eq^{n} (1), we get

$$\therefore \frac{t}{RC} = -\ln(V-V_c) + \ln V$$

$$\therefore \frac{t}{RC} = \ln\left(\frac{V}{V-V_c}\right)$$

$$\therefore \frac{V}{V-V_c} = e^{t/RC}$$

$$\therefore \frac{V}{e^{t/RC}} = V - V_c$$

$$\therefore V \cdot e^{-t/RC} = V - V_c$$

$$\therefore V_c = V - V e^{-t/RC}$$

$$\boxed{V_c = V(1 - e^{-t/RC})}$$

where $RC = \lambda$ = time constant

$$\therefore \boxed{V_c = V(1 - e^{-t/\lambda})}$$

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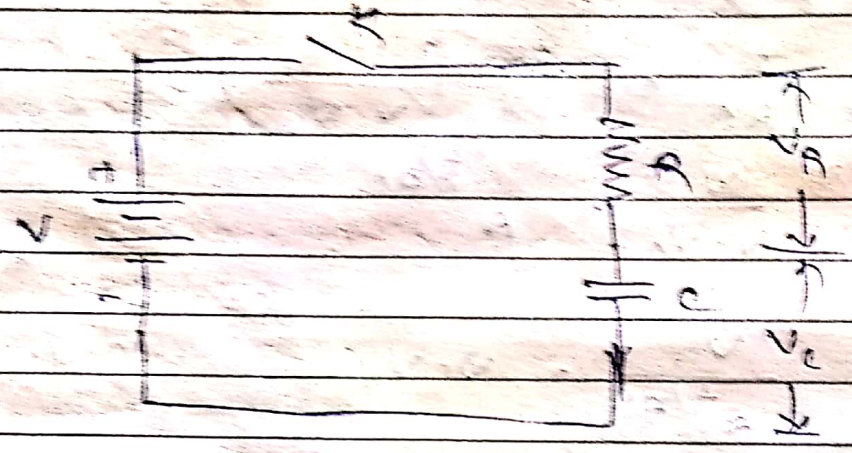
Que: Explain the discharging of a capacitor through resistor

OR

Derive the eqⁿ, $v_c = Ve^{-t/RC}$

Ans: For understanding this concept, let's consider the fig. 11 which is given below



[Fig.11 discharging of a capacitor through resistor]

→ From fig.11, it's clear that, here switch K is open i.e. supply is disconnected from the ckt.

→ Hence, capacitor starts discharging through the resistor.

→ Hence, discharging current flows through R and C will be equal but opposite.

$$i_c = -i_R \quad \text{---(1)}$$

→ As we know that, $C = Q/V$

$$\therefore Q = CV$$

$$Q = CV_c$$

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Q = charge on capacitor
C = capacitance of a capacitor
V_C = Volt. across capacitor

→ Take differentiation w.r.t time on both sides, we get,

$$\frac{dq}{dt} = C \cdot \frac{dV_C}{dt}$$

but $Q = It \Rightarrow I_C = Q/t = \frac{dq}{dt}$

$$\therefore I_C = C \cdot \frac{dV_C}{dt} \quad \text{——(2)}$$

→ While current through resistor is given by,

$$I_R = \frac{V_R}{R}$$

→ But from fig., $V_R = V_C$

$$\therefore I_R = \frac{V_C}{R} \quad \text{——(3)}$$

→ Put eq^n (2) & (3) in eq^n (1), we get,

$$C \cdot \frac{dV_C}{dt} = -\frac{V_C}{R}$$

$$RC \cdot \frac{dV_C}{dt} = -V_C$$

$$-\frac{dV_C}{V_C} = \frac{dt}{RC}$$

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→ Now, take integration on both sides, we get

$$\int \frac{dt}{RC} = \int \frac{dV_c}{-V_c}$$

$$\therefore \frac{t}{RC} = -\ln V_c + K \quad \text{---(1)}$$

where K is constant of integration. the value of K can be obtain by considering initial conditions, i.e., $t = 0$, $V_c = V$

$$\therefore \frac{0}{RC} = -\ln V + K$$

$$K = \ln V$$

→ put the value of K into eq(1), we get

$$\frac{t}{RC} = -\ln V_c + \ln V$$

$$\therefore \frac{t}{RC} = \ln (V/V_c)$$

$$\therefore V/V_c = e^{t/RC}$$

$$\therefore V = V_c \cdot e^{t/RC}$$

$$\therefore \boxed{V_c = V \cdot e^{-t/RC}}$$

where $RC = \lambda$ = time constant

$$\therefore \boxed{V_c = V \cdot e^{-t/\lambda}}$$

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* Examples based on capacitors:

