Line Drawing Algorithms.

Point - can be represented on screen by specifying its color & intensity.

Point (x, y) or Point (x, y, z)

⇒ **Properties of line**

1. It should interpolate both end points
2. Brightness of line is independent of orientation of line.
3. It should appear straight & smooth.
4. It should be drawn quickly

⇒ **Representation of Line.**

1. Implicit representation
   $ax + bx + c = 0$
2. Explicit representation.
   $y = mx + c.$
3. Parametric Representation

$$x = x_1 + (x_2 - x_1) \cdot t$$
$$y = y_1 + (y_2 - y_1) \cdot t$$

⊛ Incremental Approach

$y = mx + c$

where c is y intercept
m is slope.

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

→ next point $x_1 = x_0 + 1$

$y_1 = m(x_0) + c$

$$\boxed{c = y_0 - m x_0}$$

algorithm
Line-1 (x_1, y_1, x_2, y_2, c)

step 1 : $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

step 2 : $c = y_1 - m x_1$

for $x = x_1$ to x_2 do
    $y = mx + c$
    putpixel $(x, \text{round}(y))$
end

DDA line Algorithm

(Digital Differential Analyzer)

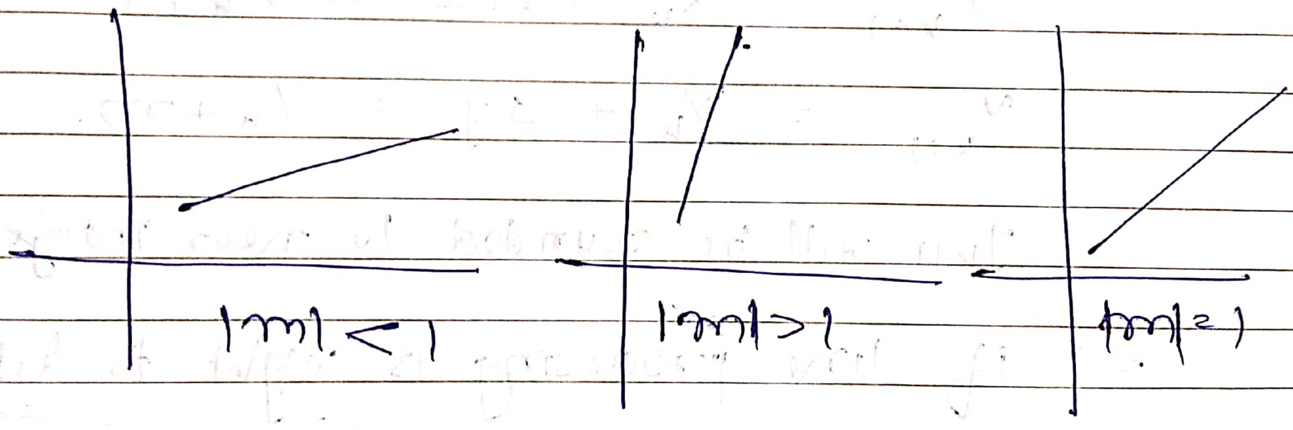
→ Horizontal & Vertical displacement is set to unit interval & corresponding displacement for other direction is calculated using slope.

→ if line makes angle < 45° with x axis.

i.e $|m| < 1$ then $x = x + 1$

→ if angle > 45° with x axis.

i.e $|m| > 1$ then $y = y + 1$



new value = old value + displacement

$x_{k+1} = x_k + \Delta x$ $\quad\quad m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

$y_{k+1} = y_k + \Delta y$

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Case I: $|m| < 1$ (line processing is left to right)

→ Δx is set to unit interval
i.e. $\Delta x = 1$

→ and corresponding coordinate is calculated by:

$$m = \Delta y / \Delta x$$

$$m = \Delta y / 1$$

$$\therefore \boxed{\Delta y = m}$$

→ So, coordinates for next pixel are.

$$x_{k+1} = x_k + \Delta x = x_k + 1$$

$$y_{k+1} = y_k + \Delta y = y_k + m.$$

y_{k+1} will be rounded to near integer.

