

Unit - 1

classmate

Date _____
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* Neural Network :

- A processing device, either an algorithm or an actual hardware
- Inspired by brains.
- Ability to learn by example
- Flexible and powerful

⇒ Artificial Neural Networks :

- An information - processing model, inspired by the way biological nervous systems (brain) process information.
- Learn by example as people
- Specific applications
- Composed of large number of highly interconnected processing elements (Neurons) working in unison to solve specific problems.
- Ex. Applications - pattern recognition
data classification

• Advantages :

- Adaptive Learning

Ability to learn how to do tasks based on data given for training.

- Self-organization

Creates its own organization/representation of info. it receives during training.

- Real-time operation:
Computations may be carried out in parallel

- Fault tolerance via redundant info. Coding:
 • Partial destruction of NN leads to
 • degradation of performance.
 • Some capabilities may be retained.

⇒ Problems:

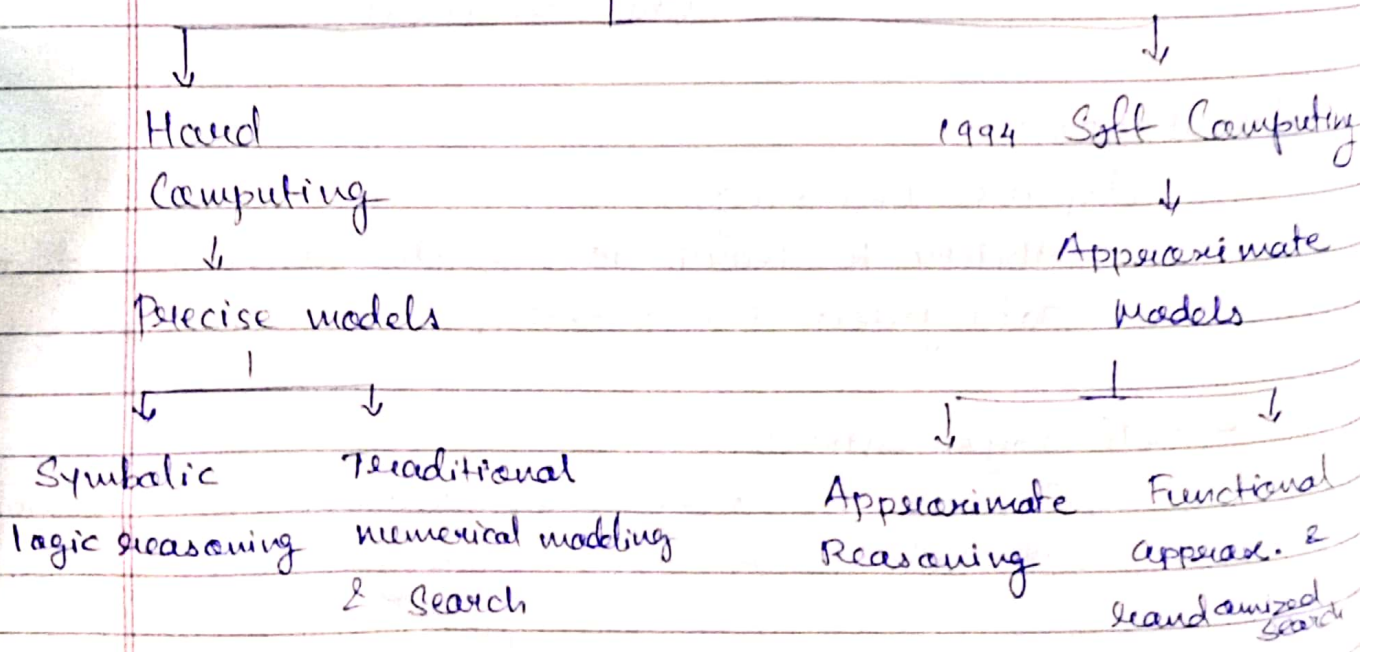
- Provide a reason behind computer generated answer - why loan denied.
- How network learned, why it recommends a particular decision - Black box

⇒ Applications:

- Bank

* Soft Computing:

→ Problem-solving technologies:



→ Combination of GA, NN, FL

→ Hybrid technique:

- good learning capacity
- better learning time
- less sensitivity to the prob. of local extremes
- Generate fuzzy knowledge base (linguistic representation & low complexity)

* ANN:

→ Neurons / Nodes / Units :

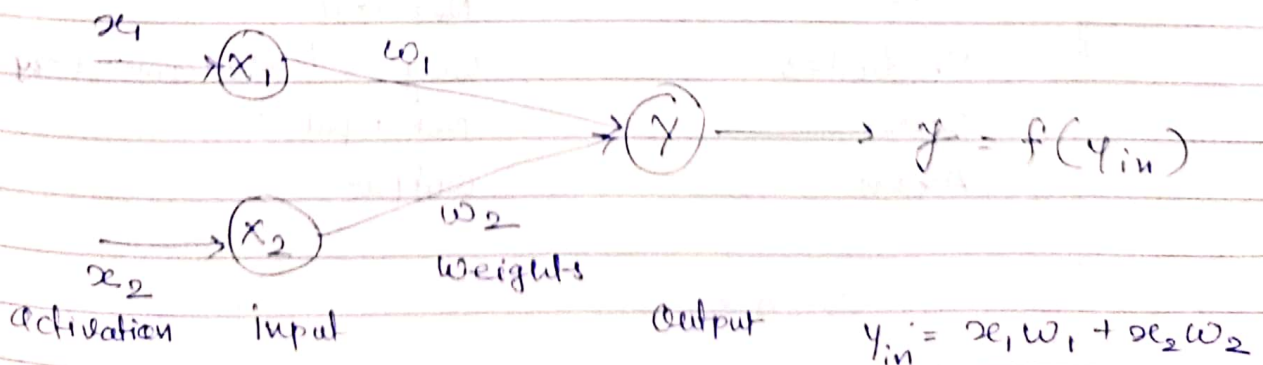
- operate in parallel

→ Connection links:

- Each neuron is connected with others
- It has associated weights - contains info. about the input signal - used by NN to solve a problem.

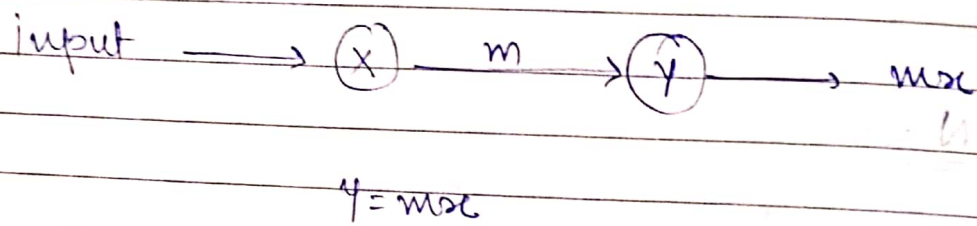
→ Activation / activity level:

- Each neuron has internal state
- function of the inputs the neuron receives
- transmits activation signal to others, only one signal at a time.



→ The output y of output neuron y can be obtained by applying activations over the net input (function of net input)

→ The function to be applied over the net input is called activation function.



* Biological Neural Network:

→ Biological Neuron:

- 3 Parts -
 - Soma / Cell body
 - Dendrites - connected
 - Axon - carries impulses

$$\rightarrow y_{in} = x_1 w_1 + \dots + x_n w_n = \sum_{i=1}^n x_i w_i$$

<u>BNN</u>	Vs	<u>ANN</u>
Cell		Neuron
Dendrites		Weights / interconnections
Soma		Net input
Axon		Output

→ Comparison:

- | | Brain | | Computer |
|---------------------|--|---|---|
| • Speed | Milliseconds | - | Nanoseconds |
| • Processing | Parallel | - | parallel - Faster |
| • Size & complexity | Higher | - | depends on design |
| • Memory | Store more
Retrieving not always good | - | Storage can be avoided
Retrieving is good
More adaptability |
| • Tolerance | Fault tolerant capability | - | No fault tolerance |
| • Central Mechanism | No central unit
Interconnections & strengths based on chemicals | - | Central unit
Simpler interconnec.
No chemical actions |

NEEDS

* Evolution of NN:

In book - Self study

* Basic models of ANN:

1) Connections

- Single layer feed forward
- Multilayer "
- Single node with its own feedback
- Single layer recurrent
- Multi layer "

2) Learning

i) Supervised -

- Input vector has corresponding target vector

- During training - input vector → output vector
- Actual output vector compare target vector
- Difference - error signal
- Adjustments of weights

ii) Unsupervised

- Input vectors of similar type are grouped without using training data.

iii) Reinforcement learning

- Only critic information is available not exact info.
- Receives feedback from its environment

3) Activation Functions

- Applied over the net input to calculate the output of an ANN.
- Why non linear?
 - Multilayer network - output obtained remains same as single layer while using linear funⁿ

- i) Identity fun.
- ii) Binary step fun
- iii) Bipolar step fun
- iv) Sigmoidal
 - Binary
- v) Ramp
 - Bipolar

* Important Terminologies :

1) Weights :

- Contains information about Input Signal
- Info. is used to solve a problem.
- Matrix form $m \times n$ - processing elements
- m no. of weights per neuron

2) Bias :

- Impact in calculating the net input
- Considered like another weight
- By adding a component $x_0 = 1 \rightarrow b_j$

3) Threshold: (θ)

- Set value based upon which the final output of the network may be calculated.

4) Learning Rate: (α)

- To control the amount of weight adjustment at each step of training. (0 to 1)

5) Momentum Factor:

- Back propagation network
- Convergence is faster if momentum added to weight updation process

6) Vigilance Parameter (ρ)

- ART network
- To control degree of similarity required for patterns to be assigned to same cluster unit.
- (0.7 to 1)

Example 1:

→ AND Function (1 - 1.9)

x_1	x_2	y	θ	y_{in}
0	0	0		
0	1	0		
1	0	0		
1	1	1		

$$\begin{aligned}
 y_{in} > \theta &\Rightarrow y = 1 \\
 y_{in} \leq \theta &\Rightarrow y = 0
 \end{aligned}$$

Input 1:

$$x_1 = 0, x_2 = 0, y = 0$$

$$\begin{aligned}
 y_{in} &= 0 & x_1 w_1 + x_2 w_2 \\
 y &= 0
 \end{aligned}$$

$$y_{in} \leq \theta \Rightarrow y = 0 \quad (0 \leq \theta)$$

$$\theta = [0 \dots 1] \quad \theta = 0.1 \dots 0.9 \quad \theta > 0$$

take $\theta = 1 \Rightarrow$ Checks for all conditions

Input 2:

$$x_1 = 0, x_2 = 1, y = 0$$

$$y_{in} = 1$$

$$y_{in} \leq \theta \Rightarrow y = 0$$

$$(1 \leq \theta) \Rightarrow \theta = 1 \text{ True}$$

$$1 \leq 1 \Rightarrow 1 > \theta$$

Input - 3

$$x_1 = 1, x_2 = 0, y = 0$$

$$y_{in} = 1$$

$$y_{in} \leq 0 \Rightarrow y = 0$$

$$\text{for } 0 = 1 \Rightarrow 1 \leq 0 = 1 \leq 1 \quad \checkmark$$

True

Input - 4

$$x_1 = 1, x_2 = 1, y = 1$$

$$y_{in} = 2$$

$$y_{in} > 0 \Rightarrow y = 1$$

$$2 > 0 \Rightarrow 2 > 1 \quad \text{True}$$

$2 > 2^1$

So the range is (1.. 1.9)

$$0 = [1 - 1.9]$$

Example 2:

→ OR Function

x_1	x_2	$y \Rightarrow 0$	y_{in}
0	0	0	
0	1	1	
1	0	1	
1	1	1	

AND

$$x_1 = [0 \ 0 \ 1 \ 1]$$

$$x_2 = [1 \ 0 \ 1 \ 0]$$

$$w = 1$$

$$t = 1$$

$$y = x_1 * w + x_2 * w$$

for $i = 1 : 4$

if $v(i) > t$

$y(i) = 1$

else

$y(i) = 0$

end

end

disp(y)

out - 0 1 1 2
0 0 0 1

OR

$t = 0$

if $v(i) > t$

$y(i) = 0$

else

$y(i) = 1$

* Linear Separability:

Exact

→ ANN does not provide a solution for nonlinear problem.

