Design of Pressure Vessel

Contents

- Introduction.
- Classification Of Pressure Vessels.
- Stresses In A Thin Cylindrical Shell Due To An Internal pressure.
- Circumferential Or Hoop Stress.
- Longitudinal Stress.
- Change In Dimensions Of a
- Thin Cylindrical Shell Due To An Internal Pressure.
- Thin Spherical Shells Subjected To An Internal Pressure
- Change In Dimensions Of A Thin Spherical Shell Due To An Internal Pressure
- Thick Cylindrical Shell Subjected To An Internal Pressure
- Compound Cylindrical Shells
- Stresses In Compound Cylindrical Shells
- Cylinder Heads And Cover Plates.

Introduction

- **Use**: To store fluids under pressure.
- The fluid being stored may undergo a change of state inside the pressure vessel as in case of steam boilers or it may combine with other reagents as in a chemical plant.
- The pressure vessels are designed with great care because rupture of a pressure vessel means an explosion which may cause loss of life and property.
- The material of pressure vessels may be brittle such as cast iron, or ductile such as mild steel.

Classification of Pressure Vessels

1. **According to the dimensions**

(a) **Thin shell:** wall thickness of the shell (t) is less than 1/10 of the diameter of the shell (d). **OR**, internal fluid pressure (*p) is less than 1/6 of the allowable* stress

Use: boilers, tanks and pipes

(b)**Thick shell**: wall thickness of the shell (t) is greater than 1/10 of the diameter of the shell (d) **OR**, internal fluid pressure (*p) is greater than 1/6 of the allowable* stress

Use: high pressure cylinders, tanks, gun barrels etc.

2. According to the end construction

(a) **open end** : A simple cylinder with a piston, such as cylinder of a press is an example of an open end vessel, In case of vessels having open ends, the circumferential or hoop stresses are induced by the fluid pressure.

(b) ***closed end:*** *A* tank is an example of a closed end vessel., whereas in case of closed ends, longitudinal stresses in addition to circumferential stresses are induced.

Stresses in a Thin Cylindrical Shell due to an Internal Pressure

Assumptions:

1. The effect of curvature of the cylinder wall is neglected.
2. The tensile stresses are uniformly distributed over the section of the walls.
3. The effect of the restraining action of the heads at the end of the pressure vessel is neglected

(a) Circumferential or hoop stress

Fail along the longitudinal section (i.e. circumferentially) splitting the cylinder into two troughs, Fig. (a).

(a) Failure of a cylindrical shell along the longitudinal section.

(b) Longitudinal stress.

Fail across the transverse section (i.e. longitudinally) splitting the cylinder into two cylindrical shells , Fig. (b).

(b) Failure of a cylindrical shell along the transverse section.

Circumferential or Hoop Stress

A tensile stress acting in a direction tangential to the circumference is called ***circumferential or***

hoop stress

➢ Total force acting on a longitudinal section (*i.e. along the diameter X-X) of the* shell

= Intensity of pressure × Projected area

= $p \times d \times l$ ...***(i)***

➢ Total resisting force acting on the cylinder walls

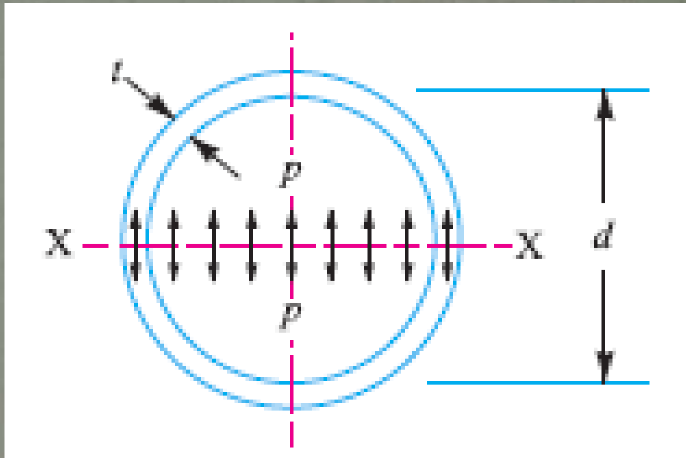
= $\sigma_{t1} \times 2t \times l$ ...*(two sections)* ...***(ii)***

➢From equations ***(i) and (ii),***

$\sigma_{t1} \times 2t$ $\times l$

$$t = \frac{p \times d}{2\,\sigma_{t1}}$$

(*a*) View of shell.



(*b*) Cross-section of shell.

Longitudinal Stress

- A tensile stress acting in the direction of the axis is called ***longitudinal stress.***

- It is a tensile stress acting on the *transverse or circumferential section Y-Y (or on the ends of the vessel).

- Total force acting on the transverse section (*i.e. along Y-Y)*

$$= p \times \frac{\pi}{4}(d)^2$$ ressure × cross-sectional area

...*(i)*

- Total resisting force

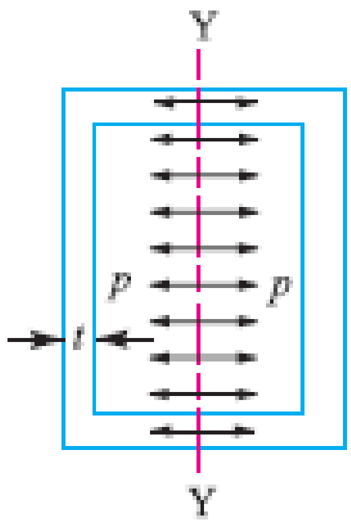
$$= \sigma_{t2} \times \pi d.t = p \times \frac{\pi}{4}(d)^2$$ ...*(ii)*

- I ons (*i*) ***and*** (

$$t = \frac{p \times d}{4\,\sigma_{t2}}$$

$$t = \frac{p \times d}{4\sigma_{t2} \times \eta_c}$$

(*a*) View of shell.



(*b*) Cross-section of shell.

Change in Dimensions of a Thin Cylindrical Shell due to an Internal Pressure

The increase in diameter of the shell due to an internal pressure is given by,

$$\delta d = \frac{p.d^2}{2\,t.E}\left(1 - \frac{\mu}{2}\right)$$

The increase in length of the shell due to an internal pressure is given by,

$$\delta l = \frac{p.d.l}{2\,t.E}\left(\frac{1}{2} - \mu\right)$$

Increase in diameter and length of the shell will also increase its volume.

The increase in volume of the shell due to an internal pressure is given by

$$\delta V = \text{Final volume} - \text{Original volume} = \frac{\pi}{4}(d + \delta d)^2 (l + \delta l) - \frac{\pi}{4} \times d^2.l$$

$$= \frac{\pi}{4}(d^2.\delta l + 2\,d.l.\delta d) \qquad \text{...(Neglecting small quantities)}$$

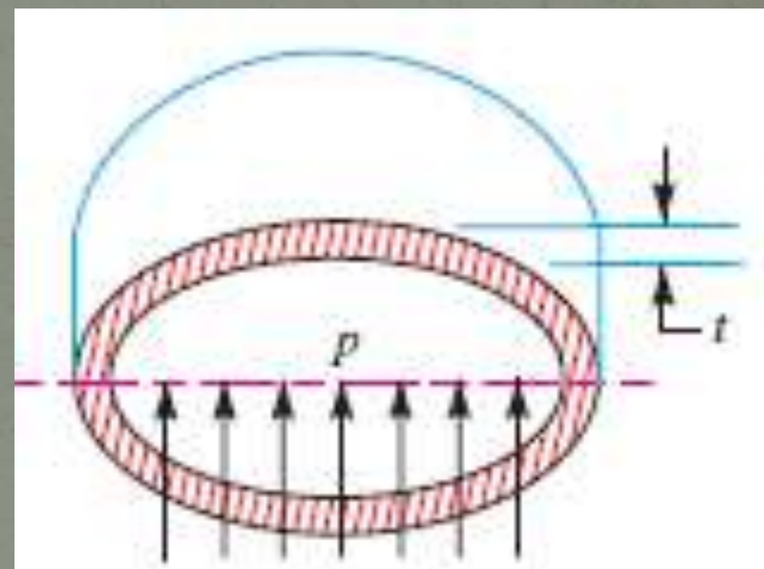
1. Diameter of the shell

Thin Spherical Shells Subjected to an
ure

The storage capacity of the shell

$$V = \frac{4}{3} \times \pi r^3 = \frac{\pi}{6} \times d^3$$

$$d = \left(\frac{6V}{\pi}\right)^{1/3}$$



2. Thickness of the shell

Forc
cen

$$= \text{Pressure} \times \text{Area} = p \times \frac{\pi}{4} \times d^2$$

Resisting force of the shell

= S d.t

Equatin

$$p \times \frac{\pi}{4} \times d^2 = \sigma_t \times \pi d.t$$

$$t = \frac{p.d}{4\,\sigma_t}$$

$$t = \frac{p.d}{4\,\sigma_t . \eta}$$

Change in Dimensions of a Thin Spherical Shell due to an Internal Pressure

Increase in diameter of the spherical shell due to an internal pressure is given by,

$$\delta d = \frac{p.d^2}{4\,t.E}(1-\mu) \qquad \text{...}(i)$$

and increase in volume of the spherical shell due to an internal pressure is given by,

$$\delta V = \text{Final volume} - \text{Original volume} = \frac{\pi}{6}(d+\delta d)^3 - \frac{\pi}{6} \times d^3$$

$$= \frac{\pi}{6}(3d^2 \times \delta d) \qquad \text{...(Neglecting higher terms)}$$