⇒ If line processing is right to left then value of x & y decreases in every iteration

So $\Delta x = -1$ & $\Delta y = -m$

So, coordinates value for next pixel is given as.

$$y_{k+1} = y_k + \Delta y = y_k - m$$

$$x_{k+1} = x_k + \Delta x = x_k - 1$$

<u>Case II</u> $|m| > 1$ (line processing left to right)

Set Δy to unit interval i.e. $\Delta y = 1$ & corresponding x coordinate is computed.

$$m = \Delta y / \Delta x = \frac{1}{\Delta x}$$

$$m = 1/\Delta x$$

$$\boxed{\Delta x = 1/m}$$

Coordinates of next pixel.

$$x_{k+1} = x_k + 1/m$$

$$y_{k+1} = y_k + 1$$

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⇒ **advantages** :-

1) Simple
2) eliminates multiplications involved in explicit line drawing algo.

i.e. ⇒ $y = mx + c$

3) DDA is faster
4) more efficient than simple line algo.

⇒ **Disadvantages**

1. floating point operation at each pixel.
2. performs roundup operation at each pixel.
3. Rounding error will be accumulated in each iteration
4. Calculated pixel position may drift from actual position due to rounding of an error.

$m > 1$ $\quad x = x + 1/m$ $\quad y = y + 1$

$m < 1$ $\quad x = x + 1$ $\quad y = y + m$

(*) Algorithm DDA

DDA_Line (x_1, y_1, x_2, y_2)

Step-1 $\quad dx = x_2 - x_1$

$dy = y_2 - y_1$

Step 2 $\quad x = x_1$

$y = y_1$

$m = dy / dx.$

Step 3: if abs (m) $\leq$ 1 then

~~steps = abs~~

incr = abs (dx)

else

incr = abs (dy)

end.

Step 4

$\Delta x = dx / incr$

$\Delta y = dy / incr$

putpixel (x, y, 1)

Step 5 for $x = x_1$ to x_2 do

$x = x + \Delta x$

$y = y + \Delta y$

putpixel (x, round(y) , 1)

end

Ex:- Draw line from $\overset{x_1\ y_1}{(2,2)}$ to $\overset{x_2\ y_2}{(10,7)}$ using DDA line drawing algo.

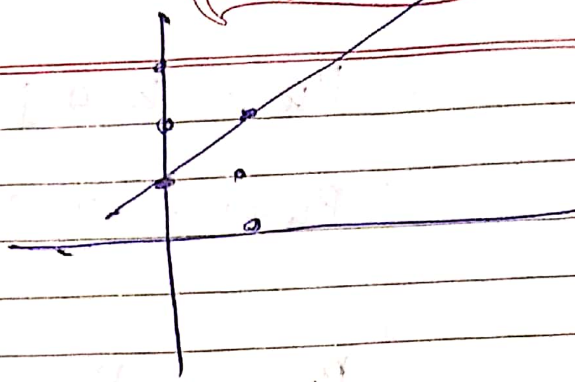
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{7-2}{10-2} = 5/8 = 0.625$$

here $|m| < 1$ so $\Delta x = 1$

$\Delta y = m = 0.625$

k	$x_k\ y_k$	$x_{k+1} = x_k + 1$	$y_{k+1} = y_k + 1$	Round(y_{k+1})
0	(2, 2)	2+1 = 3	2+0.625 = 2.625	3
1	(3, 3)	3+1 = 4	2+0.625 = 3.25	3
2	(4, 3)	5	3.25+0.625 = 3.8	4
3	(5, 4)	6	3.875 + 0.625 = 4.5	5
4	(6, 5)	7	4.5 + 0.625 = 5.125	5
5	(7, 5)	8	5.125 + 0.625 = 5.75	6
6	(8, 6)	9	5.75 + 0.625 = 6.315	6
7	(9, 6)	10	6.315 + 0.625 = 7.00	7
8	(10, 7)			



for pixel position x_{k+1}

$y = m(x_k+1)+b$

$d_1 = y - y_k$

$= m(x_k+1)+b-y_k$ — (1)