→ The separation of the input space into regions is based on whether the network response is positive or negative.

→ Decision line:

• Bipolar

$$b + \sum_{i=1}^n x_i w_i = 0$$

$$\Rightarrow y_{in} = b + x_1 w_1 + x_2 w_2$$

$$\Rightarrow x_2 = -\frac{w_1}{w_2} x_1 - \frac{b}{w_2}$$

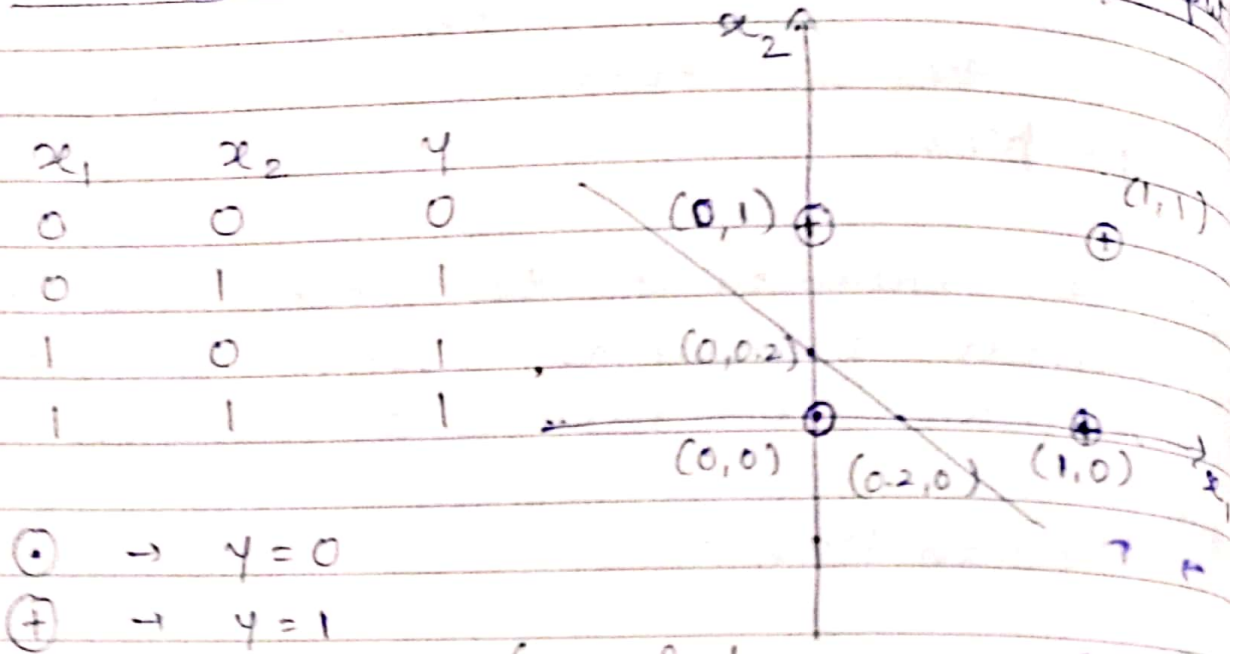
→ Find the values of w_1, w_2, b

• Threshold - 0

$$x_1 w_1 + x_2 w_2 = 0$$

$$x_2 = -\frac{w_1}{w_2} x_1 + \frac{0}{w_2} \quad (0 = -b)$$

* Example - 1: OR Function, binary input



(No. of lines = no. of neurons)

Activation Function:

$$y_{in} > 0 \Rightarrow y = 1$$

$$y_{in} \leq 0 \Rightarrow y = 0$$

Points (x_1, y_1) (x_2, y_2)
 $(0.2, 0)$ $(0, 0.2)$

Line equation - $y = mx + c$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0.2 - 0}{0 - 0.2} = -1$$

$$m = -1$$

$$\rightarrow y_1 = mx_1 + c$$

$$c = y_1 - mx_1$$

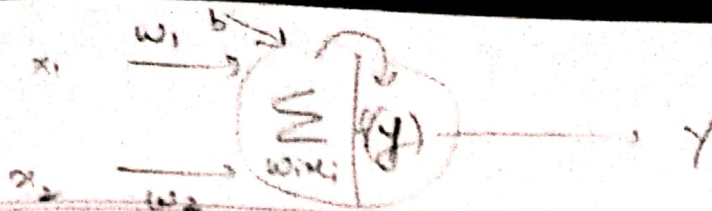
$$= 0 - (-1)(0.2)$$

$$c = 0.2$$

$$y = mx + c$$

$$= -x + 0.2$$

$$x_2 = -x_1 + 0.2$$



$$x_2 = -\frac{w_1}{w_2} x_1 - \frac{b}{w_2} \quad \text{--- (2)}$$

Comparing equation (1) & (2)

$$\frac{w_1}{w_2} = 1 \quad \frac{b}{w_2} = -0.2$$

$$w_1 = w_2 = 1 \quad , \quad b = -0.2$$

$$V(0,0) = 0 \times 1 + 0 \times 1 - 0.2 \leq 0 \Rightarrow y = 0$$

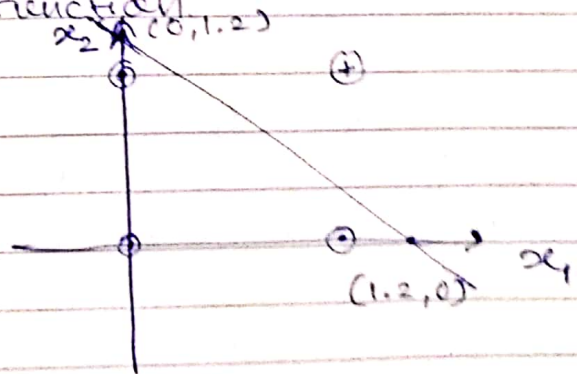
$$V(0,1) = 0 \times 1 + 1 \times 1 - 0.2 > 0 \Rightarrow y = 1$$

$$V(1,0) = 1 \times 1 + 0 \times 1 - 0.2 > 0 \Rightarrow y = 1$$

$$V(1,1) = 1 \times 1 + 1 \times 1 - 0.2 > 0 \Rightarrow y = 1$$

* Example - 2 : AND Function

x_1	x_2	y
0	0	0
0	1	0
1	0	0
1	1	1

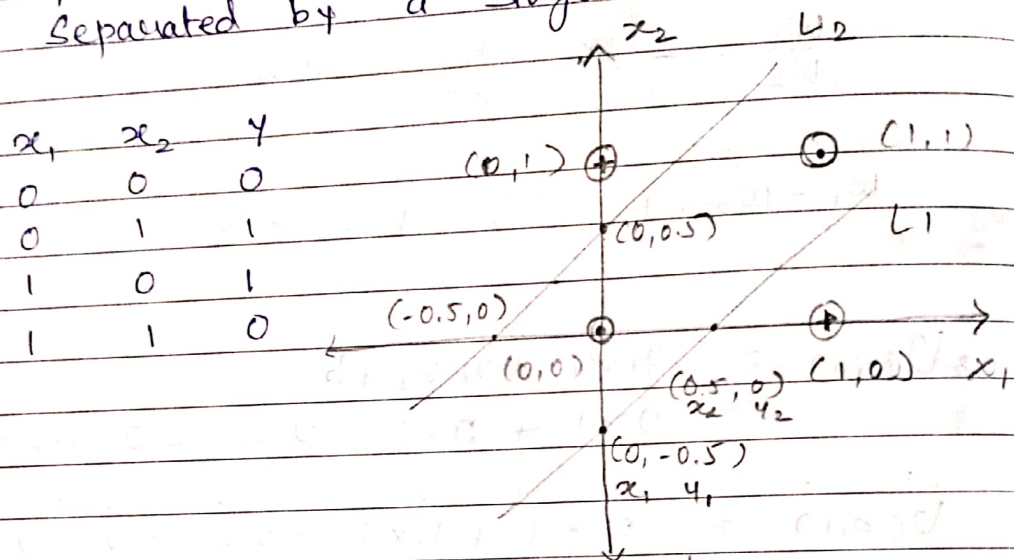


$$m = -1 \quad w_1 = w_2 = 1 \quad , \quad b = -1.1$$

$$c = 1.2$$

* Ex-3 XOR function

→ XOR is nonlinear or linearly non separable problem as its patterns cannot be separated by a single line



⇒ For line 1 = output = z_1 | $(x_1, y_1) (0, -0.5)$
 $(x_2, y_2) (0.5, 0)$

$$y = mx + c$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - (-0.5)}{0.5 - 0} = 1$$

$$m = 1$$

$$c = y_1 - mx_1$$

$$= (-0.5) - 1 \times 0$$

$$c = -0.5$$

$$y = x - 0.5$$

$$\Rightarrow x_2 = x_1 - 0.5 \quad \text{--- (1)}$$

$$x_2 = -x_1 \frac{w_1}{w_2} - \frac{b}{w_2} \quad \text{--- (2)}$$

$$\frac{w_1}{w_2} = -1 \quad \frac{b}{w_2} = -0.5$$

$$w_1 = 1, \quad w_2 = -1 \quad b = -0.5$$

Activation function :

$$y_{z_1} > 0 \Rightarrow z_1 = 1$$

$$y_{z_1} \leq 0 \Rightarrow z_1 = 0$$

$$(0,0) \rightarrow y_{z_1} = b + w_1 x_1 + w_2 x_2 \\ = -0.5 + (1) \times 0 + (-1) \times 0 < 0 \Rightarrow z_1 = 0$$

$$(0,1) \rightarrow -0.5 + (1) \times 0 + (-1) \times 1 < 0 \Rightarrow z_1 = 0$$

$$(1,0) \rightarrow -0.5 + (1) \times 1 + (-1) \times 0 > 0 \Rightarrow z_1 = 1$$

$$(1,1) \rightarrow -0.5 + (1) \times 1 + (-1) \times 1 < 0 \Rightarrow z_1 = 0$$