Substituting the value of δd from equation (i), we have

$$\delta V = \frac{3\pi d^2}{6}\left[\frac{p.d^2}{4\,t.E}(1-\mu)\right] = \frac{\pi\, p\, d^4}{8\,t.E}(1-\mu)$$

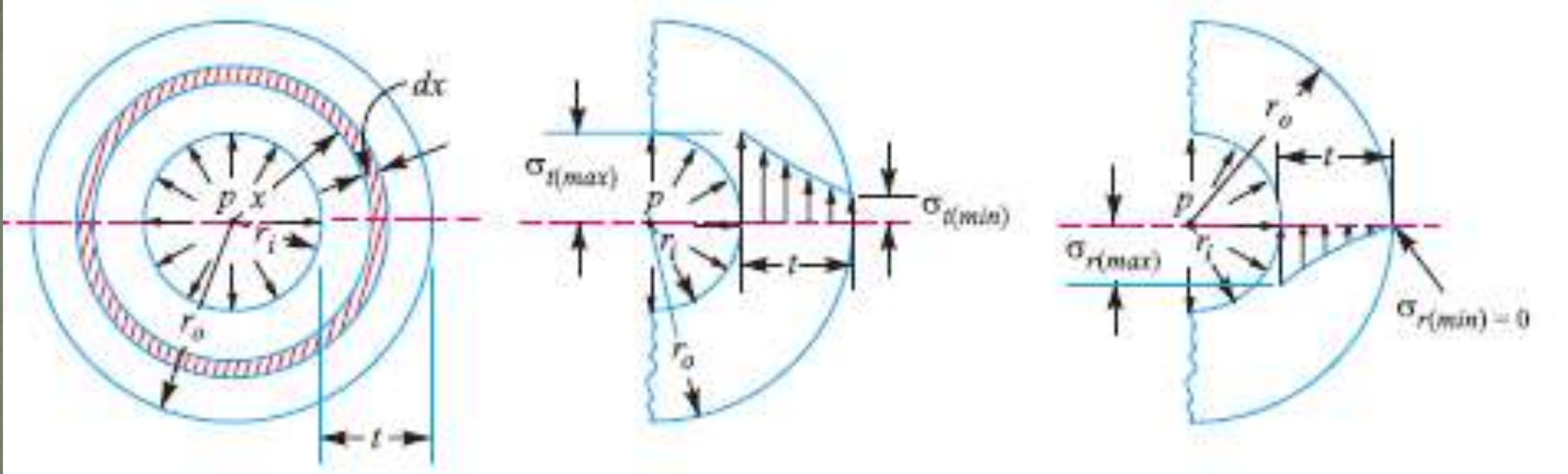
Thick Cylindrical Shells Subjected to an Internal Pressure

- When a cylindrical shell of a pressure vessel, hydraulic cylinder and a pipe is subjected to a very high internal fluid pressure, then the walls of the cylinder must be made extremely heavy or thick.

- In the case of thick wall cylinders , *the stress* over the section of the walls cannot be assumed to be uniformly distributed.

- They develop both **tangential** and **radial stresses** with values which are **dependent upon the radius** of the element under consideration.

- The **Tangential stress** is **maximum at the inner surface** and **minimum at the outer surface** of the shell.
- The **radial stress** is **maximum at the inner surface** and **zero at the outer surface** of the shell.



(a) Thick cylindrical shell. (b) Tangential stress distribution. (c) Radial stress distribution.

Let,

- *ro = Outer radius of cylindrical shell,*
- *ri = Inner radius of cylindrical shell,*
- *t = Thickness of cylindrical shell = ro – ri,*
- *p = Intensity of internal pressure,*
- μ = Poisson's ratio,
- σ*t = Tangential stress, and*
- σ*r = Radial stress.*

1. *Lame's equation*

- Assuming that the longitudinal fibres of the cylindrical shell are equally strained,
- tangential stress at any radius *x is,*

$$\sigma_t = \frac{p_i (r_i)^2 - p_o (r_o)^2}{(r_o)^2 - (r_i)^2} + \frac{(r_i)^2 (r_o)^2}{x^2} \left[\frac{p_i - p_o}{(r_o)^2 - (r_i)^2} \right]$$

- radial stress at any radius x,

$$\sigma_r = \frac{p_i (r_i)^2 - p_o (r_o)^2}{(r_o)^2 - (r_i)^2} - \frac{(r_i)^2 (r_o)^2}{x^2} \left[\frac{p_i - p_o}{(r_o)^2 - (r_i)^2} \right]$$

- Since we are concerned with the internal pressure (*pi = p) only, therefore substituting the value* of external pressure, *po* = *o.*

∴ Tangential stress at any radius x,

$$\sigma_t = \frac{p\,(r_i)^2}{(r_o)^2 - (r_i)^2}\left[1 + \frac{(r_o)^2}{x^2}\right]$$

and radial stress at any radius x,

$$\sigma_r = \frac{p\,(r_i)^2}{(r_o)^2 - (r_i)^2}\left[1 - \frac{(r_o)^2}{x^2}\right]$$

- *Maximum tangential stress at the inner surface of the shell,*

$$\sigma_{t(max)} = \frac{p\,[(r_o)^2 + (r_i)^2]}{(r_o)^2 - (r_i)^2}$$

- *Minimum tangential stress at the outer surface of the shell,*

$$\sigma_{t(min)} = \frac{2p\,(r_i)^2}{(r_o)^2 - (r_i)^2}$$

- maximum radial stress at the inner surface of the shell,

  $\sigma r(max) = -p$ *(compressive)*

- minimum radial stress at the outer surface of the shell,

  $\sigma r(min) = 0$

- In designing a thick cylindrical shell of brittle material (*e.g. cast iron, hard steel and cast* aluminium) with closed or open ends and in accordance with the maximum normal stress theory failure, the tangential stress induced in the cylinder wall,

$$\sigma_t = \sigma_{t(max)} = \frac{p\,[(r_o)^2 + (r_i)^2]}{(r_o)^2 - (r_i)^2}$$

Since ro = ri + t,

$$\sigma_t = \frac{p\,[(r_i + t)^2 + (r_i)^2]}{(r_i + t)^2 - (r_i)^2}$$

$$t = r_i \left[\sqrt{\frac{\sigma_t + p}{\sigma_t - p}} - 1 \right]$$

The value of *σt for brittle materials may be taken as 0.125 times the ultimate tensile* strength (*σu).*

- In case of cylinders made of ductile material, Lame's equation is modified according to maximum shear stress theory.
- According to this theory, the maximum shear stress at any point in a strained body is equal to one-half the algebraic difference of the maximum and minimum principal stresses at that poi
- Maximum

$$\sigma_{t\,(max)} = \frac{p\,[(r_o)^2 + (r_i)^2]}{(r_o)^2 - (r_i)^2}$$

nner surface

- Minimum principal stress at the outer surface,

$$\sigma t(min) = -p$$

- Maximum Shear Stress

$$\tau = \tau_{max} = \frac{\sigma_{t(max)} - \sigma_{t(min)}}{2} = \frac{\dfrac{p\,[(r_o)^2 + (r_i)^2]}{(r_o)^2 - (r_i)^2} - (-p)}{2}$$

$$= \frac{p\,[(r_o)^2 + (r_i)^2] + p\,[(r_o)^2 - (r_i)^2]}{2[(r_o)^2 - (r_i)^2]} = \frac{2p\,(r_o)^2}{2[(r_o)^2 - (r_i)^2]}$$

$$= \frac{p\,(r_i + t)^2}{(r_i + t)^2 - (r_i)^2} \qquad \ldots (\because r_o = r_i + t)$$

$$\tau(r_i + t)^2 - \tau(r_i)^2 = p(r_i + t)^2$$

$$(r_i + t)^2\,(\tau - p) = \tau(r_i)^2$$

$$\frac{(r_i + t)^2}{(r_i)^2} = \frac{\tau}{\tau - p}$$

$$\frac{r_i + t}{r_i} = \sqrt{\frac{\tau}{\tau - p}} \quad \text{or} \quad 1 + \frac{t}{r_i} = \sqrt{\frac{\tau}{\tau - p}}$$

$$\frac{t}{r_i} = \sqrt{\frac{\tau}{\tau - p}} - 1 \quad \text{or} \quad t = r_i\left[\sqrt{\frac{\tau}{\tau - p}} - 1\right] \qquad \ldots (iv)$$

- The value of shear stress (τ) is usually taken as one-half the tensile stress (σ*t)*.

$$t = r_i\left[\sqrt{\frac{\sigma_t}{\sigma_t - 2p}} - 1\right]$$

• From the above expression, we see that if the internal pressure (p) is equal to or greater than the allowable working stress (σt or τ), then no thickness of the cylinder wall will prevent failure.