Ex.1 A capacitor consists of two parallel square plates each 120 mm side separated by 1 mm in air. When a voltage of 1000 V is applied b/w the plates, an average current of 12 mA flows for 5 sec. calculate,

(a) The charge on the capacitor
(b) The electric flux
(c) The electric flux density
(d) The ele. field strength in the dielectric

Ans. Given,

$V = 1000\ V$

$I = 12\ mA = 12 \times 10^{-3}\ A$

$t = 5$ sec.

$d = 1\ mm = 1 \times 10^{-3}\ m$

$A = 120\ mm \times 120\ mm$

$= 144 \times 10^{2} \times 10^{-6} = 144 \times 10^{-4}\ m^2$

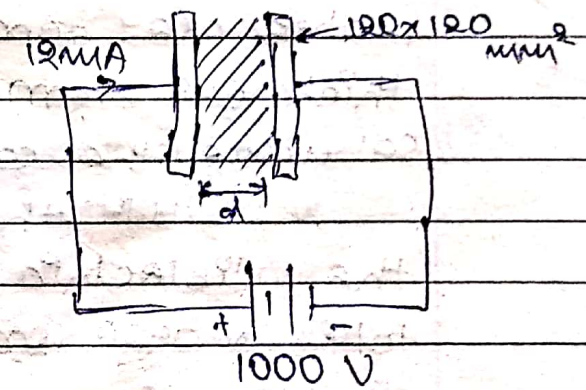
Find,

$Q = (?)$

$\psi = (?)$

$D = (?)$

$E = (?)$



(A) charge $Q = It$

$= 12 \times 10^{-3} \times 5$

$= 60 \times 10^{-3}\ C$

$\boxed{Q = 60\ mC}$ — Ans. (A)

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(B) As we know, $Q = \Psi$

$\therefore \boxed{\Psi = 60\ mC}$ — Ans (B)

(C) The ele. flux density,

$$D = \frac{\Psi}{A}$$

$$= \frac{60 \times 10^{-3}}{14 \times 10^{-1}}$$

$$\boxed{D = 4.184\ C/m^2}$$ — Ans (C)

(D) The ele. field strength,

$$E = V/d$$

$$= \frac{1000}{1 \times 10^{-3}}$$

$$\boxed{E = 1 \times 10^6\ V/m}$$ — Ans (D)

Ex. 2 A capacitor is constructed from the square metal plates each of side 120 mm. The plates are separated by a dielectric of thickness 2 mm and relative permittivity 5. Calculate the capacitance.

If the electric field strength in the dielectric is 12.5 kV/mm, calculate the total charge on each plate.

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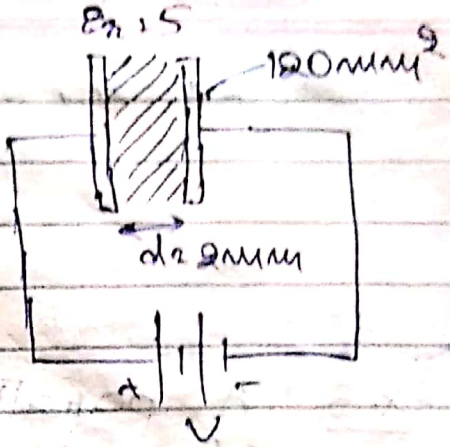

Ans: Given,

$A = 120\ mm$

$= 120 \times 120\ mm^2$ (2 plates)

$= 144 \times 10^{-4}\ m^2$

$d = 2\ mm$

$= 2 \times 10^{-3}\ m$

$\varepsilon_r = 5$

$B = 12.5\ KV/mm$

$= 12.5 \times 10^3\ V/mm = 12.5 \times 10^6\ V/m$

Find, C (1)

Q (1)