x_1	x_2	z_1
0	0	0
0	1	0
1	0	1
1	1	0

= For Line 2 - Output $z_2 = (x_1, y_1) (-0.5, 0)$
 $(x_2, y_2) (0, 0.5)$

$$m = \frac{0.5 - 0}{0 - (-0.5)} = 1$$

$$c = 0.5 \rightarrow y = x + 0.5$$

$$z_2 = x_1 + 0.5 \rightarrow (1)$$

$$x_2 = \frac{-w_1 x_1 - b}{w_2} \quad \text{--- (2)}$$

$$\frac{w_1}{w_2} = -1 \quad \frac{b}{w_2} = -0.5$$

$$w_1 = -1 \quad w_2 = 1 \quad b = -0.5$$

Activation $y_{z_2} > 0 \Rightarrow z_2 = 1$

$$y_{z_2} \leq 0 \Rightarrow z_2 = 0$$

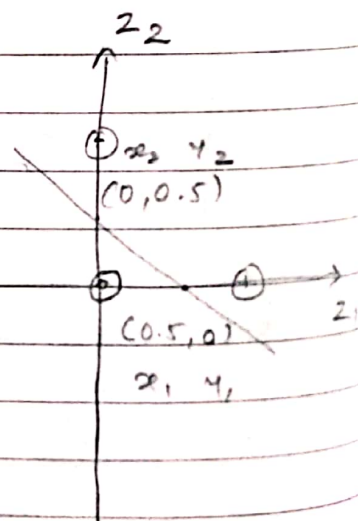
$$(0,0) \rightarrow -0.5 + (-1)(0) + (1)(0) < 0 \Rightarrow z_2 = 0$$

$$(0,1) \rightarrow -0.5 + (-1)(0) + (1)(1) > 0 \Rightarrow z_2 = 1$$

$$(1,0) \rightarrow -0.5 + (-1)(1) + (1)(0) < 0 \Rightarrow z_2 = 0$$

$$(1,1) \rightarrow -0.5 + (-1)(1) + (1)(1) < 0 \Rightarrow z_2 = 0$$

x_1	x_2	z_2	z_1	y
0	0	$\rightarrow 0$	0	0
0	1	$\rightarrow 1$	0	1
1	0	$\rightarrow 0$	1	1
1	1	$\rightarrow 0$	0	0



$$m = \frac{0.5 - 0}{0 - 0.5} = -1$$

$$c = -m x_1 - y_1$$

$$c = 0.5$$

$$x_2 = -x_1 + 0.5$$

$$x_2 = -\frac{w_1}{w_2} \cdot x_1 - \frac{b}{w_2}$$

$$\frac{w_1}{w_2} = +1$$

$$\frac{b}{w_2} = -0.5$$

$$w_1 = w_2 = 1$$

$$b = -0.5$$

Activation function

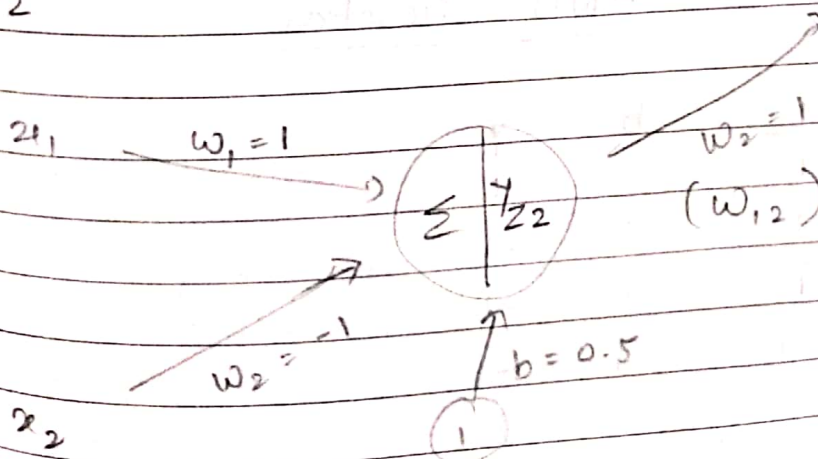
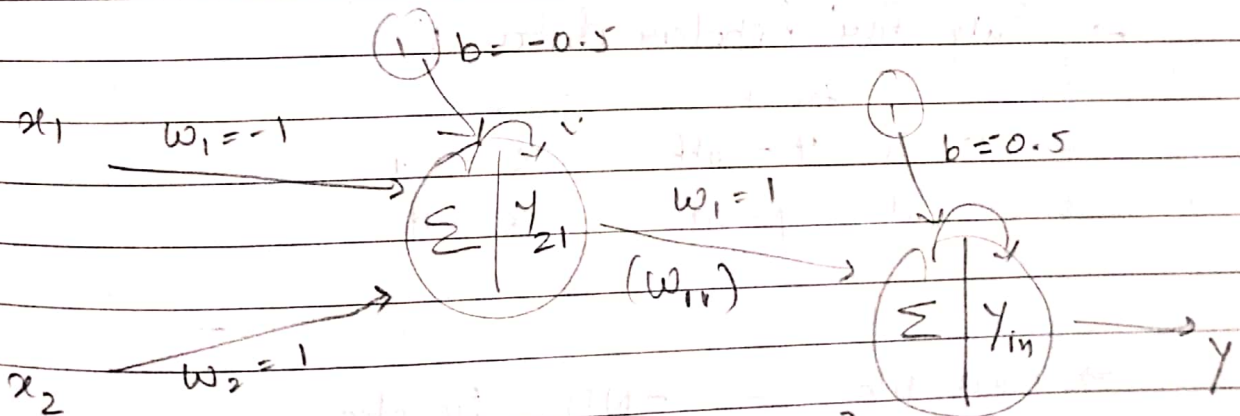
$$y_{in} > 0 \Rightarrow y = 1$$

$$y_{in} \leq 0 \Rightarrow y = 0$$

$$(0, 0) \rightarrow -0.5 + 1 \times 0 + 1 \times 0 < 0 \Rightarrow y = 0$$

$$(0, 1) \rightarrow -0.5 + 1 \times 0 + 1 \times 1 > 0 \Rightarrow y = 1$$

$$(1, 0) \rightarrow -0.5 + 1 \times 1 + 1 \times 0 > 0 \Rightarrow y = 1$$



* Hebb Network

→ Synaptic Gap (brain)

→ Weight factor is found to increase proportionately to the product of the input and the learning signal.

→ If two interconnected neurons are 'on' simultaneously then weight associated with these neurons can be increased by the modification made in their synaptic gap.

$$w_i(\text{new}) = w_i(\text{old}) + x_i y$$

$$b(\text{new}) = b(\text{old}) + y$$

→ Only for bipolar data :

- | | | |
|------|---------|---------|
| | target | input |
| - if | y - off | x - on |
| - if | y - off | x - off |

⇒ Example - AND Function

x_1	x_2	b	y
1	1	1	1
1	-1	1	-1
-1	1	1	-1
-1	-1	1	-1

Initially, $w_1 = w_2 = b = 0$

→ Input - 1 $\begin{matrix} \alpha_1 & \alpha_2 & b \\ (1 & 1 & 1) \end{matrix}$ target y (1)

$$w_i(n) = w_i(0) + \alpha_i y$$

$$\Delta b = y = 1$$

$$\Delta w_1 = \alpha_1 y = 1$$

$$\Delta w_2 = \alpha_2 y = 1$$

$$w_1(n) = w_1(0) + \alpha_1 y = 0 + 1 \times 1 = 1$$

$$w_2(n) = w_2(0) + \alpha_2 y = 0 + 1 \times 1 = 1$$

$$b(n) = b(0) + y = 0 + 1 = 1$$

→ Input - 2 $\begin{matrix} \alpha_1 & \alpha_2 & b \\ (1 & -1 & 1) \end{matrix}$ $y = -1$ $\Delta b = -1$

$$[w_1, w_2, b] = [1, 1, 1]$$

$$\Delta w_1 = -1$$

$$\Delta w_2 = 1$$

$$w_1(n) = 1 + 1 \times (-1) = 0$$

$$w_2(n) = 1 + (-1) \times (-1) = 2$$

$$b(n) = 1 + (-1) = 0$$

→ Input - 3 $\begin{matrix} \alpha_1 & \alpha_2 & b \\ (-1 & 1 & 1) \end{matrix}$ $y = -1$ $\Delta b = -1$

$$[w_1, w_2, b] = [0, 2, 0]$$

$$\Delta w_1 = 1$$

$$\Delta w_2 = -1$$

$$w_1(n) = w_1(0) + \alpha_1 y$$

$$w_1(n) = 0 + (-1) \times (-1) = 1$$

$$w_2(n) = 2 + (1) \times (-1) = 1$$

$$b(n) = 0 + (-1) = -1$$

→ Input - 4 $(-1, -1, 1)$ $y = -1$ $\Delta b = -1$

$$[w_1, w_2, b] = [1, 1, -1]$$

$$\Delta w_1 = 1$$

$$\Delta w_2 = 1$$

$$w_1(n) = 1 + (-1) \times (-1) = 2$$

$$w_2(n) = 1 + (-1) \times (-1) = 2$$

$$b(n) = -1 + (-1) = -2$$

$$x_2 = -\frac{w_1}{w_2} x_1 - \frac{b}{w_2}$$

I4 \rightarrow $w_1 = w_2 = 2$ $b = -2$

\checkmark $x_2 = -x_1 + 1$ — (1)

I3 \rightarrow $w_1 = 1$ $w_2 = 1$ $b = -1$

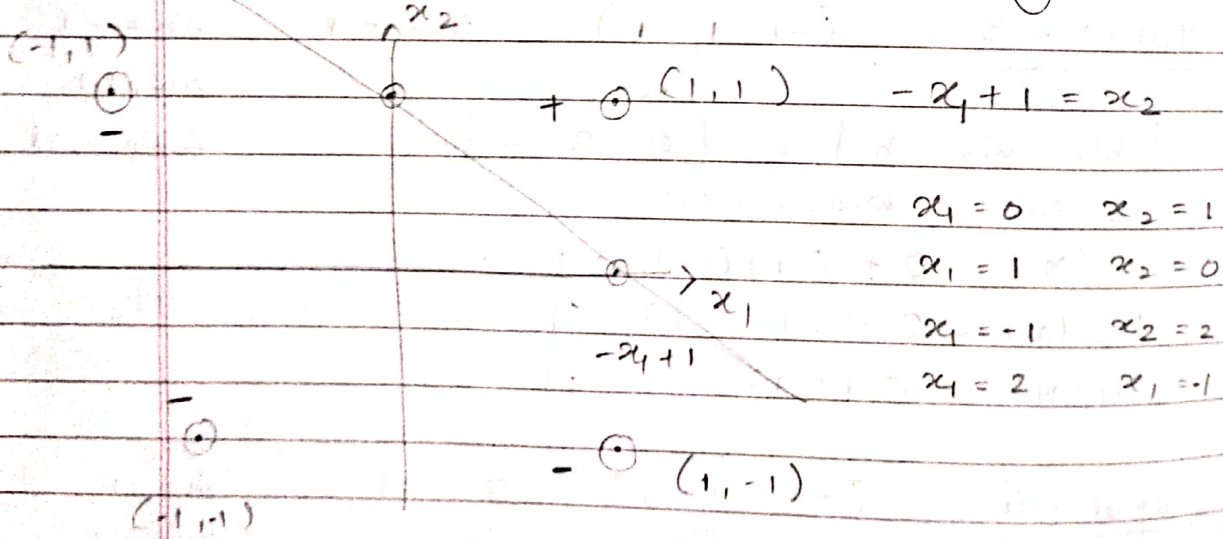
\checkmark $x_2 = -x_1 + 1$ — (2)