• So, it is impossible to design a cylinder to withstand fluid pressure greater than the allowable working stress for a given material.

•This difficulty is overcome by using compound cylinders

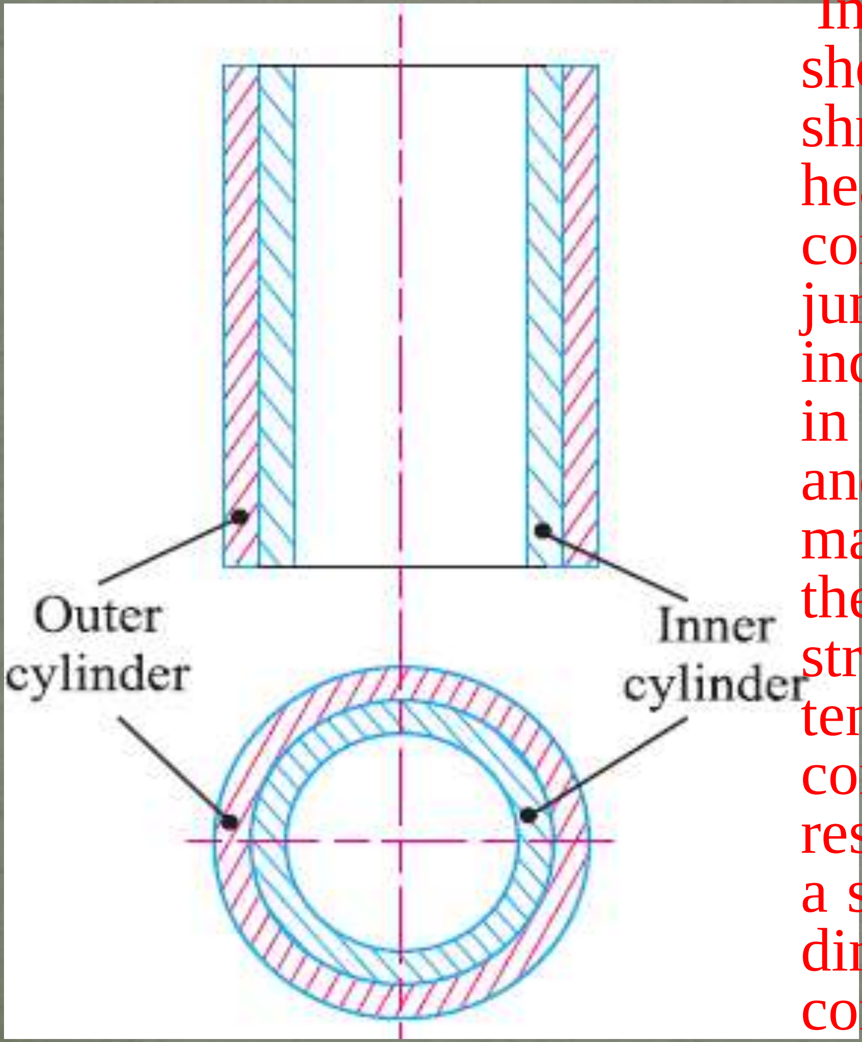
Compound cylinder

- According to Lame's equation, the thickness of a cylindrical shell is given by

$$t = r_i \left(\sqrt{\frac{\sigma_t + p}{\sigma_t - p}} - 1 \right)$$

It is impossible to design a cylinder to withstand internal pressure equal to or greater than the allowable working stress.
This difficulty is overcome by inducing an initial compressive stress on the wall of the cylindrical shell.
This may be done by the following two methods:
1. By using compound cylindrical shells, and
2. By using the theory of plasticity.

By using compound cylindrical shells,

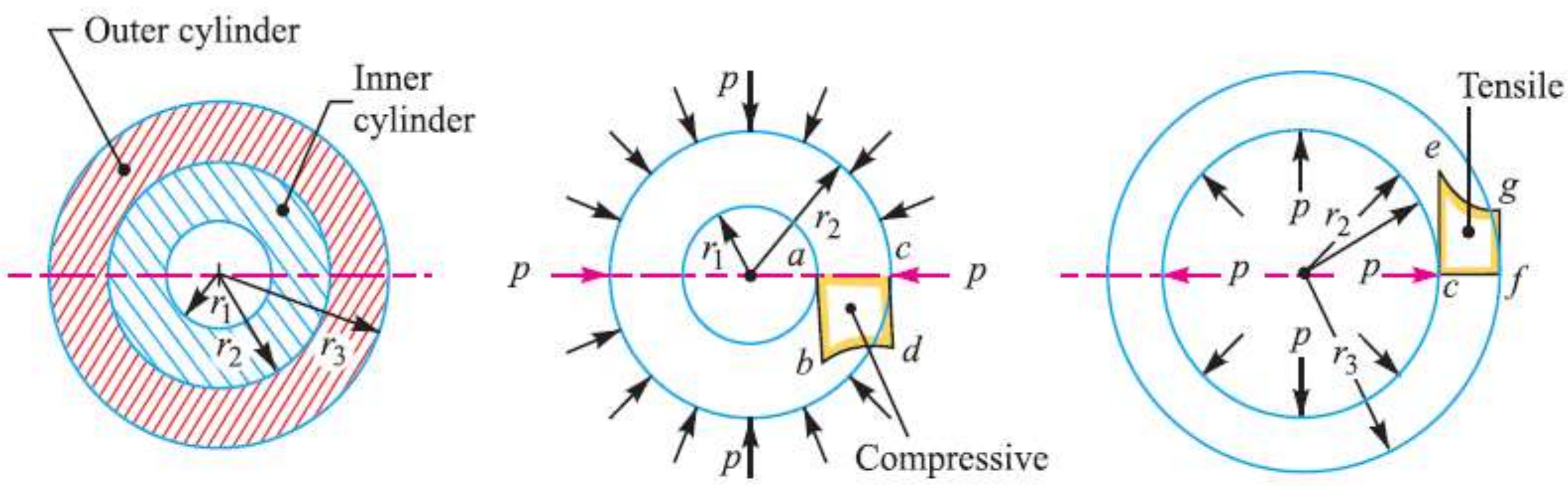


In a compound cylindrical shell, as shown in Fig. the outer cylinder is shrunk fit over the inner cylinder by heating and cooling. On cooling, the contact pressure is developed at the junction of the two cylinders, which induces compressive tangential stress in the material of the inner cylinder and tensile tangential stress in the material of the outer cylinder. When the cylinder is loaded, the compressive stresses are first relieved and then tensile stresses are induced. Thus, a compound cylinder is effective in resisting higher internal pressure than a single cylinder with the same overall dimensions. The principle of compound cylinder is used in the design of gun tubes.

By using the theory of plasticity.

- In the theory of plasticity, a temporary high internal pressure is applied till the plastic stage is reached near the inside of the cylinder wall.
- This results in a residual compressive stress upon the removal of the internal pressure, thereby making the cylinder more effective to withstand a higher internal pressure.