$d_2 = (y_k+1) - y$

$= y_k+1-m(x_k+1)-b$ — (2)

$d_1 - d_2 = 2m(x_k+1) - 2y_k + 2b - 1$ — (3)

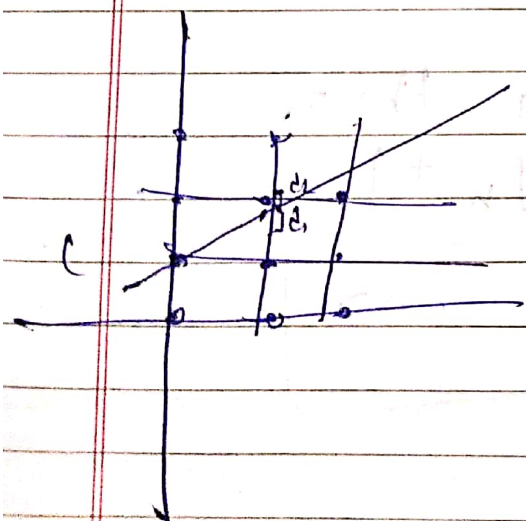
$P_k = \Delta x\,(d_1 - d_2)$

$$\boxed{P_k = 2\Delta y\cdot x_k - 2\Delta x\cdot y_k + C}$$

$C = 2\Delta y + 2\Delta x b$
$\quad - 2b\Delta x$

$= 2\Delta y - \cancel{\Delta x}\,C$

$= 2\Delta y + \Delta x(2b-1)$



$$P_k = 2\Delta y\, x_k - 2\Delta x\, y_k + C$$

$$P_{k+1} = 2\Delta y\,(x_{k+1}) - 2\Delta x\, y_{k+1} + C$$

$$P_{k+1} - P_k = 2\Delta y\,(x_{k+1} - x_k) - 2\Delta x\,(y_{k+1} - y_k)$$

here $x_{k+1} = x_k + 1$

$$= 2\Delta y - 2\Delta x\,(y_{k+1} - y_k)$$

$$\boxed{P_{k+1} = P_k + 2\Delta y - 2\Delta x\,(y_{k+1} - y_k)}$$

if next pixel is y_k then

$$y_{k+1} = y_k$$

$$P_{k+1} = P_k + 2\Delta y - 2\Delta x.(0)$$

$$= \boxed{P_k + 2\Delta y}$$

if next pixel is y_{k+1} then

$$y_{k+1} = y_k + 1$$

$$\boxed{P_{k+1} = P_k + 2\Delta y - 2\Delta x}$$

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(4) Generalized Bresenham's Algorithm

Step 1: read the end point (x_1, y_1) & (x_2, y_2)

Step 2:
Calculate $dx = |x_2 - x_1|$
$dy = |y_2 - y_1|$

Step 3 Initialize.

$x = x_1$
$y = y_1$

$x = x - 1$ $y = y + 1$	$x = x + 1$ $y = y + 1$
$x = x - 1$ $y = y - 1$	$x = x + 1$ $y = y - 1$

Step 4: $S_1 = Sign(x_2 - x_1)$
$S_2 = Sign(y_2 - y_1)$ } return 1 if +ve
0 if 0
-1 if -ve

Step 5 if $dy > dx$
then exchange dx & dy
ex-change = 1
else
ex-change = 0

Step 6 $e = 2 * dy - dx$

Step 7 $i = 1$

Step 8 plot (x, y)

step-9 while (e ≥ 0)

If ex-change = 1 then

$x = x + S1$

else

$y = y + S2$

$e = e - 2 * dx$

step-10

if ex-change ≠ 1

$x = x + S1$

else

$y = y + S2$

$e = e + 2 * dy$

$i = i + 1$

if

$c = c + 2\Delta y$
$- 2\Delta x$

Generalized Bresenham's

Ex: (0,0) to (6,7)

$x = 0$ $\Delta x = 6$ $S_1 = 1$
$y = 0$ $\Delta y = 7$ $S_2 = 1$

$\Delta y > \Delta x$ so, exchange $= 1$

$\Delta x = 7$ $\Delta y = 6$

plot (0,0)