I2 \rightarrow $w_1 = 0$ $w_2 = 2$ $b = 0$

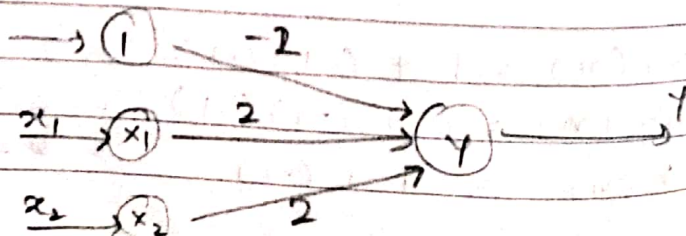
$x_2 = 0$ — (3)

I1 \rightarrow $w_1 = 1$ $w_2 = 1$ $b = 1$

$x_2 = -x_1 - 1$ — (4)



Hebb Network



2) OR Function (bipolar)

x_1	x_2	y
1	1	1
1	-1	1
-1	1	1
-1	-1	-1

⇒ Initial weights $w_1 = w_2 = 0$, bias = 0

Input - 1 $[x_1 \ x_2 \ b] = [1 \ 1 \ 0]$ $\Delta b = 1$
 $\Delta w_1 = \Delta w_2 = 1$

$$w_1(n) = w_1(0) + x_1 y (\Delta w_1) = 1$$

$$w_2(n) = w_2(0) + x_2 y (\Delta w_2) = 1$$

$$b(n) = b(0) + y (\Delta b) = 1$$

Input - 2 $[1 \ -1 \ 1]$, $w_1 = 1$, $w_2 = 1$, $b = 1$

$$\Delta w_1 = 1 \times 1 = 1$$

$$w_1(n) = w_1(0) + \Delta w_1 = 2$$

$$\Delta w_2 = -1 \times 1 = -1$$

$$w_2(n) = w_2(0) + \Delta w_2 = 0$$

$$\Delta b = 1 = 1$$

$$b(n) = b(0) + \Delta b = 2$$

Input - 3 $[-1 \ 1 \ 1]$ $w_1 = 2$ $w_2 = 0$, $b = 2$

$$\Delta w_1 = (-1) \times 1 = -1$$

$$w_1(n) = w_1(0) + \Delta w_1 = 1$$

$$\Delta w_2 = 1 \times 1 = 1$$

$$w_2(n) = w_2(0) + \Delta w_2 = 1$$

$$\Delta b = 1 = 1$$

$$b(n) = b(0) + \Delta b = 3$$

Input - 4 $[-1 \ -1 \ 1]$ $w_1 = 1$ $w_2 = 1$ $b = 3$ $y = -1$

$$\Delta w_1 = -1 \times -1 = 1$$

$$w_1(n) = 1 + 1 = 2$$

$$\Delta w_2 = -1 \times -1 = 1$$

$$w_2(n) = 1 + (1) = 2$$

$$\Delta b = -1 = -1$$

$$b(n) = 3 + (-1) = 2$$

$$x_2 = -\frac{w_1}{w_2} x_1 + \frac{b}{w_2}$$

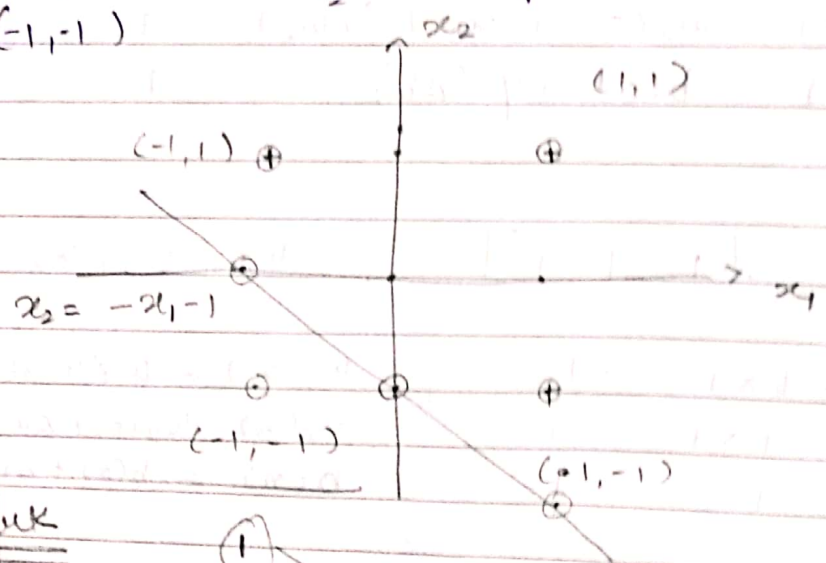
L1 : $x_2 = -x_1 - 1$ ✓
(1, 1)

$x_1 = 0 \quad x_2 = 1$
 $x_1 = 1 \quad x_2 = 2$
 $x_1 = -1 \quad x_2 = 0$

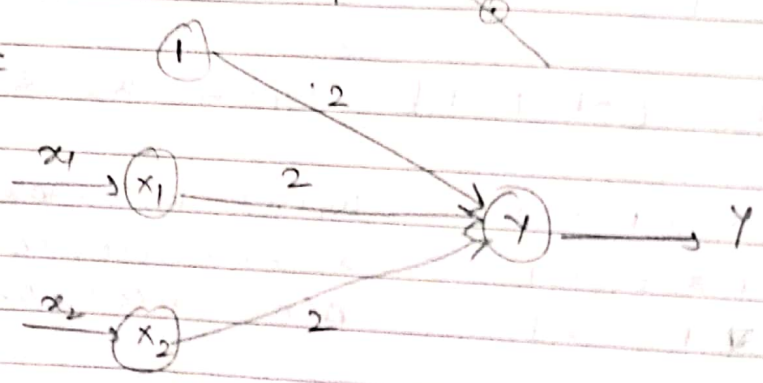
L2 $x_2 = -\frac{2}{0} x_1 - \frac{2}{0}$
(1, -1)

L3 $x_2 = -x_1 - 3$
(-1, 1)

L4 $x_2 = -x_1 - 1$
(-1, -1)



Network



3) Hebb rule for following pattern classification

'+' Symbol \rightarrow Value 1
Empty \rightarrow Value 0

+	+	+
	+	
+	+	+

+	+	+
+		+
+	+	+

\rightarrow I belongs to members of class 'I' '0'

Pattern	Inputs									target
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	y
I	1	1	1	-1	1	-1	1	1	1	1
0	1	1	1	1	-1	1	1	1	1	-1

\Rightarrow Single Layer Network, Nine input, one bias

Initial weights & bias = 0

Case-1 : Pattern 'I'

$\rightarrow \Delta w_i = x_i y \quad i = 1 \text{ to } 9$

$\Delta w_1 = 1 \times 1 = 1$

$\Delta w_2 = 1 \times 1 = 1$

$\Delta w_3 = 1 \times 1 = 1$

$\Delta w_4 = -1 \times 1 = -1$

$\Delta w_5 = 1 \times 1 = 1$

$\Delta w_6 = -1 \times 1 = -1$

$\Delta w_7 = 1 \times 1 = 1$

$\Delta w_8 = 1 \times 1 = 1$

$\Delta w_9 = 1 \times 1 = 1$

$\Delta b = y = 1$

$w_i(n) = w_i(0) + \Delta w_i$

$w_1(n) = 0 + 1 = 1$

$w_6(n) = -1$

$w_2(n) = 0 + 1 = 1$

$w_7(n) = 1$

$w_3(n) = 0 + 1 = 1$

$w_8(n) = 1$

$w_4(n) = -1$

$w_9(n) = 1$

$w_5(n) = 1$

$b(n) = 1$

Case-2 : Pattern '0'

$$W_i = [1 \ 1 \ 1 \ -1 \ 1 \ -1 \ 1 \ 1 \ 1 \ 1] \quad y = -1$$

$$W_i(n) = W_i(0) + \alpha_i y$$

$$W_1(n) = 1 + 1x-1 = 0$$

$$W_6(n) = -1 + 1x-1 = -2$$

$$W_2(n) = 1 + 1x-1 = 0$$

$$W_7(n) = 1 + 1x-1 = 0$$

$$W_3(n) = 1 + 1x-1 = 0$$

$$W_8(n) = 1 + 1x-1 = 0$$

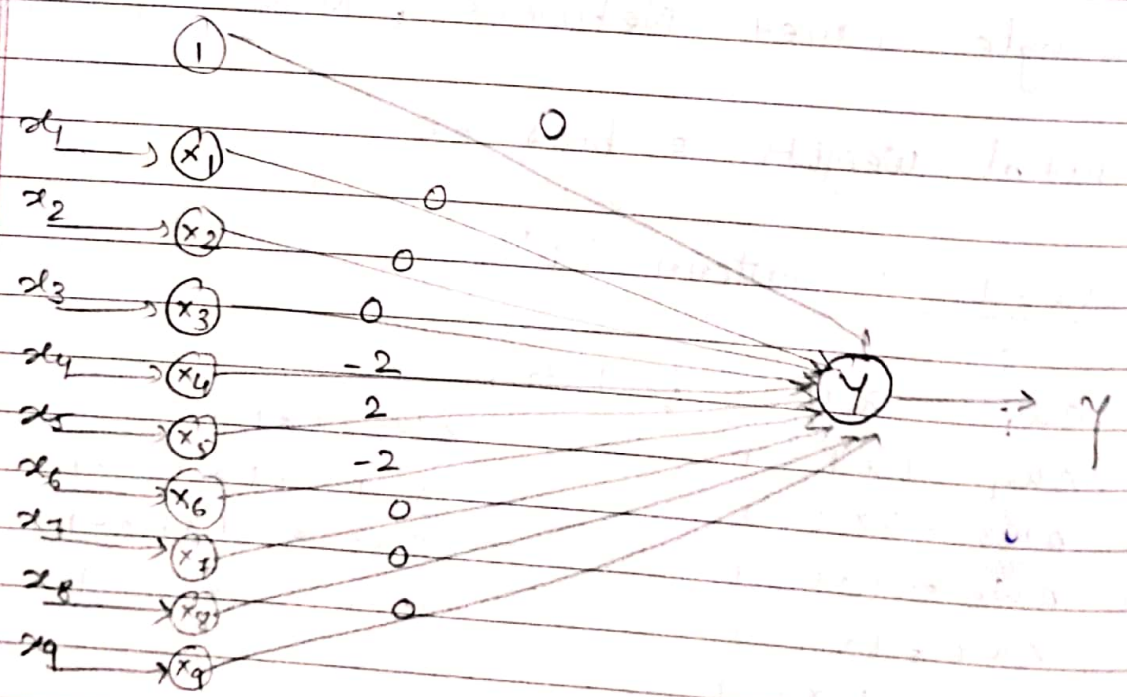
$$W_4(n) = -1 + 1x-1 = -2$$

$$W_9(n) = 1 + 1x-1 = 0$$

$$W_5(n) = 1 + -1x-1 = 2$$

$$b(n) = 1 + 1x-1 = 0$$

$$W_n = [0 \ 0 \ 0 \ -2 \ 2 \ -2 \ 0 \ 0 \ 0 \ 0]$$



* McCulloch - Pitts Neuron : (M-P Neuron)

- Earliest neural network - 1943
- Neurons connected by directed weighted paths
- Activation is binary - Fire or not fire
- Excitatory weights - positive - Same weights
- Inhibitory weights - Negative
- Threshold - If net input to the neuron is greater than the threshold then neuron fires.