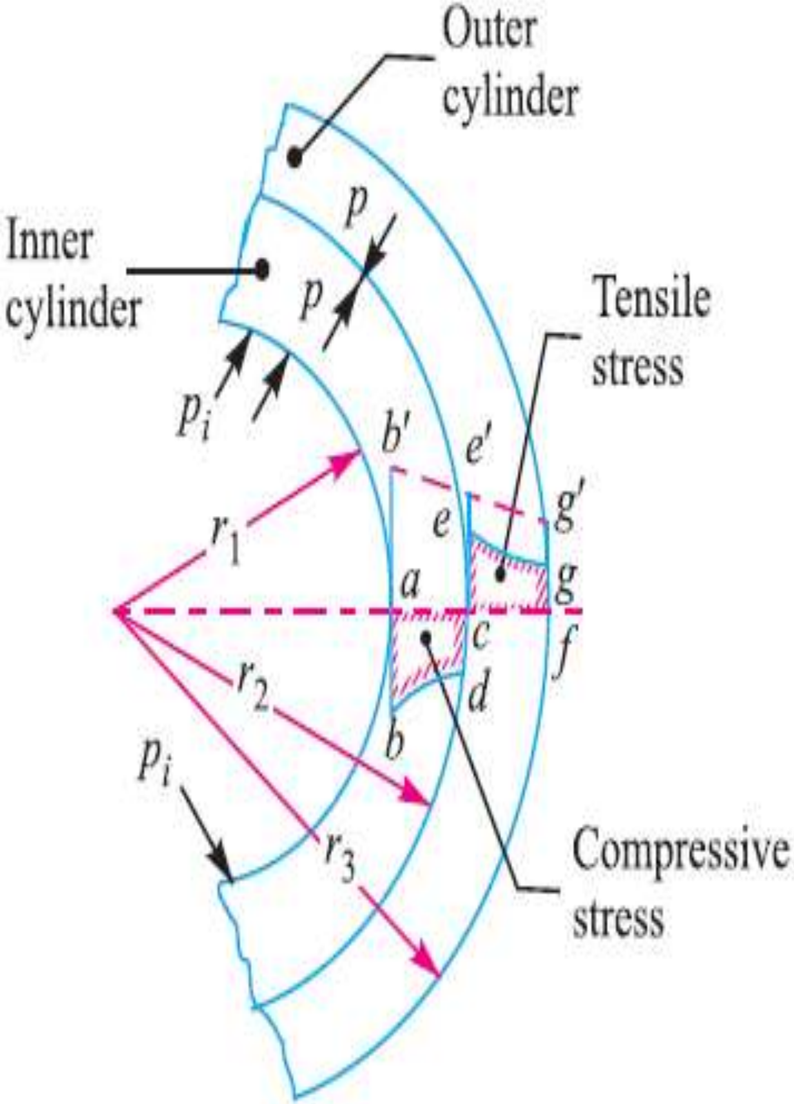
Stresses in Compound Cylindrical Shells



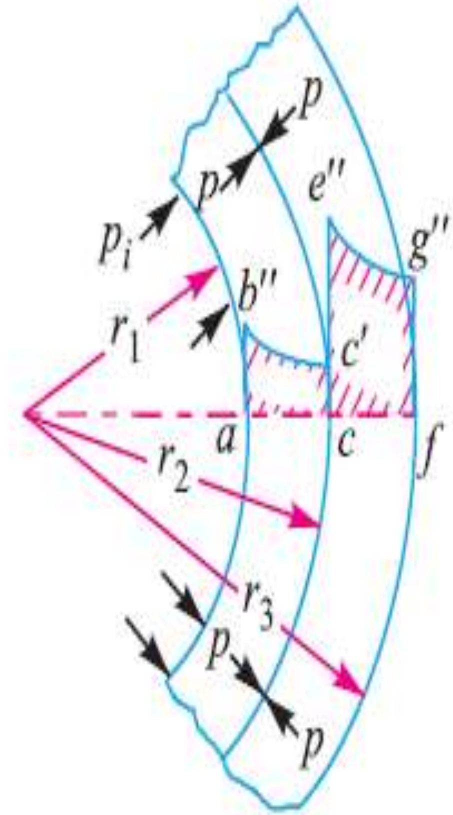
(*a*) Compound cylinder.

(*b*) Inner cylinder.

(*c*) Outer cylinder.



(*d*) Tangential stress distribution due to shrinkage fitting and internal fluid pressure.



(*e*) Resultant tangential stress distribution across a compound cylindrical shell.

the tangential stress at any radius x is

$$\sigma_t = \frac{p_i (r_i)^2 - p_o (r_o)^2}{(r_o)^2 - (r_i)^2} + \frac{(r_i)^2 (r_o)^2}{x^2}\left[\frac{p_i - p_o}{(r_o)^2 - (r_i)^2}\right] \quad ...(i)$$

and radial stress at any radius x,

$$\sigma_r = \frac{p_i (r_i)^2 - p_o (r_o)^2}{(r_o)^2 - (r_i)^2} - \frac{(r_i)^2 (r_o)^2}{x^2}\left[\frac{p_i - p_o}{(r_o)^2 - (r_i)^2}\right] \quad ...(ii)$$

Considering the external pressure only,

$$\sigma_t = \frac{-p_o (r_o)^2}{(r_o)^2 - (r_i)^2}\left[1 + \frac{(r_i)^2}{x^2}\right] \quad ...(iii)$$

...[Substituting $p_i = 0$ in equation (*i*)]

and $$\sigma_r = \frac{-p_o (r_o)^2}{(r_o)^2 - (r_i)^2}\left[1 - \frac{(r_i)^2}{x^2}\right] \quad ...(iv)$$

Considering the internal pressure only,

$$\sigma_t = \frac{p_i (r_i)^2}{(r_o)^2 - (r_i)^2}\left[1 + \frac{(r_o)^2}{x^2}\right] \quad ...(v)$$

...[Substituting $p_o = 0$ in equation (*i*)]

and

$$\sigma_r = \frac{p_i (r_i)^2}{(r_o)^2 - (r_i)^2}\left[1 - \frac{(r_o)^2}{x^2}\right] \quad ...(vi)$$

Since the inner cylinder is subjected to an external pressure (p) caused by the shrink fit and the outer cylinder is subjected to internal pressure (p), therefore from equation (*iii*), we find that the tangential stress at the inner surface of the inner cylinder,

$$\sigma_{t1} = \frac{-p (r_2)^2}{(r_2)^2 - (r_1)^2}\left[1 + \frac{(r_1)^2}{(r_1)^2}\right] = \frac{-2p (r_2)^2}{(r_2)^2 - (r_1)^2} \text{ (compressive)} \quad ...(vii)$$

... [Substituting $p_o = p$, $x = r_1$, $r_o = r_2$ and $r_i = r_1$]

Radial stress at the inner surface of the inner cylinder,

$$\sigma_{r1} = \frac{-p\,(r_2)^2}{(r_2)^2 - (r_1)^2}\left[1 - \frac{(r_1)^2}{(r_1)^2}\right] = 0 \qquad \text{...[From equation } (iv)]$$

Similarly from equation (*iii*), we find that tangential stress at the outer surface of the inner cylinder,

$$\sigma_{t2} = \frac{-p\,(r_2)^2}{(r_2)^2 - (r_1)^2}\left[1 + \frac{(r_1)^2}{(r_2)^2}\right] = \frac{-p\,[(r_2)^2 + (r_1)^2]}{(r_2)^2 - (r_1)^2} \text{ (compressive)} \quad ...(viii)$$

...[Substituting $p_o = p$, $x = r_2$, $r_o = r_2$ and $r_i = r_1$]

Radial stress at the outer surface of the inner cylinder,

$$\sigma_{r2} = \frac{-p\,(r_2)^2}{(r_2)^2 - (r_1)^2}\left[1 - \frac{(r_1)^2}{(r_2)^2}\right] = -p$$

Now let us consider the outer cylinder subjected to internal pressure (p). From equation (v), we find that the tangential stress at the inner surface of the outer cylinder,

$$\sigma_{t3} = \frac{p\,(r_2)^2}{(r_3)^2 - (r_2)^2}\left[1 + \frac{(r_3)^2}{(r_2)^2}\right] = \frac{p\,[(r_3)^2 + (r_2)^2]}{(r_3)^2 - (r_2)^2} \text{ (tensile)} \qquad ...(ix)$$

...[Substituting $p_i = p$, $x = r_2$, $r_o = r_3$ and $r_i = r_2$]

This stress is tensile and is shown by ce in Fig. 7.9 (c).