$d_0 = 2\Delta y - \Delta x$
$= 12 - 7 = 5$

$d > 0$, $x = x + S_1 = 0 + 1 = 1$
$d = d - 2\Delta x$
$= 5 - 14 = -9$

$d < 0$, exit for loop.

exchange $= 1$ $y = y + S_2$
$y = 0 + 1 = 1$
$d = d + 2\Delta y$
$= -9 + 2(6)$
$\boxed{d = 3}$

plot (1,1)

$e > 0$
$x = x + S_1 = 1 + 1 = 2$
$e = e - 2\Delta x = 3 - 2(7) = -11$
~~$y = y + S_2 = 1 + 1 = 2$~~
$e = e + 2\Delta y = -11 + 2 * 6 = 1$
$\boxed{e = 1}$

plot (2,2)

$e > 0$

$x = x + S1 = 2 + 1 = 3$

$e = e - 2\Delta x = 1 - 2(7)$

$= -13$

$y = 2 + 1 = 3$

$e = e + 2\Delta y$

$= -13 + 2(6) = -1$

plot (3, 3)

$\boxed{e = -1}$ $e < 0$

$y = y + S2$

$= 3 + 1 = 4$

$e = e + 2 * \Delta y$

$= -1 + 2 * (6) = 11$

$\boxed{e = 11}$

plot (3, 4)