Activation Function:

$$f(y_{in}) = \begin{cases} 1 & \text{if } y_{in} \geq \theta \\ 0 & \text{if } y_{in} < \theta \end{cases}$$

For inhibition to be absolute, threshold should

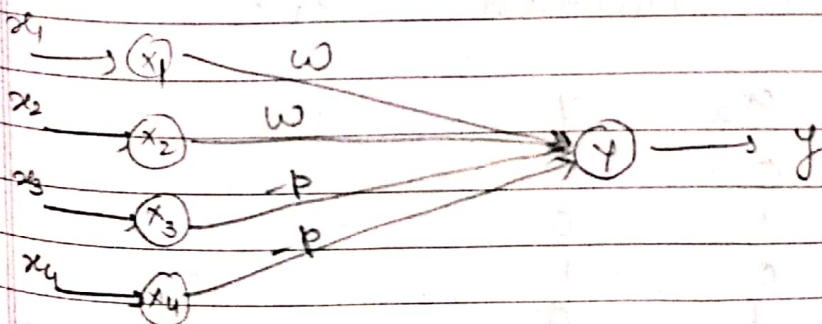
$$\theta \geq nW - P$$

n - no. of neurons

W - Magnitude of +ve weights

P - Magnitude of -ve weights

- No training algorithm
- Analysis - to determine values of weights and threshold.



(1)

Ex-1

AND Function

(binary data)

x_1	x_2	y
0	0	0
0	1	0
1	0	0
1	1	1

→ Output is high if both inputs are high

Let's take $w_1 = 1$ $w_2 = 1$

$$y_{in} = x_1 w_1 + x_2 w_2$$

$$[0, 0] \rightarrow y_{in} = 0 + 0 = 0$$

$$[0, 1] \rightarrow y_{in} = 0 + 1 = 1$$

$$[1, 0] \rightarrow y_{in} = 1 + 0 = 1$$

$$[1, 1] \rightarrow y_{in} = 1 + 1 = 2 \rightarrow 0 = 1$$

$$\theta \geq n w - p$$
$$2 \times 1 - 0$$

$$\theta \geq 2$$

$$y = f(y_{in}) = \begin{cases} 1 & \text{if } y_{in} \geq 2 \\ 0 & \text{if } y_{in} < 2 \end{cases}$$

(2)

AND NOT

Function

(x_1, \bar{x}_2)

x_1	x_2	y
0	0	0
0	1	0
1	0	1
1	1	0

→ Case-1 $w_1 = 1$ $w_2 = 1$

$$[0, 0] \rightarrow y_{in} = 0 + 0 = 0$$

$$[0, 1] \rightarrow = 0 + 1 = 1$$

$$[1, 0] \rightarrow = 1 + 0 = 1$$

$$[1, 1] \rightarrow = 1 + 1 = 2$$

→ It is not possible to fix the newton first input (1, 0) Only. These weights are not suitable.

→ Case-2 $w_1 = 1$ $w_2 = -1$

$$[0, 0] \rightarrow y_{in} = 0 + 0 = 0$$

$$[0, 1] = 0 + (-1) = -1$$

$$[1, 0] = 1 + 0 = 1 \leftarrow \textcircled{0}$$

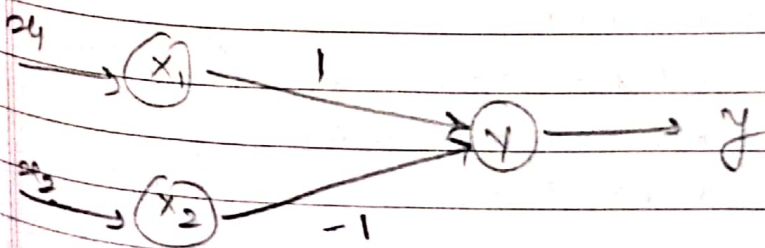
$$[1, 1] = 1 - 1 = 0$$

Brain Output is high if first input is high and second input is low

$$\textcircled{0} \geq \frac{nw - p}{2 \times 1 - 1}$$

$$\underline{\textcircled{0} \geq 1}$$

$$y = f(y_{in}) = \begin{cases} 1 & \text{if } y_{in} \geq 1 \\ 0 & \text{if } y_{in} < 1 \end{cases}$$



③ Ex-3 XOR Function

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0

$y = \underbrace{x_1 \bar{x}_2}_{z_1} + \underbrace{\bar{x}_1 x_2}_{z_2}$

$y = z_1 \text{ (OR) } z_2$

→ Single layer Net is not sufficient

① $z_1 = x_1 \bar{x}_2$ $w_{11} = w_{21} = 1$

x_1	x_2	z_1
0	0	0
0	1	0
1	0	1
1	1	0

Case-1 → $w_{11} = w_{21} = 1$

$(0,0) \rightarrow x_1 w_1 + x_2 w_2 = 0$

$(0,1) \rightarrow 0 \times 1 + 1 \times 1 = 1$

$(1,0) \rightarrow 1 \times 1 + 0 \times 1 = 1$

$(1,1) \rightarrow 1 \times 1 + 1 \times 1 = 2$

Case-2 → $w_{11} = 1$ $w_{21} = -1$

$(0,0) \rightarrow 0 \times 1 + 0 \times -1 = 0$

$(0,1) \rightarrow 0 \times 1 + 1 \times -1 = -1$

$(1,0) \rightarrow 1 \times 1 + 0 \times -1 = 1$

$(1,1) \rightarrow 1 \times 1 + 1 \times -1 = 0$

$\theta \geq n w - 1$
 $2 \times 1 - 1$
 $\theta \geq 1$

$$(2) \quad Z_2 = \bar{x}_1 x_2$$

x_1	x_2	Z_2
0	0	0
0	1	1
1	0	0
1	1	0

Case-1 : $w_{12} = w_{22} = 1$

$$(0,0) \rightarrow x_1 w_{12} + x_2 w_{22} = 0 \times 1 + 0 \times 1 = 0$$

$$(0,1) \rightarrow 1$$

$$(1,0) \rightarrow 0$$

$$(1,1) \rightarrow 0$$

Case-2 : $w_{12} = -1$ $w_{22} = 1$

$$(0,0) \rightarrow 0$$

$$(0,1) \rightarrow 1 \quad \leftarrow \quad \theta \geq 1$$

$$(1,0) \rightarrow -1$$

$$(1,1) \rightarrow 0$$

$$(3) \quad Y = Z_1 \text{ OR } Z_2$$

x_1	x_2	Y	z_1	z_2	<u>Case-1</u>	$w_1 = w_2 = 1$
-------	-------	-----	-------	-------	---------------	-----------------

$$0 \quad 0 \quad 0 \quad 0 \quad 0$$

$$0 \quad 1 \quad 1 \quad 0 \quad 1 \quad (0,0) \rightarrow 0 \times 1 + 0 \times 1 = 0$$

$$1 \quad 0 \quad 1 \quad 1 \quad 0 \quad (0,1) \rightarrow 0 \times 1 + 1 \times 1 = 1$$

$$1 \quad 1 \quad 0 \quad 0 \quad 0 \quad (1,0) \rightarrow 1 \times 1 + 0 \times 1 = 1$$

$$(1,1) \rightarrow 0 \times 1 + 0 \times 1 = 0$$

Supervised Learning Networks

* Perceptron Networks :

→ Single layer feed-forward

→ Three units :

Sensory unit (Input)

Associator unit (Hidden) - Feature Predicates

Response unit (Output) - Pattern Recognizers

→ Sensory - Associator - Fixed weights (1, 0, -1)
- Binary activation (0, 1)

→ Response unit → Activation 0, 1, -1
→ Binary step - threshold 0

→ Associator - Response → Binary output signal

→ Output - $y = f(y_{in})$

$$f(y_{in}) = \begin{cases} 1 & y_{in} > 0 \\ 0 & -\theta \leq y_{in} \leq \theta \\ -1 & y_{in} < -\theta \end{cases}$$

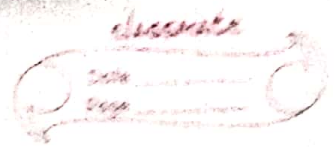
→ Weight updation betⁿ Associator & Response

→ Error calculation - comparison betⁿ target & output

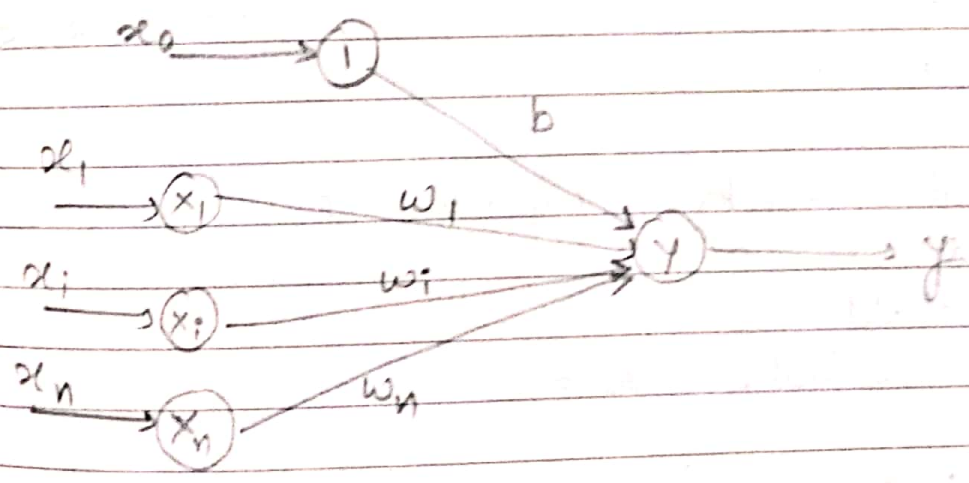
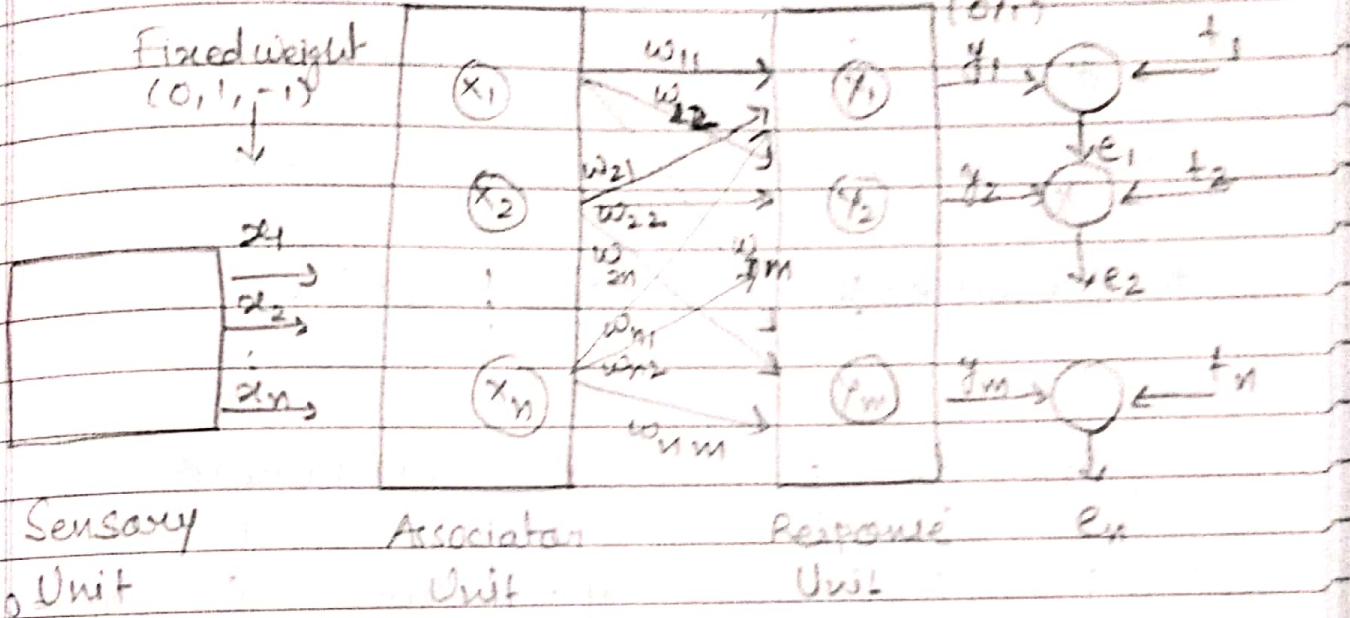
→ Weight updation - If $(y \neq t)$ then