Radial stress at the inner surface of the outer cylinder,

$$\sigma_{r3} = \frac{p\,(r_2)^2}{(r_3)^2 - (r_2)^2}\left[1 - \frac{(r_3)^2}{(r_2)^2}\right] = -\,p \qquad \text{...[From equation } (vi)]$$

Similarly from equation (v), we find that the tangential stress at the outer surface of the outer cylinder,

$$\sigma_{t4} = \frac{p\,(r_2)^2}{(r_3)^2 - (r_2)^2}\left[1 + \frac{(r_3)^2}{(r_3)^2}\right] = \frac{2\,p\,(r_2)^2}{(r_3)^2 - (r_2)^2} \text{ (tensile)} \qquad ...(x)$$

...[Substituting $p_i = p$, $x = r_3$, $r_o = r_3$ and $r_i = r_2$]

Radial stress at the outer surface of the outer cylinder,

$$\sigma_{r4} = \frac{p\,(r_2)^2}{(r_3)^2 - (r_2)^2}\left[1 - \frac{(r_3)^2}{(r_3)^2}\right] = 0$$

The equations ***(vii) to (x) cannot be solved until the contact pressure (p) is known. In obtaining*** a shrink fit, the outside diameter of the inner cylinder is made larger than the inside diameter of the outer cylinder. This difference in diameters is called the **interference and is** the deformation which the two cylinders must experience. Since the diameters of the cylinders are usually known, therefore the deformation should be calculated to find the contact pressure.

Let δ_0 = Increase in inner radius of the outer cylinder,

δ_1 = Decrease in outer radius of the inner cylinder,

E_0 = Young's modulus for the material of the outer cylinder,

E_1 = Young's modulus for the material of the inner cylinder, and

μ = Poisson's ratio.

We know that the tangential strain in the outer cylinder at the inner radius (r_2),

$$\varepsilon_{to} = \frac{\text{Change in circumference}}{\text{Original circumference}} = \frac{2\pi(r_2 + \delta_0) - 2\pi r_2}{2\pi r_2} = \frac{\delta_0}{r_2} \qquad ...(xi)$$

Also the tangential strain in the outer cylinder at the inner radius (r_2),

$$\varepsilon_{to} = \frac{\sigma_{to}}{E_0} - \frac{\mu.\sigma_{ro}}{E_0} \qquad ...(xii)$$

We have discussed above that the tangential stress at the inner surface of the outer cylinder (or at the contact surfaces),

$$\sigma_{to} = \sigma_{t3} = \frac{p\,[(r_3)^2 + (r_2)^2]}{(r_3)^2 - (r_2)^2} \qquad \text{...[From equation } (ix)]$$

and radial stress at the inner surface of the outer cylinder (or at the contact surfaces),

$$\sigma_{ro} = \sigma_{r3} = -p$$

Substituting the value of σ_{to} and σ_{ro} in equation (*xii*), we get

$$\varepsilon_{to} = \frac{p\,[(r_3)^2 + (r_2)^2]}{E_o\,[(r_3)^2 - (r_2)^2]} + \frac{\mu.p}{E_o} = \frac{p}{E_o}\left[\frac{(r_3)^2 + (r_2)^2}{(r_3)^2 - (r_2)^2} + \mu\right] \qquad \text{...}(xiii)$$

From equations (*xi*) and (*xiii*),

$$\delta_o = \frac{p.r_2}{E_o}\left[\frac{(r_3)^2 + (r_2)^2}{(r_3)^2 - (r_2)^2} + \mu\right] \qquad \text{...}(xiv)$$

Similarly, we may find that the decrease in the outer radius of the inner cylinder,

$$\delta_i = \frac{-p.r_2}{E_i}\left[\frac{(r_2)^2+(r_1)^2}{(r_2)^2-(r_1)^2}-\mu\right] \qquad \text{...}(xv)$$

$\therefore$ Difference in radius,

$$\delta_r = \delta_o - \delta_i = \frac{p.r_2}{E_o}\left[\frac{(r_3)^2+(r_2)^2}{(r_3)^2-(r_2)^2}+\mu\right] + \frac{p.r_2}{E_i}\left[\frac{(r_2)^2+(r_1)^2}{(r_2)^2-(r_1)^2}-\mu\right]$$

If both the cylinders are of the same material, then $E_o = E_i = E$. Thus the above expression may be written as

$$\delta_r = \frac{p.r_2}{E}\left[\frac{(r_3)^2+(r_2)^2}{(r_3)^2-(r_2)^2}+\frac{(r_2)^2+(r_1)^2}{(r_2)^2-(r_1)^2}\right]$$

$$= \frac{p.r_2}{E}\left[\frac{[(r_3)^2+(r_2)^2]\,[(r_2)^2-(r_1)^2]+[(r_2)^2+(r_1)^2]\,[(r_3)^2-(r_2)^2]}{[(r_3)^2-(r_2)^2]\,[(r_2)^2-(r_1)^2]}\right]$$

$$= \frac{p.r_2}{E}\left[\frac{2(r_2)^2[(r_3)^2-(r_1)^2]}{[(r_3)^2-(r_2)^2][(r_2)^2-(r_1)^2]}\right]$$

or $$p = \frac{E.\delta_r}{r_2}\left[\frac{[(r_3)^2-(r_2)^2][(r_2)^2-(r_1)^2]}{2(r_2)^2[(r_3)^2-(r_1)^2]}\right]$$

Substituting this value of p in equations (*vii*) to (*x*), we may obtain the tangential stresses at the various surfaces of the compound cylinder.

Now let us consider the compound cylinder subjected to an internal fluid pressure (p_i). We have discussed above that when the compound cylinder is subjected to internal pressure (p_i), then the tangential stress at any radius (x) is given by

$$\sigma_t = \frac{p_i (r_i)^2}{(r_o)^2-(r_i)^2}\left[1+\frac{(r_o)^2}{x^2}\right]$$

$\therefore$ Tangential stress at the inner surface of the inner cylinder,

$$\sigma_{t5} = \frac{p_i (r_1)^2}{(r_3)^2-(r_1)^2}\left[1+\frac{(r_3)^2}{(r_1)^2}\right] = \frac{p_i[(r_3)^2+(r_1)^2]}{(r_3)^2-(r_1)^2} \text{ (tensile)}$$

... [Substituting $x = r_1$, $r_o = r_3$ and $r_i = r_1$]

This stress is tensile and is shown by *ab′* in Fig. 7.9 (*d*).

Tangential stress at the outer surface of the inner cylinder or inner surface of the outer cylinder,

$$\sigma_{t6} = \frac{p_i (r_1)^2}{(r_3)^2 - (r_1)^2}\left[1 + \frac{(r_3)^2}{(r_2)^2}\right] = \frac{p_i (r_1)^2}{(r_2)^2}\left[\frac{(r_3)^2 + (r_2)^2}{(r_3)^2 - (r_1)^2}\right] \text{ (tensile)}$$

... [Substituting $x = r_2$, $r_o = r_3$ and $r_i = r_1$]

This stress is tensile and is shown by *ce′* in Fig. 7.9 (*d*),

and tangential stress at the outer surface of the outer cylinder,

$$\sigma_{t7} = \frac{p_i (r_1)^2}{(r_3)^2 - (r_1)^2}\left[1 + \frac{(r_3)^2}{(r_3)^2}\right] = \frac{2\, p_i (r_1)^2}{(r_3)^2 - (r_1)^2} \text{ (tensile)}$$

...[Substituting $x = r_3$, $r_o = r_3$ and $r_i = r_1$]

This stress is tensile and is shown by *fg′* in Fig. 7.9 (*d*).

Now the resultant stress at the inner surface of the compound cylinder,

$$\sigma_{ti} = \sigma_{t1} + \sigma_{t5} \quad \text{or} \quad ab' - ab$$

This stress is tensile and is shown by *ab''* in Fig. 7.9 (*e*).

Resultant stress at the outer surface of the inner cylinder

$$= \sigma_{t2} + \sigma_{t6} \quad \text{or} \quad ce' - cd \text{ or } cc'$$

Resultant stress at the inner surface of the outer cylinder

$$= \sigma_{t3} + \sigma_{t6} \quad \text{or} \quad ce + ce' \text{ or } c'e''$$

∴ Total resultant stress at the mating or contact surface,

$$\sigma_{tm} = \sigma_{t2} + \sigma_{t6} + \sigma_{t3} + \sigma_{t6}$$

This stress is tensile and is shown by *ce''* in Fig. 7.9 (*e*),

and resultant stress at the outer surface of the outer cylinder,

$$\sigma_{to} = \sigma_{t4} + \sigma_{t7} \quad \text{or} \quad fg + fg'$$

This stress is tensile and is shown by *fg''* in Fig. 7.9 (*e*).