(A) As we know that, the eqⁿ for parallel plate capacitor is given by,

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

$$= \frac{8.854 \times 10^{-12} \times 5 \times 144 \times 10^{-4}}{2 \times 10^{-3}}$$

$$= 318.7 \times 10^{-12}\ F$$

$$C = 318.7\ pF$$ — Ans (1)

(B) charge, $Q = CV$

Here, voltage is not given but electric field strength is given. Hence we know that,

$E = V/d$

$\therefore V = Ed$

$= 12.5 \times 10^3 \times 2$ ← (in mm)

$V = 25 \times 10^3\ V$ (as E is in V/mm)

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Hence, $Q = CV$

$= 313.2 \times 10^{-12} \times 25 \times 10^{3}$

$= 7.983 \times 10^{-6}$ C

$\boxed{Q = 7.983\ \mu C}$ — Ans (a)

Ex. 3 A parallel plate capacitor has plates of area $1.5\ m^2$. It has three dielectrics 1 mm, 1.6 mm and 2 mm thick. The relative permittivities of these dielectrics are 2, 4 and 5 respectively; calculate the capacitance of the capacitor and the electric field strengths in the dielectrics if a voltage of 2.5 kV is applied b/w the plates

Ans: Given,

$A = 1.5\ m^2$

$d_1 = 1\ mm = 1 \times 10^{-3}\ m$

$d_2 = 1.6\ mm = 1.6 \times 10^{-3}\ m$

$d_3 = 2\ mm = 2 \times 10^{-3}\ m$

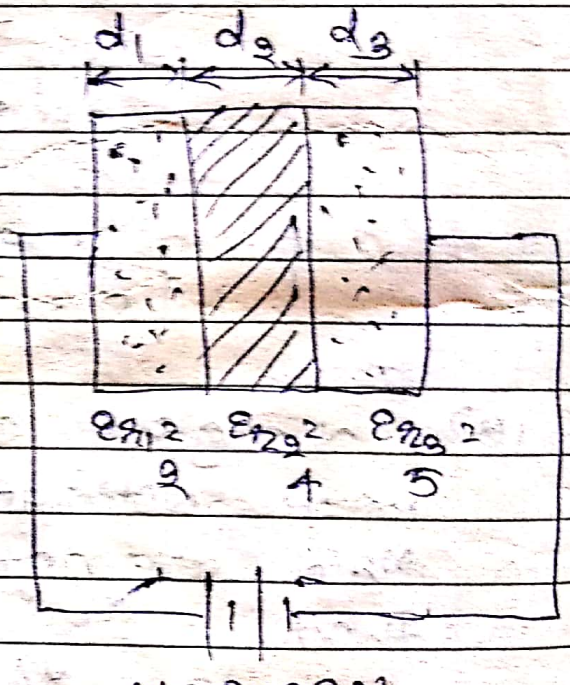
$\varepsilon_{r_1} = 2$

$\varepsilon_{r_2} = 4$

$\varepsilon_{r_3} = 5$

$V = 2.5\ kV = 2.5 \times 10^{3}\ V$

$= 2500\ V$



Find,

$C = (?)$

$E_1 = (?)$

$E_2 = (?)$

$E_3 = (?)$

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Ans. (1) As we know the eqⁿ of a capacitance of a parallel plate with composite dielectrics and it is given by,

$$C = \frac{\varepsilon_0 A}{\frac{d_1}{\varepsilon_{r_1}} + \frac{d_2}{\varepsilon_{r_2}} + \frac{d_3}{\varepsilon_{r_3}}}$$

$$= \frac{8.854 \times 10^{-12} \times 1.5}{\left(\frac{1}{2} + \frac{1.6}{4} + \frac{2}{5}\right) \times 10^{-3}}$$

$$= 10.216 \times 10^{-9} \text{ F}$$

$$\boxed{C = 10.216 \text{ nF}}$$ Ans (1)

(2) As we know that, electric field strength for dielectric is given by,

$$E = \frac{D}{\varepsilon_0 \varepsilon_r}$$

where, $D = \frac{Q}{A}$

but $Q = CV$

$$= 10.216 \times 10^{-9} \times 2500$$

$$Q = 25.54 \times 10^{-6} \text{ C}$$

$$\therefore D = \frac{25.54 \times 10^{-6}}{1.5}$$

$$D = 17.027 \times 10^{-6} \text{ C/m}^2$$

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∴ Hence, $E_1 = \frac{D}{\varepsilon_0 \varepsilon_{r1}}$

