CONM-Unit-II

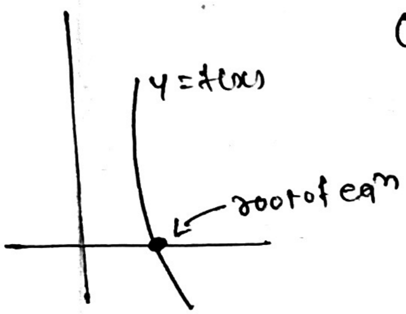
Algebric equation : The equation $f(x)=0$ is called an algebric equation if $f(x)$ is purely a polynomical in x.

e.g. $x^3-5x+6=0$ is an elligebric eqn.

Transcendental equation : The eqn including trigonometric or exponential or logarithmic function is known as transcendental eqn.

e.g. $x-\log x=0$.

⇒ The root of equation $f(x)=0$ is a value of x which satisfies the given equation



Geometric Interpolation of roots of eqn : Graphically root of a equation $y=f(x)$ is a point on x axis where x axis and that curve intersect or one can say that it's a value of x where the graph of $y=f(x)$ crosses the x axis.

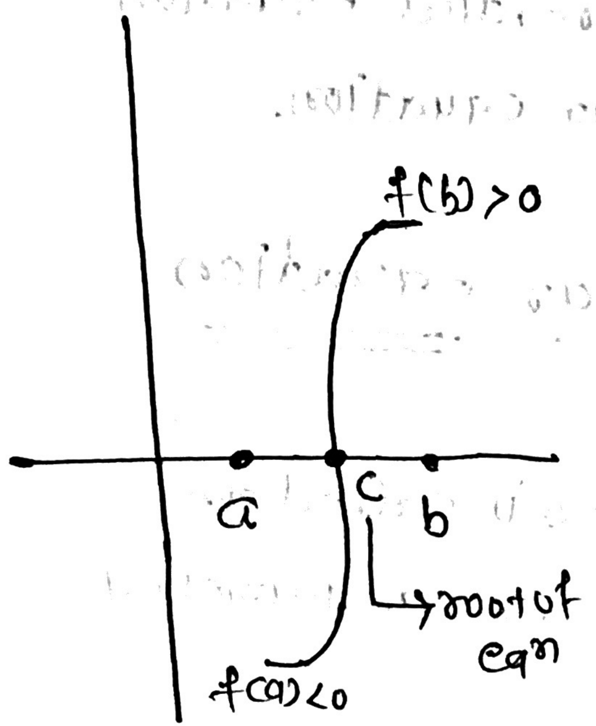
⇒ Generally numerical methods are divided into two part

1. Direct method
2. Indirect method. (Iterative method).

- In direct method we'll use particular formula and we get the solution of mathematical problem.

- In iterative method we have to repeate one procedure no. of times and we have the solution of mathematical problems.

* Intermediate Value Theorm: If f is continues on close interval of a to b and if f(a) and f(b) have opposite signs then there exist one point c in open interval a,b such that f(c) = 0.

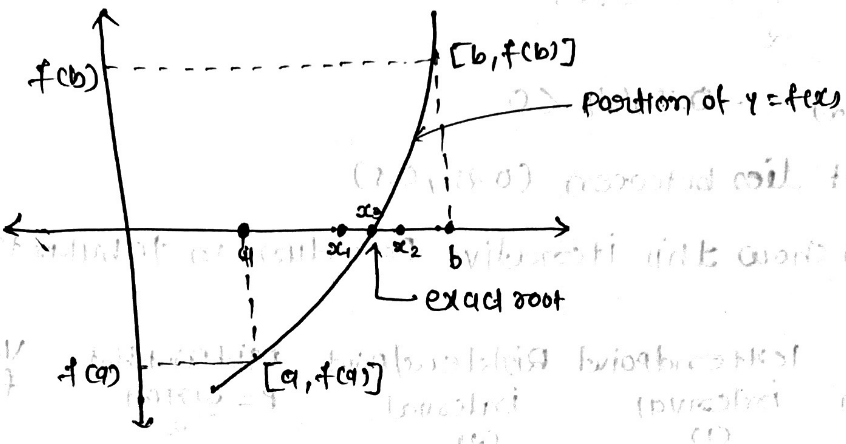


* BISECTION METHOD / HALF INTERVAL METHOD / BOLZANO METHOD.