$$w_i(\text{new}) = w_i(\text{old}) + \alpha t x_i$$

$$b(\text{new}) = b(\text{old}) + \alpha t$$



t - target output (1, -1)
 α - Learning rate



Goal : Classify the input patterns as member or not member to a particular class

Single output :
$$y_{in} = b + \sum_{i=1}^n x_i w_i$$

Multiple output :
$$y_{in_j} = b_j + \sum_{i=1}^n x_i w_{ij}$$

output unit - $j = 1$ to m

⇒ Algorithms are not sensitive to initial values of weights & learning rate.

* Example - 1 AND Function

x_1	x_2	t	Activation Function
1	1	1	$y = f(y_{in}) \begin{cases} 1 & y_{in} > 0 \\ 0 & y_{in} = 0 \\ -1 & y_{in} < 0 \end{cases}$
1	-1	-1	
-1	1	-1	
-1	-1	-1	

- ⇒ Initial weights - $w_1 = w_2 = 0$
- ⇒ Bias $b = 0$
- ⇒ Threshold = 0
- ⇒ Learning rate $\alpha = 1$

Epoch-1

Input 1 : $x_1 = 1, x_2 = 1, t = 1$

$$y_{in} = b + x_1 w_1 + x_2 w_2 = 0$$

$$\text{As } y = 0 \Rightarrow y_{in} = 0$$

$$\text{Check } \rightarrow y = t, t = 1 \Rightarrow y = 0 \Rightarrow y \neq t$$

So weight updation

$$w_i(\text{new}) = w_i(\text{old}) + \alpha t x_i$$

$$b(\text{new}) = b(\text{old}) + \alpha t$$

Here, $\Delta w_i = \alpha t x_i$
 $\Delta b = \alpha t$

$$\Delta w_1 = 1 \times 1 \times 1 = 1 \quad \Rightarrow w_1(\text{new}) = 0 + 1 = 1$$

$$\Delta w_2 = 1 \times 1 \times 1 = 1 \quad \Rightarrow w_2(\text{new}) = 0 + 1 = 1$$

$$\Delta b = 1 \times 1 = 1 \quad \Rightarrow b(\text{new}) = 0 + 1 = 1$$

$\Rightarrow w_1 = 1, w_2 = 1, b = 1$ - These are final weights after first input pattern is presented.

\Rightarrow The same process is repeated for all the input patterns. Process is stopped when all the targets become equal to calculated output.

\Rightarrow When $y = t$, weight changes Δw_i will not perform and $\Delta w_i = \Delta b = 0$.

Input 2 : $x_1 = 1, x_2 = -1, t = -1$
 $w_1 = 1, w_2 = 1, b = 1$

$$y_{in} = 1 + 1 + (-1) = 1$$

$$y = 1 \quad \& \quad t = -1 \quad \therefore y \neq t$$

$$\Delta w_1 = 1 \times (-1)(1) = -1 \quad w_1 = 1 - 1 = 0$$

$$\Delta w_2 = 1 \times (-1)(-1) = 1 \quad w_2 = 1 + 1 = 2$$

$$\Delta b = 1 \times (-1) = -1 \quad b = 1 - 1 = 0$$

Input-3 $x_1 = -1$ $x_2 = 1$ $t = -1$
 $w_1 = 0$ $w_2 = 2$ $b = 0$

$$y_{in} = 0 + (-1) \times 0 + 1 \times 2 = 2 > 0$$

$$y = 1 \text{ \& } t = -1 \Rightarrow y \neq t$$

$$\Delta w_1 = 1 \times (-1) \times (-1) = 1 \quad w_1 = 1$$

$$\Delta w_2 = 1 \times (-1) \times (1) = -1 \quad w_2 = 1$$

$$\Delta b = 1 \times (-1) = -1 \quad b = -1$$

Input-4 $x_1 = -1$ $x_2 = -1$ $t = -1$
 $w_1 = 1$ $w_2 = 1$ $b = -1$

$$y_{in} = -1 + (-1)(1) + (-1)(1) = -3 < 0$$

$$y = -1, t = -1 \Rightarrow y = t$$

No weight updation $\Delta w_i = \Delta b = 0$

$$w_1 = 1 \quad w_2 = 1 \quad b = -1 \quad \text{Same weights}$$

EPOCH-2

$$w_1 = 1 \quad w_2 = 1 \quad b = -1 \quad \theta = 0 \quad \Delta \theta = 1$$

Input-1 $x_1 = 1$ $x_2 = 1$ $t = 1$

$$y_{in} = -1 + 1 \times 1 + 1 \times 1 = 1 > 0$$

$$y = 1, t = 1 \Rightarrow y = t$$

$$\Rightarrow w_1 = 1 \quad w_2 = 1 \quad b = -1 \Rightarrow \text{No weight change}$$

Input-2 $x_1 = 1$ $x_2 = -1$ $t = -1$

$$y_{in} = -1 + (1)(1) + (-1)(1) = -1 < 0 \Rightarrow y = -1$$

$$y = -1, t = -1 \Rightarrow y = t \quad \text{No weight change}$$

Input-3 $x_1 = -1$ $x_2 = 1$ $t = -1$

$$y_{in} = -1 + (-1)(1) + (1)(1) = -1 < 0 \Rightarrow y = -1$$

$$y = -1, t = -1 \Rightarrow y = t \quad \text{No weight change}$$

Input-4 $x_1 = -1$ $x_2 = -1$ $t = -1$

$$y_{in} = -1 + (-1) \times 1 + (-1) \times 1 = -3 < 0 \Rightarrow y = -1$$

$$y = -1, t = -1 \Rightarrow y = t \quad \text{No weight change}$$

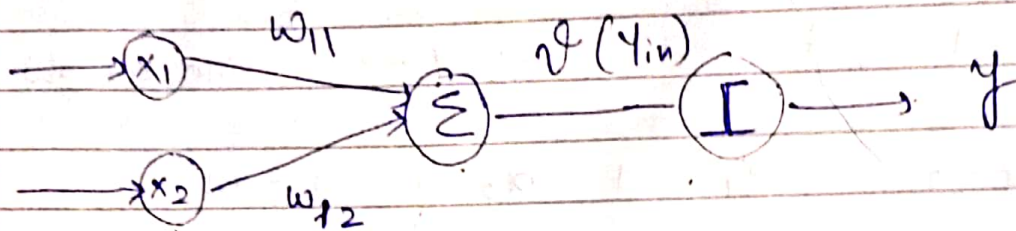
Final weights & Bias $w_1 = 1$ $w_2 = 1$ $b = -1$

$$x_2 = -\frac{w_1}{w_2} x_1 - \frac{b}{w_2}$$

$$\boxed{x_2 = -x_1 + 1}$$

NAND
Same for OR, ANDNOT,

* Finding the equation of Δw_{ji} (Error) in neural network.



$$\text{Take } y_{in} = v = \sum x_i w_{ji} \Rightarrow \frac{\partial v}{\partial w_{ji}} = x_i$$

$$y = f(v) \Rightarrow \frac{\partial y}{\partial v} = f'(v)$$

$$E = (t - y)^2 \Rightarrow \frac{\partial E}{\partial y} = -2(t - y)$$

$$\frac{\partial E}{\partial w_{ji}} = \frac{\partial E}{\partial y} \times \frac{\partial y}{\partial v} \times \frac{\partial v}{\partial w_{ji}}$$

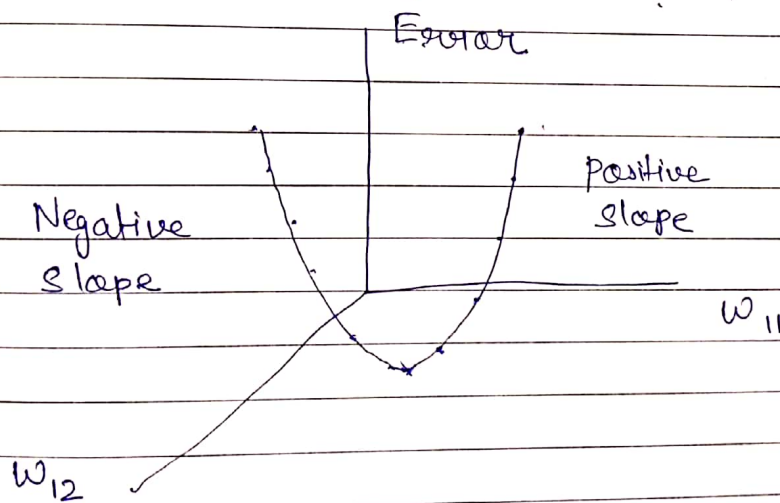
$$\Delta w_{ji} = -2(t - y) \times f'(v) \times x_i$$

So,

$$w_{ji}(\text{new}) = w_{ji}(\text{old}) + \Delta w_{ji}$$

* Gradient Descent Method (Steepest Descent)

- A gradient measures how much an output function changes if you change in input.
- Gradient is partial derivative with respect to its learning.
- Higher the gradient, steeper the slope and faster the model.
- But if slope is 0, model stops learning.