The hydraulic press, having a working pressure of water as 16 N/mm2 and exerting a force of 80 kN is required to press materials upto a maximum size of 800 mm × 800 mm and 800 mm high, the stroke length is 80 mm. Design and draw the following parts of the press : 1. Design of ram; 2. Cylinder; 3. Pillars; and 4. Gland

1. *Design of ram*

Let d_r = Diameter of ram.

We know that the maximum force to be exerted by the ram (F),

$$80 \times 10^3 = \frac{\pi}{4}(d_r)^2 p = \frac{\pi}{4}(d_r)^2\, 16 = 12.57\,(d_r)^2$$

$\therefore$ $(d_r)^2 = 80 \times 10^3/12.57 = 6364$ or $d_r = 79.8$ say 80 mm **Ans.**

$$\sigma_{t(max)} = \frac{-p_o (d_{ro})^2}{(d_{ro})^2 - (d_{ri})^2}\left[1 + \frac{(d_{ri})^2}{(d_{ro})^2}\right] = -p_o\left[\frac{(d_{ro})^2 + (d_{ri})^2}{(d_{ro})^2 - (d_{ri})^2}\right] \text{ (compressive)}$$

and maximum radial stress,

$$\sigma_{r(max)} = -p_o \text{ (compressive)}$$

where d_{ro} = Outer diameter of ram = d_r = 80 mm

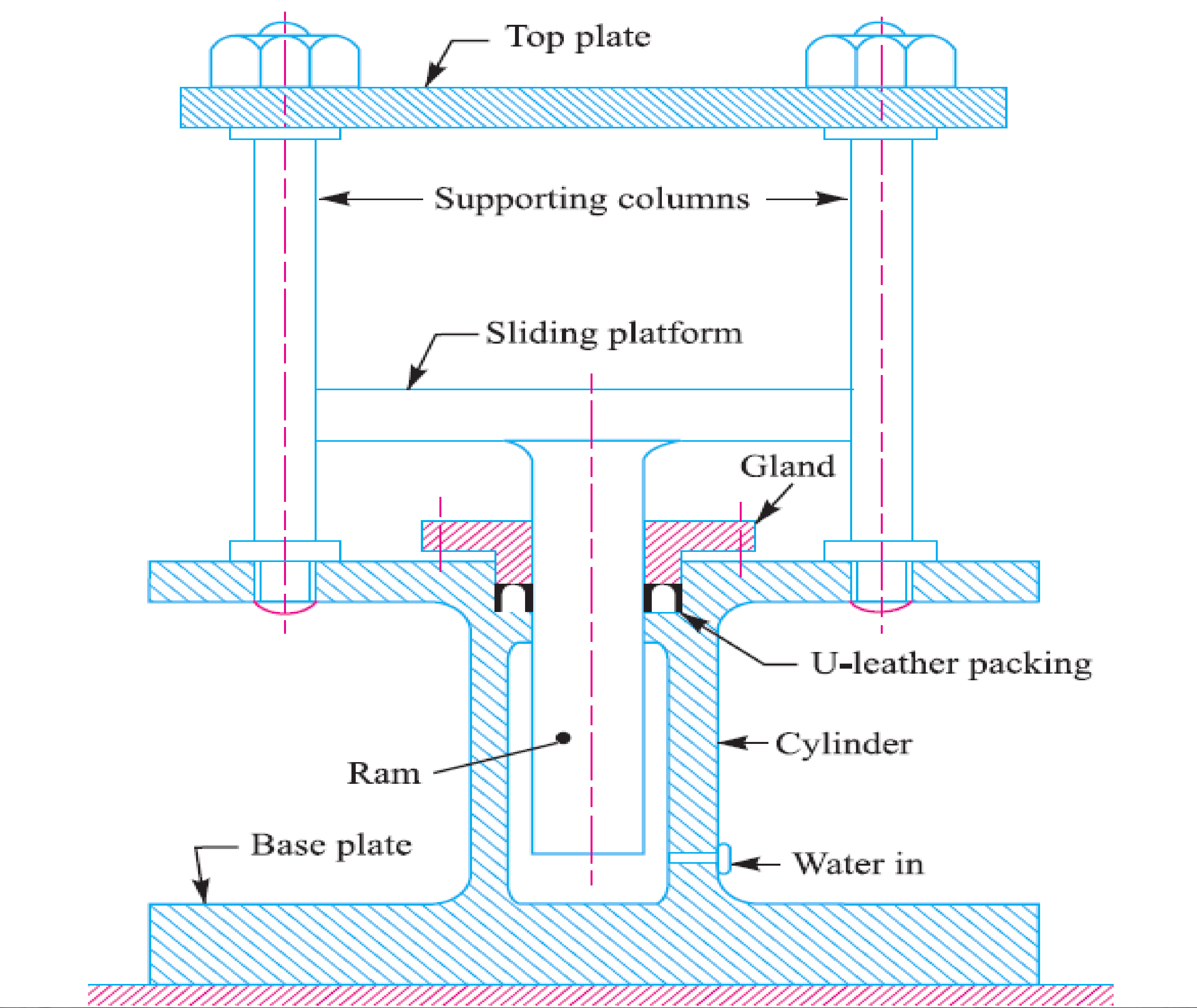
d_{ri} = Inner diameter of ram, and

p_o = External pressure = p = 16 N/mm^2 ...(Given)

Now according to maximum shear stress theory for ductile materials, maximum shear stress is

$$\tau_{max} = \frac{\sigma_{t(max)} - \sigma_{r(max)}}{2} = \frac{-p_o\left[\frac{(d_{ro})^2 + (d_{ri})^2}{(d_{ro})^2 - (d_{ri})^2}\right] - (-p_o)}{2}$$

$$= -p_o\left[\frac{(d_{ri})^2}{(d_{ro})^2 - (d_{ri})^2}\right]$$


Top plate
Supporting columns
Sliding platform
Gland
U-leather packing
Cylinder
Ram
Base plate
Water in

Since the maximum shear stress is one-half the maximum principal stress (which is compressive), therefore

$$\sigma_c = 2\,\tau_{max} = 2\,p_o \left[\frac{(d_{ri})^2}{(d_{ro})^2 - (d_{ri})^2} \right]$$

The ram is usually made of mild steel for which the compressive stress may be taken as 75 N/mm^2. Substituting this value of stress in the above expression, we get

$$75 = 2 \times 16 \left[\frac{(d_{ri})^2}{(80)^2 - (d_{ri})^2} \right] = \frac{32\,(d_{ri})^2}{6400 - (d_{ri})^2}$$

or $$\frac{(d_{ri})^2}{6400 - (d_{ri})^2} = \frac{75}{32} = 2.34$$

$$(d_{ri})^2 = 2.34\,[6400 - (d_{ri})^2] = 14\,976 - 2.34\,(d_{ri})^2$$

$$3.34\,(d_{ri})^2 = 14\,976 \qquad \text{or} \qquad (d_{ri})^2 = 14\,976/3.34 = 4484$$

$\therefore$ $$d_{ri} = 67 \text{ mm } \textbf{Ans.}$$

and $$d_{ro} = d_r = 80 \text{ mm } \textbf{Ans.}$$

2. *Design of cylinder*

Let d_{ci} = Inner diameter of cylinder, and

d_{co} = Outer diameter of cylinder.

Assuming a clearance of 15 mm between the ram and the cylinder bore, therefore inner diameter of the cylinder,

$$d_{ci} = d_{ro} + \text{Clearance} = 80 + 15 = 95 \text{ mm}$$ **Ans.**

The cylinder is usually made of cast iron for which the tensile stress may be taken as 30 N/mm^2. According to Lame's equation, we know that wall thickness of a cylinder,

$$t = \frac{d_{ci}}{2}\left[\sqrt{\frac{\sigma_t + p}{\sigma_t - p}} - 1\right] = \frac{95}{2}\left[\sqrt{\frac{30 + 16}{30 - 16}} - 1\right] \text{mm}$$

$$= 47.5\,(1.81 - 1) = 38.5 \text{ say } 40 \text{ mm}$$

and outside diameter of the cylinder,

$$d_{co} = d_{cl} + 2\,t = 95 + 2 \times 40 = 175 \text{ mm } \textbf{Ans.}$$

3. *Design of pillars*

Let $\qquad d_p$ = Diameter of the pillar.

The function of the pillars is to support the top plate and to guide the sliding plate. When the material is being pressed, the pillars will be under direct tension. Let there are four pillars and the load is equally shared by these pillars.