⇒ Bisection method is also known as bolzano method or half interval method and it is one of the simplest iterative method.

⇒ To start with two initial approximation say a and b such that $f(a) \times f(b) < 0$, which ensures that root lies between a and b.

⇒ Graphical representation of bisection method shown in fig.



⇒ Now according to this method first bisect the interval [a, b] at $x_1 = a+b/2$

- If $f(x_1) = 0$, then x_1 is the root of given equation
- If $f(x_1)$ = positive ($f(a)f(x_1) < 0$), then root lies between a and x_1
- If $f(x_1)$ = negative ($f(x_1)f(b) < 0$), then root lies between x_1 and b.

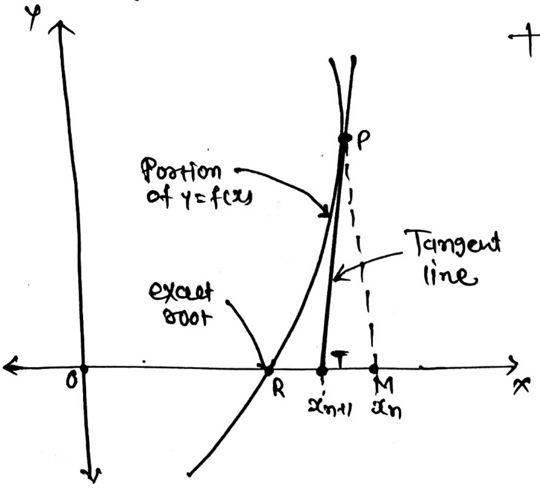
⇒ Then again bisect this interval and repeat the above procedure till the root upto desired accuracy is obtained.

NEWTON RAPHSON METHOD (N-R METHOD)

⇒ Newton Raphson method is faster due to the use of local behaviour of funn f(x) like derivative of funn.

⇒ It requires only one intial value for the root.

⇒ The method can be derived by two ways graphically and analytically.



$$\tan\theta = \frac{PM}{TM} = \frac{f(x_n)}{x_n - x_{n+1}} = f'(x_n)$$

$$\therefore x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \; ; n = 0,1,2,3.$$

⇒ This is the general iteration Scheme for NR Method.

<u>Analytical Approach</u> : The expansion of $f(x)=0$ by taylor's series in powers of $(x-x_0)$ is

$$f(x) = f(x_0) + (x-x_0)f'(x_0) + \frac{1}{2}(x-x_0)^2 f''(x_0) + \cdots = 0.$$

Where x = exact root.
x_0 = approximate root.

$(x-x_0)f'(x_0) = -f(x_0)$

$x - x_0 = -\frac{f(x_0)}{f'(x_0)}$

$x = -\frac{f(x_0)}{f'(x_0)} + x_0$

$x = x_0 - \frac{f(x_0)}{f'(x_0)}$

$$\therefore f(x_0) + (x-x_0)f'(x_0) = 0$$

$$\boxed{\therefore x = x_0 - \frac{f(x_0)}{f'(x_0)}} \; ; f'(x_0) \neq 0.$$

⇒ The next approximation can be obtained by

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \; ; f'(x_1) \neq 0.$$

By repeating this procedure we have general formula

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \; ; f(x_n) \neq 0 ; n = 0,1,2,3\ldots}$$

* Derive the formula for finding $\sqrt[q]{N}$ using Newton Raphson formula $N \in R$, $q \in N$.