$e \geq 0$

$x = 3 + 1 = 4$

$e = e - 2\Delta x$

$= 11 - 2(7)$

$= 11 - 14$

$= -3$

$y = 4 + 1 = 5$

$P = P + 2 * \Delta y = -3 + 2(6)$

$= -3 + 12$

$= 9$

plot $(4, 5)$

$\boxed{P = 9 > 0}$

$x = 4 + 1 = 5$

$P = P - 2\Delta x$

$= 9 - 2(7) = -5$

$y = y + 1 = 5 + 1 = 6.$

$P = P + 2\Delta y$

$= -5 + 12 = 7$

plot $(5, 6)$

$P = 7 > 0$

$x = 5 + 1 = 6$

$P = P - 2\Delta x$

$= 7 - 14 = -7$

$y = y + 1 = 6 + 1 = 7$

$P = P + 2 * \Delta y$

$= -7 + 12 = 5$

plot $(6, 7)$

$x_1 \quad y_1 \qquad\qquad x_2 \quad y_2$

Ex: $(-1, -1)$ to $(-5, -6)$

$dx = -5 - (-1)$
$= -5 + 1 = |-4| = 4$

$dy = -6 - (-1) = |-5| = 5$

$S_1 = -1 \qquad S_2 = -1$

$\Rightarrow x = -1 \quad y = -1$

$\Delta y > \Delta x$. exchange $= 1$

$\boxed{\Delta x = 5} \quad \boxed{\Delta y = 4.}$

$d = 2\Delta y - \Delta x.$
$= 2(4) - 5 = 3$

plot $(-1, -1)$

$\Rightarrow d > 0$

$x = x + S_1 = -1 - 1 = -2$

$d = d - 2\Delta x = 3 - 10 = -7$

$y = y + S_2 = -1 - 1 = -2$

$d = d + 2\Delta y = -7 + 8$
$= 1$

plot $(-2, -2)$

$\Rightarrow$ $d \geq 0$

$x = x + s1 = -3$

$d = d - 2ax = 1 - 10 = -9$

$y = -3$

$d = d + 2ay = -9 + 8 = \boxed{-1}$

plot $(-3, -3)$

$\Rightarrow$ $d < 0$

$y = -4$

$d = d + 2ay = -1 + 8 = 7$

plot $(-3, -4)$

$\Rightarrow$ $d > 0$

$x = -4$

$d = d - 2ax = +\cancel{7} - 10 = -3$

$y = -5$

$d = d + 2ay = -3 + 8 = 5$

plot $(-4, -5)$

$\rightarrow$ $d > 0$

$x = -5$

$d = d - 2ax = 5 - 10 = -5$

$y = -6$

$d = d + 2ay = -5 + 8 = 3$

$x_1 \; y_1 \qquad\qquad x_2 \; y_2$

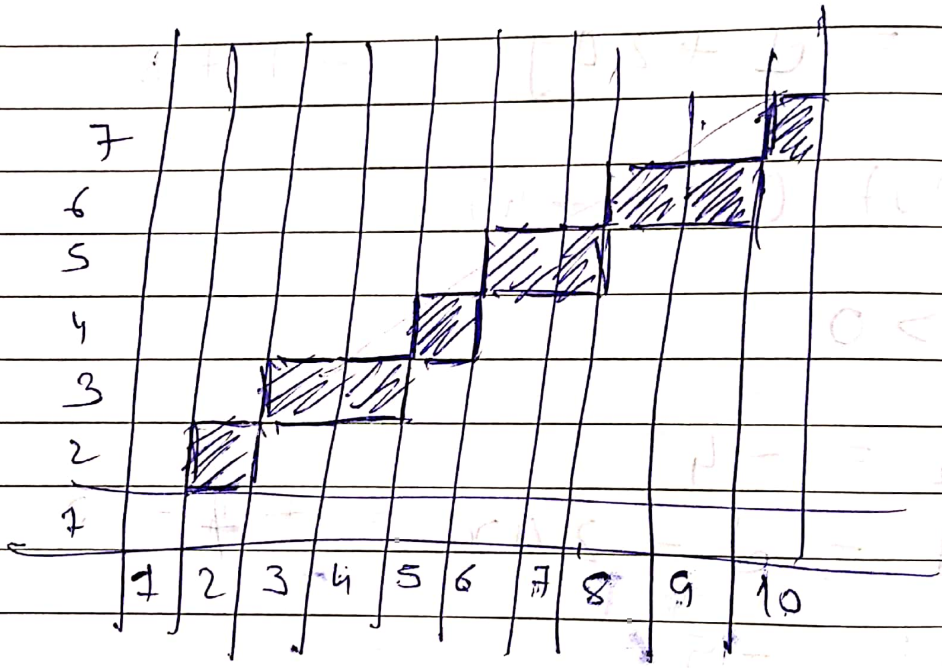
Ex:- $(0,0)$ to $(4,6)$

$|dx| = |4-0| = |4| = 4$

$dy = 6 - 0 = 6.$

$S_1 = 1 \qquad S_2 = 1$

$dy > dx.$



Ex

$x_1 \; y_1 \qquad x_2 \; y_2$

$(2, 2)$ to $(10, 7)$

$\Delta x = abs(10 - 2) = 8$

$\Delta y = abs(7 - 2) = 5$

$s_1 = 1 \qquad s_2 = 1$

$dy \not> dx \qquad dy < dx.$

$5 \qquad 8$

$x = 2, \; y = 2$

→ exchange $= 0$

$d = 2\Delta y - \Delta x$

$= 10 - 8 = 2$

plot $(2, 2)$

(1) $d \geq 0$

$y = y + s_2 = 3$

$d = d - 2\Delta x = 2 - 16 = -14$

$d < 0$ exit loop.

$x = x + s_1 = 3$

$d = d + 2\Delta y = -14 + 10 = -4$

plot $(3, 3)$

(2) $d < 0$

$x = 3 + 1 = 4$

$x = x + S1 = 3 + 1 = 4$

$d = d + 2\Delta y$

$= -4 + 10 = 6$

plot $(4, 3)$

(4) $d \geq 0$

$y = y + S1 = 3 + 1 = 4$

$d = d - 2\Delta x$

$= 6 - 16 = -10$

$x = x + S1 = 5$

$d = d + 2\Delta y = -10 + 10$

plot $(5, 4)$ $= 0$

(5) $d \geq 0$

$y = 6$

$d = 0 - 16 = -16$

$x = 5$

$d = \cancel{0}^{-16} + 10 = \cancel{10}\ -6$

plot (6, 5)

(6) $d < 0$

$x = x + 5, = 7$

$d = d + 2\Delta y = -6 + 10$

$= 4$

plot (7, 5)

(7) $d > 0$

$y = 6$

$d = d - 2\Delta x$

$= 4 - 16 = -12$

$x = 8$

$d = -12 + 10 = -2$

plot (8, 6)