$= \frac{17.097 \times 10^{-6}}{8.854 \times 10^{-12} \times 2}$

$\boxed{E_1 = 0.9815 \times 10^6 \text{ V/m}}$ — Ans (2)

$E_2 = \frac{D}{\varepsilon_0 \varepsilon_{r2}}$

$= \frac{17.097 \times 10^{-6}}{8.854 \times 10^{-12} \times 4}$

$\boxed{E_2 = 0.481 \times 10^6 \text{ V/m}}$ — Ans (3)

$E_3 = \frac{D}{\varepsilon_0 \varepsilon_{r3}}$

$= \frac{17.097 \times 10^{-6}}{8.854 \times 10^{-12} \times 5}$

$\boxed{E_3 = 0.3848 \times 10^6 \text{ V/m}}$ — Ans (4)

Ex. 9.4 Two capacitors having capacitances of 6 μF and 10 μF respectively are connected in parallel. A 16 μF capacitor is connected in series with the parallel combination and the complete ckt is connected across a 400 V DC supply. Calculate,

(A) total capacitance of the ckt

(B) The voltage across each capacitor

(C) Total charge in the circuit

(D) charge on capacitor.

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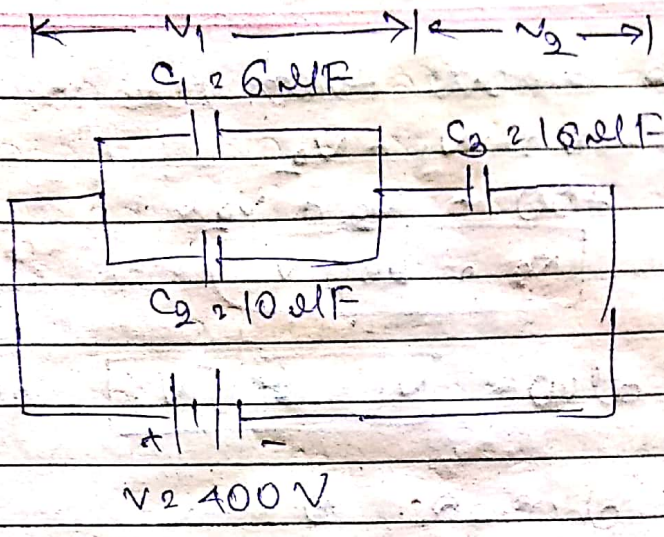
Ans.



(1) Here, C_1 and C_2 are connected in parallel.

Hence, $C_{12} = 6 + 10$

$= 16\ \mu F$

and $C_3 = 16\ \mu F$ capacitance is connected in series with parallel combination.

Hence,

total capacitance $= \frac{16 \times 16}{16+16}$

$\boxed{C_T = 8\ \mu F}$ — Ans (1)

(2) As we know that, $Q = CV$

$Q = C_p V_1 = C_3 V_2$

$\frac{V_1}{V_2} = \frac{C_3}{C_p}$

$= \frac{16}{16}$

$\frac{V_1}{V_2} = 1$

$\therefore V_1 = V_2$

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As we know that;

$V = V_1 + V_2$

$400 = V_2 + V_2$

$\boxed{V_2 = 200\ V}$ ——— Ans. (2)

and also $\boxed{V_1 = 200\ V}$ $(V_1 = V_2)$ ——— Ans. (2)

(3) Total charge $Q = CV$

$= 8\ \mu F \times 400$

$= 8 \times 10^{-6} \times 400$

$= 3.2 \times 10^{-3}\ C$

$\boxed{Q = 3.2\ mC}$ ——— Ans. (3)

(4) charge on capacitor (Q_1)

$Q_1 = C_1 V_1$

$= 6 \times 10^{-6} \times 200$

$\boxed{Q_1 = 1.2\ mC}$ ——— Ans. (4)

charge on capacitor (C_2)

$Q_2 = C_2 V_2$

$= 10 \times 10^{-6} \times 200$

$\boxed{Q_2 = 2\ mC}$ ——— Ans (4)

charge on capacitor (C_3)

$Q_3 = C_3 V_3$

$= 16 \times 10^{-6} \times 200$

$\boxed{Q_3 = 3.2\ mC}$ ——— Ans. (4)

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