⇒ Widrow - Hoff Rule / Delta Rule

$$\Delta w_i = \alpha (t - y_{in}) x_i$$

If initial weights are taken as 0

$$y_{in} = 0 \Rightarrow y = 0$$

$$\Delta w_{ji} = \alpha (t_j - y_j) x_{ji}$$

$$\therefore \boxed{\Delta w_{ji} = \alpha t x_{ji}}$$

For single output $j=1$

$$\boxed{\Delta w_i = \alpha t x_i}$$

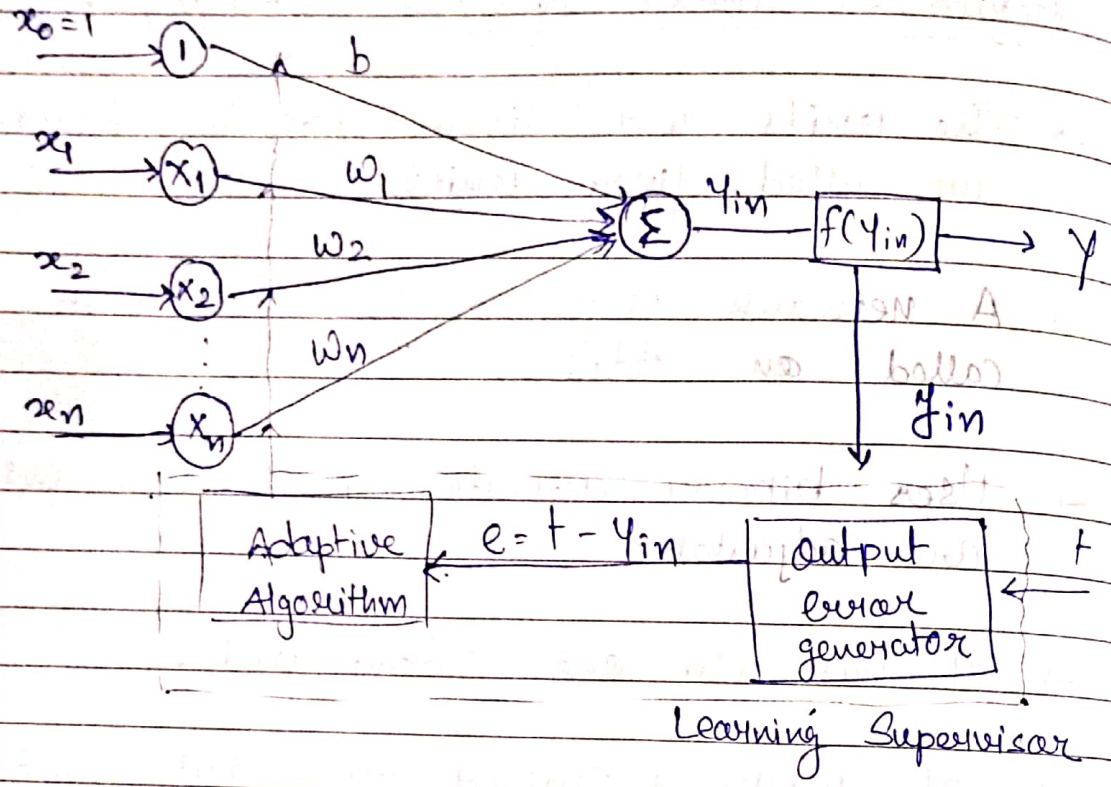
* Adaptive Linear Neuron (ADALINE)

- The units with linear activation function are called linear units.
- A network with single linear unit is called an Adaline.
- Uses bipolar activation, weights and bias are adjustable.
- It has only one output unit.
- It works & trained using delta rule or Least Mean Square (LMS) rule or Widrow-Hoff rule.
- Delta rule is derived from gradient descent method, its training continues forever converging only asymptotically to the solution.
- Delta rule updates weights to minimize the difference between net input to the output unit and the target value & minimize the error.
- Delta rule for adjusting the weight of i^{th} pattern

$$\Delta W_i = \alpha (t - y_{in}) x_i$$

- For several output units adjusting weight from i^{th} input unit, to j^{th} output unit

$$\Delta W_{ij} = \alpha (t_j - y_{inj}) x_i$$



* Example - 1 - OR Function :

	x_1	x_2	1	t
→	1	1	1	1
	1	-1	1	1
	-1	1	1	1
	-1	-1	1	-1

Step-1 Initialize weights, bias & learning rate random values.

* Input - 1

i) $w_1 = w_2 = b = 0.1$, $\alpha = 0.1$

ii) Net input $y_{in} = b + \sum x_i w_i$

$= b + x_1 w_1 + x_2 w_2$

$= 0.1 + 1 \times 0.1 + 1 \times 0.1$

$y_{in} = 0.3$

iii) Now compute $(t - y_{in}) = 1 - 0.3 = 0.7$

Update the weights, $w_i(\text{new}) = w_i(\text{old}) + \alpha(t - y_{in})x_i$

iv) Here, $\Delta w_i = \alpha(t - y_{in})x_i$

$$\Delta w_1 = \alpha(t - y_{in})x_1 = 0.1 \times (0.7) \times 1 = 0.07$$

$$\Delta w_2 = \alpha(t - y_{in})x_2 = 0.1 \times (0.7) \times 1 = 0.07$$

$$\Delta b = \alpha(t - y_{in}) = 0.1(0.7) = 0.07$$

v) $w_1(\text{new}) = w_1(\text{old}) + \Delta w_1 = 0.1 + 0.07 = 0.17$ 2

$$w_2(\text{new}) = w_2(\text{old}) + \Delta w_2 = 0.1 + 0.07 = 0.17$$
 5

$$b(\text{new}) = b(\text{old}) + \Delta b = 0.1 + 0.07 = 0.17$$
 3

vi) Error $E = (t - y_{in})^2 = (0.7)^2 = 0.49$ 1

* Input-2

$$w_1 = 0.17 \quad w_2 = 0.17 \quad b = 0.17 \quad t = 1 \quad \alpha = 0.1 \quad x_1 = 1 \quad x_2 = -1$$

$$y_{in} = b + x_1 w_1 + x_2 w_2$$

$$= 0.17 + 1 \times 0.17 + (-1) \times 0.17$$

$$= 0.17$$

$$t - y_{in} = 1 - 0.17 = 0.83$$

$$\Delta w_1 = \alpha(t - y_{in})x_1 = 0.1 \times 0.83 \times 1 = 0.083$$

$$\Delta w_2 = 0.1 \times 0.83 \times (-1) = -0.083$$

$$\Delta b = 0.1 \times 0.83 = 0.083$$

$$w_1(\text{new}) = 0.17 + 0.083 = 0.253$$

$$w_2(\text{new}) = 0.17 + (-0.083) = 0.087$$

$$b(\text{new}) = 0.17 + 0.083 = 0.253$$

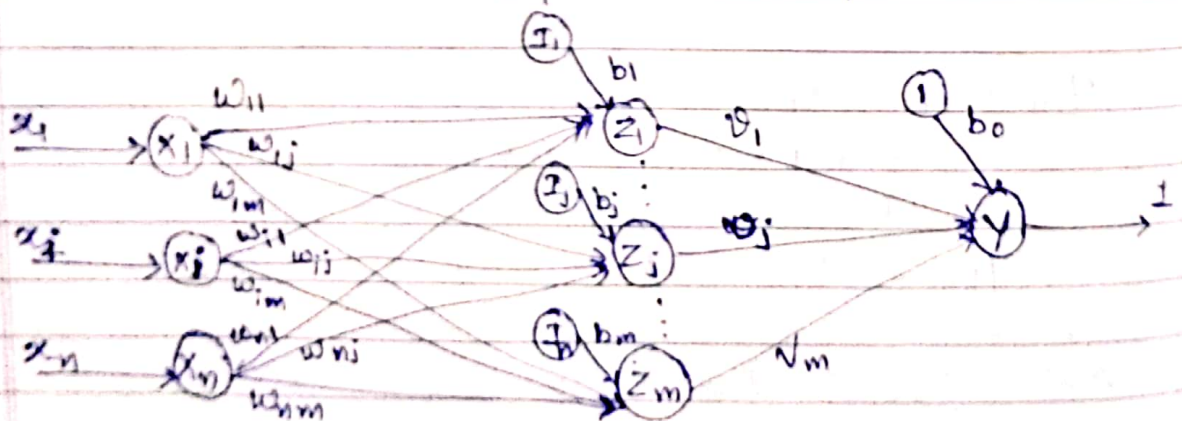
$$E = (t - y_{in})^2 = 0.83^2 = 0.6889$$

* Multiple Adaptive Linear Neurons (MADALINE)

- Many Adalines in parallel with single output unit whose value is based on certain Selection Rule
- Majority Vote Rule - True or False
- AND Rule - True iff both inputs True
- Weights betⁿ Adaline & Madaline layer are fixed, positive & same value (equal value)
- Weights betⁿ Input & Adaline are adjusted

• Architecture :

- n - input layer units
- m - Adaline layer units
- 1 - Madaline layer unit (output)



- The weights v_1, v_2, \dots, v_m & bias b_0 that enter into output unit y are determined so that Response of unit y is 1

$$v_1 = v_2 = \dots = v_m = \frac{1}{2} \quad \& \quad b_0 = \frac{1}{2}$$

→ Activation for Adaline & Madaline

$$f(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$$

→ Steps:

1) Initialize the weights w_{ij} and v_i
Here weights betⁿ hidden & output layer are fixed. (v_i)

2) Calculate net input to hidden units

$$Z_{in_j} = b_j + \sum_{i=1}^n x_i w_{ij} \quad , \quad j = 1 \text{ to } m$$

3) Calculate output of each hidden unit

$$f(Z_{in}) = \begin{cases} 1 & \text{if } Z_{in} \geq 0 \\ -1 & \text{if } Z_{in} < 0 \end{cases}$$

4) Calculate the net input entering into output unit

$$Y_{in} = b_0 + \sum_{j=1}^m Z_j v_j$$

5) Calculate the output y by applying activation function over net input Y_{in}

$$y = f(Y_{in}) = \begin{cases} 1 & \text{if } Y_{in} \geq 0 \\ -1 & \text{if } Y_{in} < 0 \end{cases}$$

6) Calculate error and update weights

i) \rightarrow If $t = y$ - no updation

ii) If $t \neq y$ and $t = +1$,

- updates weights on Z_j , whose net input is close to 0 ($\approx 0.xxx$)

$$b_j(\text{new}) = b_j(\text{old}) + \alpha(1 - Z_{in_j})$$

$$w_{ij}(\text{new}) = w_{ij}(\text{old}) + \alpha(1 - Z_{in_j}) \alpha_i$$

iii) If $t \neq y$ and $t = -1$,

update weights on Z_k , whose net input is +ve

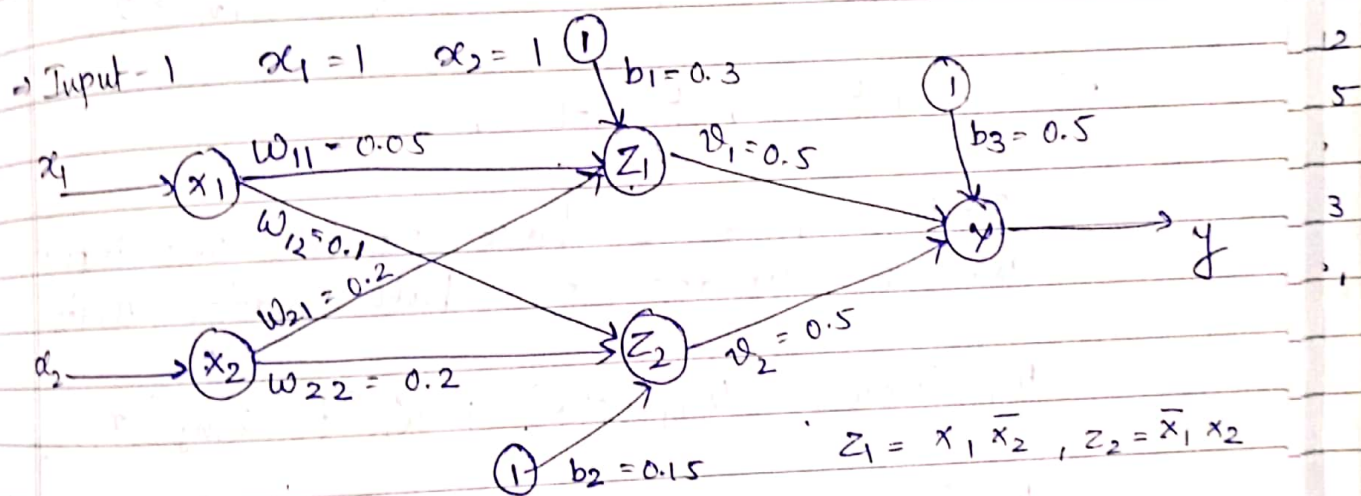
$$w_{ik}(\text{new}) = w_{ik}(\text{old}) + \alpha(-1 - Z_{in_k}) \alpha_i$$