$\therefore$ Load on each pillar

$$= 80 \times 10^3/4 = 20 \times 10^3 \text{ N} \qquad \ldots(i)$$

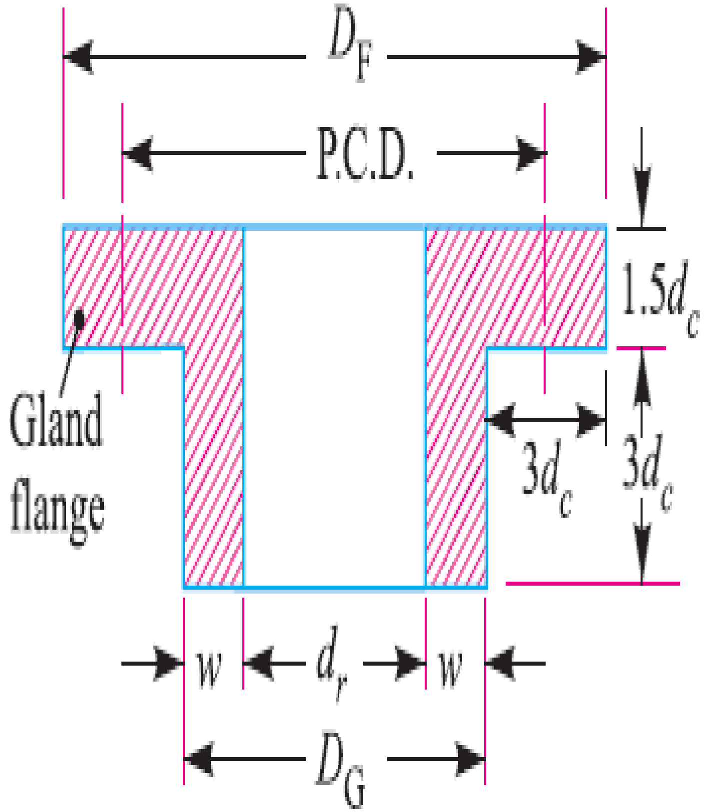
We know that load on each pillar

$$= \frac{\pi}{4}\,(d_p)^2\,\sigma_t = \frac{\pi}{4}\,(d_p)^2\,75 = 58.9\,(d_p)^2 \qquad \ldots(ii)$$

From equations (*i*) and (*ii*),

$$(d_p)^2 = 20 \times 10^3/58.9 = 340 \quad \text{or} \quad d_p = 18.4 \text{ mm}$$

From fine series of metric threads, let us adopt the threads on pillars as M 20×1.5 having major diameter as 20 mm and core diameter as 18.16 mm. **Ans.**



4. Design of gland

The gland is shown in Fig 7.11. The width (w) of the U-leather packing for a ram is given empirically as $2\sqrt{d_r}$ to $2.5\sqrt{d_r}$, where d_r is the diameter (outer) of the ram in mm. Let us take width of the packing as $2.2\sqrt{d_r}$.

$\therefore$ Width of packing,

$$w = 2.2\sqrt{80} = 19.7 \text{ say } 20 \text{ mm} \quad \textbf{Ans.}$$

and outer diameter of gland,

$$D_G = d_r + 2\,w = 80 + 2 \times 20 = 120 \text{ mm} \quad \textbf{Ans.}$$

We know that total upward load on the gland

$$= \text{Area of gland exposed to fluid pressure} \times \text{Fluid pressure}$$

$$= \pi\,(d_r + w)\,w.p = \pi\,(80 + 20)\,20 \times 16 = 100\,544 \text{ N}$$

Let us assume that 8 studs equally spaced on the pitch circle of the gland flange are used for holding down the gland.

$\therefore$ Load on each stud = 100 544 / 8 = 12 568 N

If d_c is the core diameter of the stud and σ_t is the permissible tensile stress for the stud material, then

Load on each stud,

$$12\,568 = \frac{\pi}{4}(d_c)^2\sigma_t = \frac{\pi}{4}(d_c)^2\,75 = 58.9\,(d_c)^2 \qquad \text{... (Taking } \sigma_t = 75 \text{ N/mm}^2\text{)}$$

$$\therefore \qquad (d_c)^2 = 12\,568 / 58.9 = 213.4 \quad \text{or} \quad d_c = 14.6 \text{ mm}$$

From fine series of metric threads, let us adopt the studs of size M 18 × 1.5 having major diameter as 18 mm and core diameter (d_c) as 16.16 mm. **Ans.**

Pitch circle diameter of the gland flange,

$$\text{P.C.D.} = D_G + 3\,d_c = 120 + 3 \times 16.16 = 168.48 \quad \text{or} \quad 168.5 \text{ mm } \textbf{Ans.}$$

Outer diameter of the gland flange,

$$D_F = D_G + 6\,d_c = 120 + 6 \times 16.16 = 216.96 \quad \text{or} \quad 217 \text{ mm } \textbf{Ans.}$$

and thickness of the gland flange $= 1.5\,d_c = 1.5 \times 16.16 = 24.24$ or 24.5 mm **Ans.**

$$p = \frac{E.\delta_r}{r_2}\left[\frac{[(r_3)^2 - (r_2)^2]\,[(r_2)^2 - (r_1)^2]}{2\,(r_2)^2\,[(r_3)^2 - (r_1)^2]}\right]$$

$$\sigma_{to} = \frac{-\,p\,[(r_3)^2 + (r_2)^2]}{(r_3)^2 - (r_2)^2} =$$

$$\sigma_{ti} = \frac{-\,p\,[(r_2)^2 + (r_1)^2]}{(r_2)^2 - (r_1)^2}$$

Cylinder Heads and Cover Plates

The heads of cylindrical pressure vessels and the sides of rectangular or square tanks may have flat plates or slightly dished plates. The plates may either be cast integrally with the cylinder walls or fixed by means of bolts, rivets or welds. The design of flat plates forming the heads depend upon the following two factors.

(*a*) *Type of connection between the head* and the cylindrical wall, (*i.e. freely* supported or rigidly fixed);***(b) Nature of loading (i.e. uniformly distributed*** or concentrated).

Since the stress distribution in the cylinder heads and cover plates are of complex nature, therefore empirical relations based on the work of Grashof and Bach are used in the design of flat plates. Let us consider the following cases:

1. *Circular flat plate with uniformly distributed load.* The thickness (t_1) of a plate with a diameter (d) supported at the circumference and subjected to a pressure (p) uniformly distributed over the area is given by

$$t_1 = k_1 . d \sqrt{\frac{p}{\sigma_t}}$$

where σ_t = Allowable design stress.

The coefficient k_1 depends upon the material of the plate and the method of holding the edges. The values of k_1 for the cast iron and mild steel are given in Table 7.2.

2. ***Circular flat plate loaded centrally.*** The thickness (t_1) of a flat cast iron plate supported freely at the circumference with a diameter (d) and subjected to a load (F) distributed uniformly over an area $\frac{\pi}{4}(d_0)^2$, is given by

$$t_1 = 3\sqrt{\left(1 - \frac{0.67\, d_0}{d}\right)\frac{F}{\sigma_t}}$$

If the plate with the above given type of loading is fixed rigidly around the circumference, then

$$t_1 = 1.65\sqrt{\frac{F}{\sigma_t}\log_e\left(\frac{d}{d_0}\right)}$$

3. ***Rectangular flat plate with uniformly distributed load.*** The thickness (t_1) of a rectangular plate subjected to a pressure (p) uniformly distributed over the total area is given by

$$t_1 = a.b.k_2 \sqrt{\frac{p}{\sigma_t (a^2 + b^2)}}$$

where a = Length of the plate; and

b = Width of the plate.

The values of the coefficient k_2 are given in Table 7.2.

4. ***Rectangular flat plate with concentrated load.*** The thickness (t_1) of a rectangular plate subjected to a load (F) at the intersection of the diagonals is given by

$$t_1 = k_3 \sqrt{\frac{a.b.F}{\sigma_t (a^2 + b^2)}}$$

The values of coefficient k_3 are given in Table 7.2.

5. ***Elliptical plate with uniformly distributed load.*** The thickness (t_1) of an elliptical plate subjected to a pressure (p) uniformly distributed over the total area, is given by

$$t_1 = a.b.k_4 \sqrt{\frac{p}{\sigma_t (a^2 + b^2)}}$$

where a and b = Major and minor axes respectively.

The values of coefficient k_4 are given in Table 7.2.

Table 7.2. Values of coefficients k_1, k_2, k_3 and k_4.