(8) $d < 0$

$x = 8$

$d = d + 2\Delta y$

$= -2 + 10 = 8$

plot (9, 6)

(9) $d \geq 0$

$x = 10$

$y = 7$

The Circle is represented as.

$$(x - x_c)^2 + (y - y_c)^2 = r^2$$

(mathematically) — ①

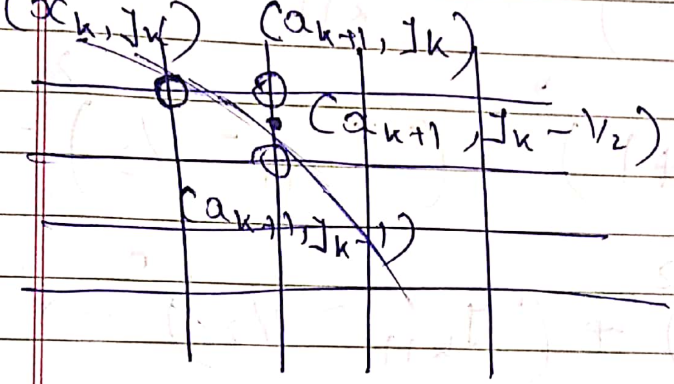
$$f(x,y) = (x - x_c)^2 + (y - y_c)^2 - r^2$$ — ②

here if

$f(x,y) = 0$ ⇒ The point lies on the circle.

> 0 ⇒ point lies outside the circle.

< 0 ⇒ point lies inside the circle.



Co-ordinates of mid point of two pixels are,

$$(x_k + 1 \; , \; y_k - 1/2)$$

Decision parameter

$$d_k = f(x_k + 1, y_k - 1/2)$$

$$= (x_k + 1)^2 + (y_k - 1/2)^2 - r^2 \quad \text{--- (3)}$$

$\cancel{d_{k+1} = f(x_{k+1} + 1)}$

$$d_{k+1} = f(x_{k+1} + 1, y_{k+1} - 1/2)$$

$$= (x_{k+1} + 1)^2 + (y_{k+1} - 1/2)^2 - r^2$$

$$= (x_k + 1 + 1)^2 + (y_{k+1} - 1/2)^2 - r^2$$

$$= (x_k + 2)^2 + (y_{k+1} - 1/2)^2 - r^2 \quad \text{--- (4)}$$

$\Rightarrow$ (4) $-$ (3) gives

$$d_{k+1} - d_k = (x_k + 2)^2 - (x_k + 1)^2 + (y_{k+1} - 1/2)^2 - (y_k - 1/2)^2 - r^2 + r^2$$

$$d_{k+1} = d_k + (2x_k + 3) + (y_{k+1} + y_k - 1)(y_{k+1} - y_k)$$

(5)

if $d_k \leq 0$, midpoint lies inside

here we have $\boxed{y_{k+1} = y_k}$

$$\boxed{d_{k+1} = d_k + 2x_k + 3}$$

(6).

if $d_k > 0$ midpoint lies outside

$$y_{k+1} = y_k - 1.$$

$$d_{k+1} = d_k + 2x_k + 3 + (y_k - 1 + y_k - 1)(y_k - 1 - y_k)$$

$$= d_k + 2x_k - 2y_k + 5$$

$$= d_k + 2(x_k - y_k) + 5$$

$\underline{x = 0}$, $\underline{y = r}$

Initial Decision parameter

$(x_c, y_c) = (0,0) \rightarrow r = r$

$P_0 = f_{circle}(1, r - 1/2)$

$= 1 + (r - 1/2)^2 - r^2$

$$\boxed{P_0 = 5/4 - r}$$

i.e. $\boxed{P_0 = 1 - r}$

if $P_K \leq 0$ — $(x_{k+1}, y_n$

$y_{k+1} = y_k$ &

$P_{k+1} = P_k + 2x_k + 3$

else

$y_{k+1} = y_k - 1$

$P_{k+1} = P_k + 2x_k - 2y_k + 5$

Scanned with CamScanner