$$b_k(\text{new}) = b_k(\text{old}) + \alpha(-1 - Z_{in_k})$$

7) Stopping condition - weight converges

Example - XOR Function

x_1	x_2	1	1	Initial values: $\alpha = 0.5$
1	1	1	-1	$[w_{11}, w_{21}, b_1] = [0.05, 0.2, 0.3]$
1	-1	1	1	$[w_{12}, w_{22}, b_2] = [0.1, 0.2, 0.15]$
-1	1	1	1	
-1	-1	1	-1	$[v_1, v_2, b_3] = [0.5, 0.5, 0.5]$



i) Net input Calculation for Adaline layer

$$Z_{in1} = b_1 + x_1 w_{11} + x_2 w_{21} = 0.55$$

$$Z_{in2} = b_2 + x_1 w_{12} + x_2 w_{22} = 0.45$$

ii) Output of Adaline layer Neurons - Activation Funⁿ

$$z_1 = f(Z_{in1}) = f(0.55) = 1 \quad (\because Z_{in1} > 0)$$

$$z_2 = f(Z_{in2}) = f(0.45) = 1 \quad (\because Z_{in2} > 0)$$

iii) Output goes into Madaline, net input

$$\begin{aligned} \text{Net input} - Y_{in} &= b_3 + z_1 v_1 + z_2 v_2 \\ &= 0.5 + 1 \times 0.5 + 1 \times 0.5 \\ &= 1.5 \end{aligned}$$

iv) Output of Madaline $y = f(Y_{in})$

$$y = f(1.5) = 1 \quad (\because Y_{in} > 0)$$

v) $y \neq t$, $t = -1$,

Both z_1 & z_2 have positive net input, weight updates on both hidden units

$$W_{ij}(\text{new}) = W_{ij}(\text{old}) + \alpha (t - z_{in_j}) x_i$$

$$W_{11}(\text{new}) = W_{11}(\text{old}) + \alpha (t - z_{in_1}) x_1 = -0.725$$

$$W_{12}(\text{new}) = W_{12}(\text{old}) + \alpha (t - z_{in_2}) x_1 = -0.625$$

$$W_{21}(\text{new}) = W_{21}(\text{old}) + \alpha (t - z_{in_1}) x_2 = -0.575$$

$$W_{22}(\text{new}) = W_{22}(\text{old}) + \alpha (t - z_{in_2}) x_2 = 0.525$$

$$b_1(\text{new}) = b_1(\text{old}) + \alpha (t - z_{in_1}) = -0.475$$

$$b_2(\text{new}) = b_2(\text{old}) + \alpha (t - z_{in_2}) = -0.575$$

$$\text{Input} = 2 \quad x_1 = 1 \quad x_2 = -1 \quad t = 1$$

$$z_{in1} = b_1 + x_1 w_{11} + x_2 w_{21} = -0.625$$

$$z_{in2} = b_2 + x_1 w_{12} + x_2 w_{22} = -0.675$$

$$z_1 = f(-0.625) = -1$$

$$z_2 = f(-0.675) = -1$$

$$y_{in} = b_3 + z_1 v_1 + z_2 v_2 = 0.5 + (-0.5) + (-0.5) = -0.5$$

$$y = f(y_{in}) = -1, \quad t = 1$$

$\Rightarrow y \neq t$ & $t = 1$, values closest to 0 should be updated, therefore w_{ij} both will be updated.

* Back - Propagation Network :

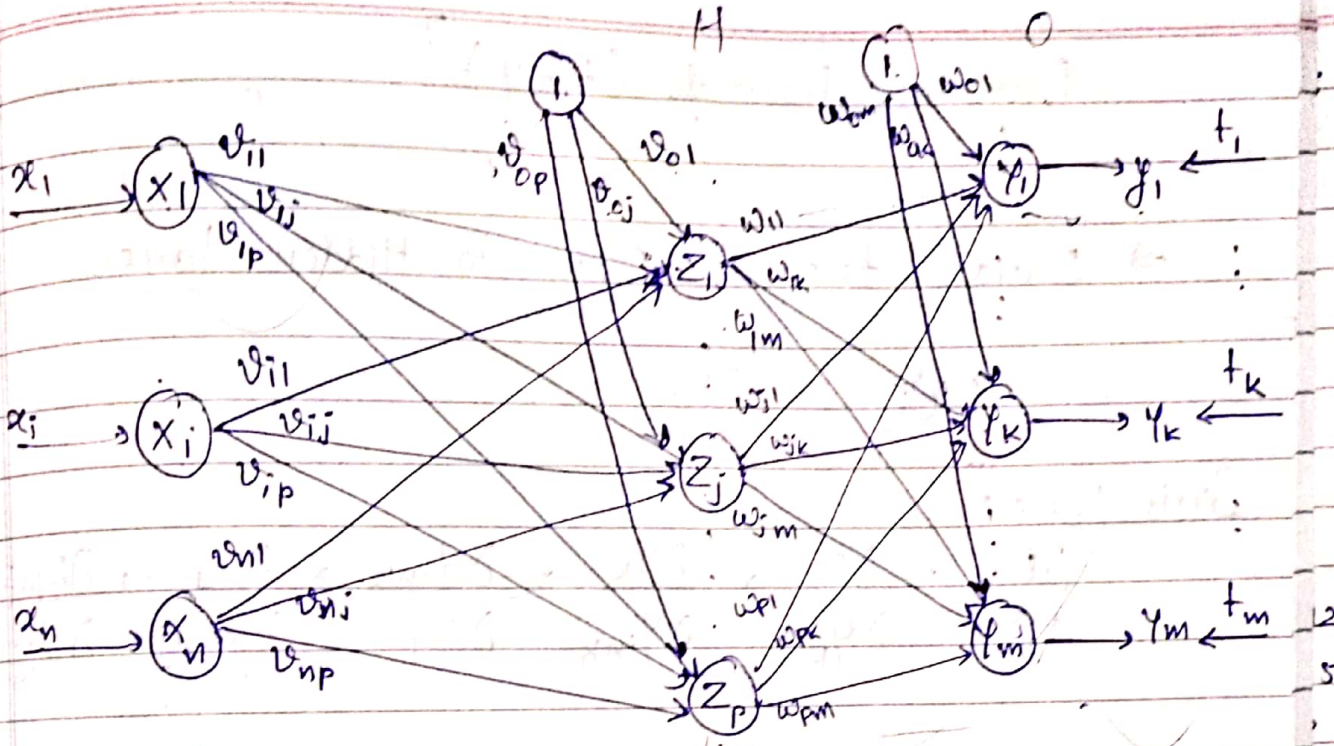
- Multilayer feed-forward networks
- Processing elements with continuous differentiable activation functions.
- Weight update algo - gradient - descent
- Error is propagated back to the hidden unit

- Aim : Train the NN to achieve a balance betⁿ net's ability to respond (Memorization) and ability to give reasonable responses to the input that is similar but not identical to the ones used in training (generalization)

- Weights are calculated during training
- To update weights, error must be calculated
- At the hidden layer - no direct info. of error
- Minimize the output error
- Three stages :
 - Feed-forward of input training pattern
 - Calculation & back-propagation of error
 - Update of weights.

Architecture :

- Input, Hidden, Output layer
- Biases at hidden & output layer acts as weights
- During back propagation - signals are sent in reverse direction
- Output - binary or bipolar
- Activation - Any



x - Input training vector $(x_1 \dots x_i \dots x_n)$

t - target output vector $(t_1 \dots t_k \dots t_m)$

x_i - Input unit i

v_{0j} - Bias on j^{th} hidden unit

w_{0k} - Bias on k^{th} output unit

z_j - Hidden unit j

Net input to z_j
$$Z_{inj} = v_{0j} + \sum_{i=1}^n x_i \cdot v_{ij}$$

$$z_j = f(Z_{inj})$$

Net input to y_k
$$Y_{ink} = w_{0k} + \sum_{j=1}^p z_j \cdot w_{jk}$$

y_k = Output unit k

$$y_k = f(Y_{ink})$$

δ_k = error correction weight adjustment for w_{jk} (output to hidden) - error of y_k

δ_j = error correction weight adjustment for v_{ij} error of hidden unit z_j

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$$\text{Error } E = \frac{1}{2} (t - y)^2$$

⇒ Error from Input to Hidden layer

$$\Delta w_{ij} = \frac{\partial E}{\partial v_{ij}}$$

Chain Rule:

$$\frac{\partial E}{\partial v_{ij}} = \frac{\partial E}{\partial y_k} \times \frac{\partial y_k}{\partial y_{ink}} \times \frac{\partial y_{ink}}{\partial z_j} \times \frac{\partial z_j}{\partial z_i} \times \frac{\partial z_i}{\partial z_{inj}} \times \frac{\partial z_{inj}}{\partial v_{ij}}$$

Now, $\frac{\partial E}{\partial y_k} = -(t_k - y_k)$

$$\frac{\partial y_k}{\partial y_{ink}} = f'(y_{ink})$$

$$\frac{\partial y_{ink}}{\partial z_j} = w_{jk}$$

$$\frac{\partial z_j}{\partial z_{inj}} = f'(z_{inj})$$

$$\frac{\partial z_{inj}}{\partial v_{ij}} = x_i$$

So,
$$\frac{\partial E}{\partial v_{ij}} = -(t_k - y_k) \times w_{jk} \times f'(y_{ink}) \times f'(z_{inj}) \times x_i$$

→ Error from Hidden to output layer:

$$\Delta W_{jk} = \frac{\partial E}{\partial W_{jk}}$$

$$= \frac{\partial E}{\partial Y_k} \times \frac{\partial Y_k}{\partial Y_{ink}} \times \frac{\partial Y_{ink}}{\partial W_{jk}}$$

$$= -(t_k - y_k) \times f'(Y_{ink}) \times z_j$$

→ Error correction term for output unit Y_k ($k=1$ to m)

$$\Rightarrow \underline{\delta_k} = (t_k - y_k) \cdot f'(Y_{ink})$$

Updates in weight & bias

$$\underline{\Delta W_{jk}} = \alpha \delta_k \cdot z_j \quad , \quad \underline{\Delta W_{ok}} = \alpha \delta_k$$

→ Error correction term for hidden unit Z_j ($j=1$ to p)

$$\delta_{inj} = \sum_{k=1}^m \delta_k \cdot W_{jk}$$

$$\delta_j = \delta_{inj} \cdot f'(Z_{inj})$$

Updates in weight & bias

$$\underline{\Delta V_{ij}} = \alpha \delta_j \cdot x_i \quad ; \quad \underline{\Delta V_{oj}} = \alpha \delta_j$$

→ Activation function -

Binary Sigmoidal (0 to 1)

Bipolar Sigmoidal (-1 to 1)

→ Features:

- Continuity
- Differentiability
- Non decreasing monotony