$x = \sqrt[q]{N}$

$x = N^{1/q}$

$x^q = N$

$\boxed{f(x) = x^q - N = 0}$

$\therefore f'(x) = q x^{q-1}$

We know the N-R formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{x_n^q - N}{q x_n^{q-1}}$$

$$= x_n - \frac{x_n^q}{q x_n^{q-1}} - \frac{N}{q x_n^{q-1}}$$

$$= x_n - \frac{x_n}{q} + \frac{N}{q x_n^{q-1}}$$

$$= x_n \left[1 - 1/q\right] + \frac{N}{q x_n^{q-1}}$$

$$= x_n \left[\frac{q-1}{q}\right] + \frac{N}{q x_n^{q-1}}$$

$$\boxed{= \frac{1}{q}\left[x_n (q-1) + \frac{N}{x_n^{q-1}}\right]}$$

$n = 0, 1, 2, 3.$

Find the formula for finding reciprocal of a given number $x = 1/N$ or $\frac{1}{x} - N = 0$ using newton Raphson method.

$$f(x) = \frac{1}{x} - N$$

$$f'(x) = -\frac{1}{x^2}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

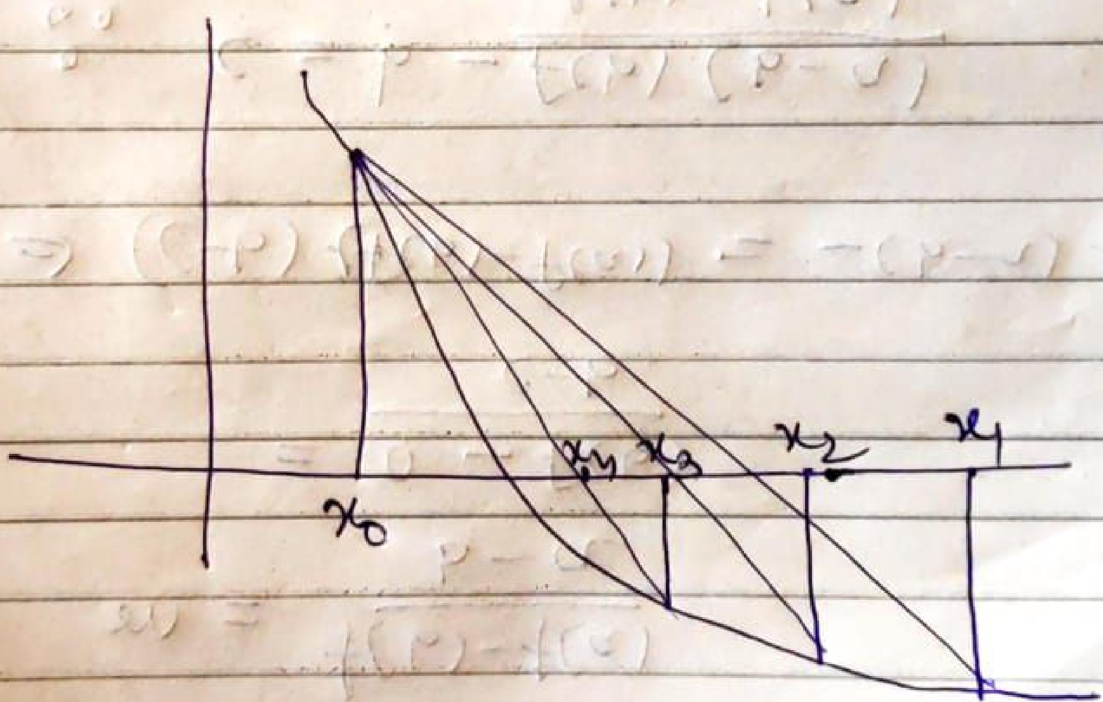
$$= x_n - \frac{\left(\frac{1}{x_n}\right) - N}{\left(-1/x_n^2\right)}$$

$$\boxed{= x_n\,(2 - N x_n)}$$

$f(b)-f(a)$

Regula falsi method

In the bisection method the convergence process depends only on the choice of the end points of the interval [a, b]. The function $f(x)$ does not have any role in finding the point c. A better approx. to c can be obtained by taking the st. line L Joining the points $(a, f(a))$ & $(b, f(b))$ intersecting the x-axis



$$m = \frac{f(b)-f(a)}{b-a}$$

$$= \frac{0 - f(b)}{c-b}$$

$$\Rightarrow (c-b)(f(b)-f(a)) = -(b-a)f(b)$$

$$\therefore \quad c = b - \frac{f(b)(b-a)}{f(b)-f(a)}$$

$$= \frac{a f(b) - b f(a)}{f(b) - f(a)}$$