Material of the cover plate	*Type of connection*	*Circular plate*	*Rectangular plate*		*Elliptical plate*
		k_1	k_2	k_3	k_4
Cast iron	Freely supported	0.54	0.75	4.3	1.5
	Fixed	0.44	0.62	4.0	1.2
Mild Steel	Freely supported	0.42	0.60	3.45	1.2
	Fixed	0.35	0.49	3.0	0.9

6. *Dished head with uniformly distributed load.* Let us consider the following cases of dished head:

(*a*) ***Riveted or welded dished head.*** When the cylinder head has a dished plate, then the thickness of such a plate that is riveted or welded as shown in Fig. 7.12 (*a*), is given by

$$t_1 = \frac{4.16\ p.R}{\sigma_u}$$

where p = Pressure inside the cylinder,

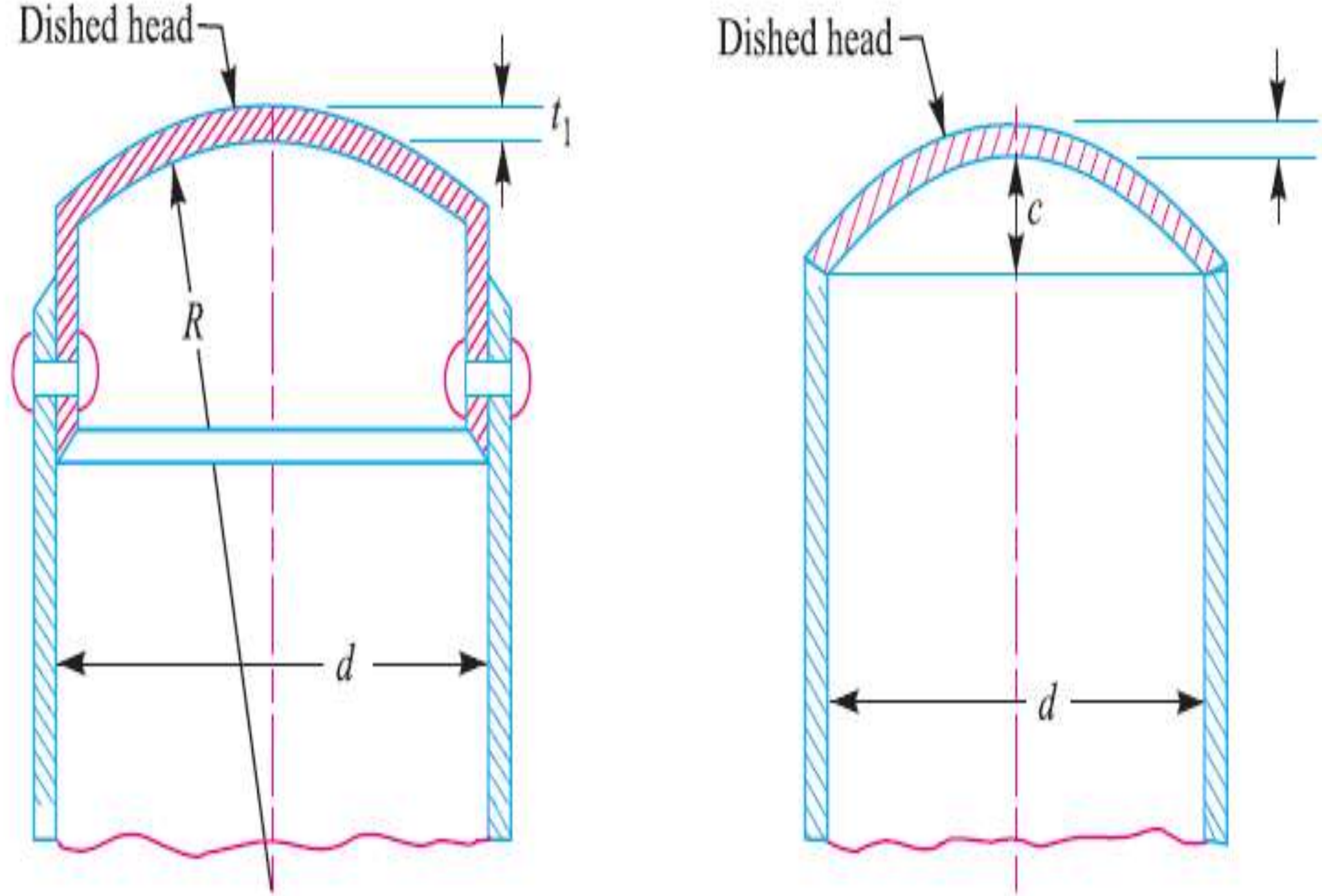
R = Inside radius of curvature of the plate, and

σ_u = Ultimate strength for the material of the plate.

When there is an opening or manhole in the head, then the thickness of the dished plate is given by

$$t_1 = \frac{4.8\ p.R}{\sigma_u}$$

It may be noted that the inside radius of curvature of the dished plate (R) should not be greater than the inside diameter of the cylinder (d).



(*a*) Riveted or welded dished head.

(*b*) Integral or welded dished head.

(*b*) ***Integral or welded dished head.*** When the dished plate is fixed integrally or welded to the cylinder as shown in Fig. 7.12 (*b*), then the thickness of the dished plate is given by

$$t_1 = \frac{p\,(d^2 + 4c^2)}{16\,\sigma_t \times c}$$

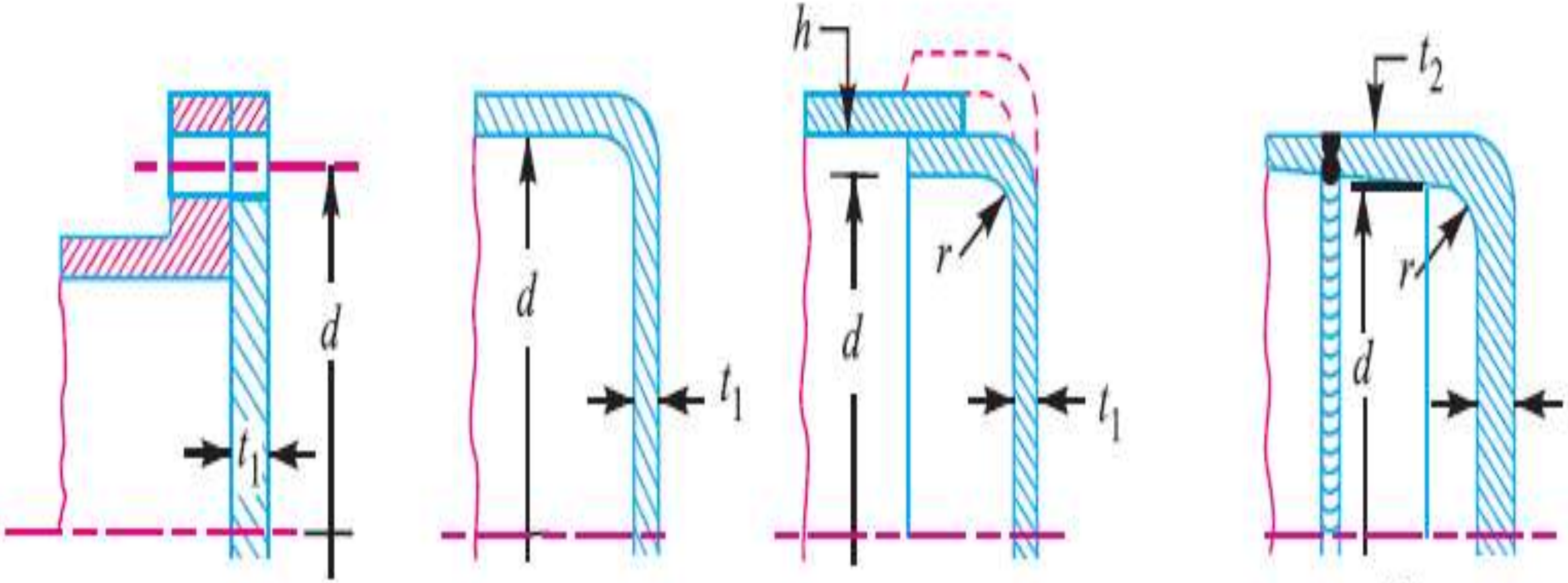
where c = Camber or radius of the dished plate.

Mostly the cylindrical shells are provided with hemispherical heads. Thus for hemispherical heads, $c = \dfrac{d}{2}$. Substituting the value of c in the above expression, we find that the thickness of the hemispherical head (fixed integrally or welded),

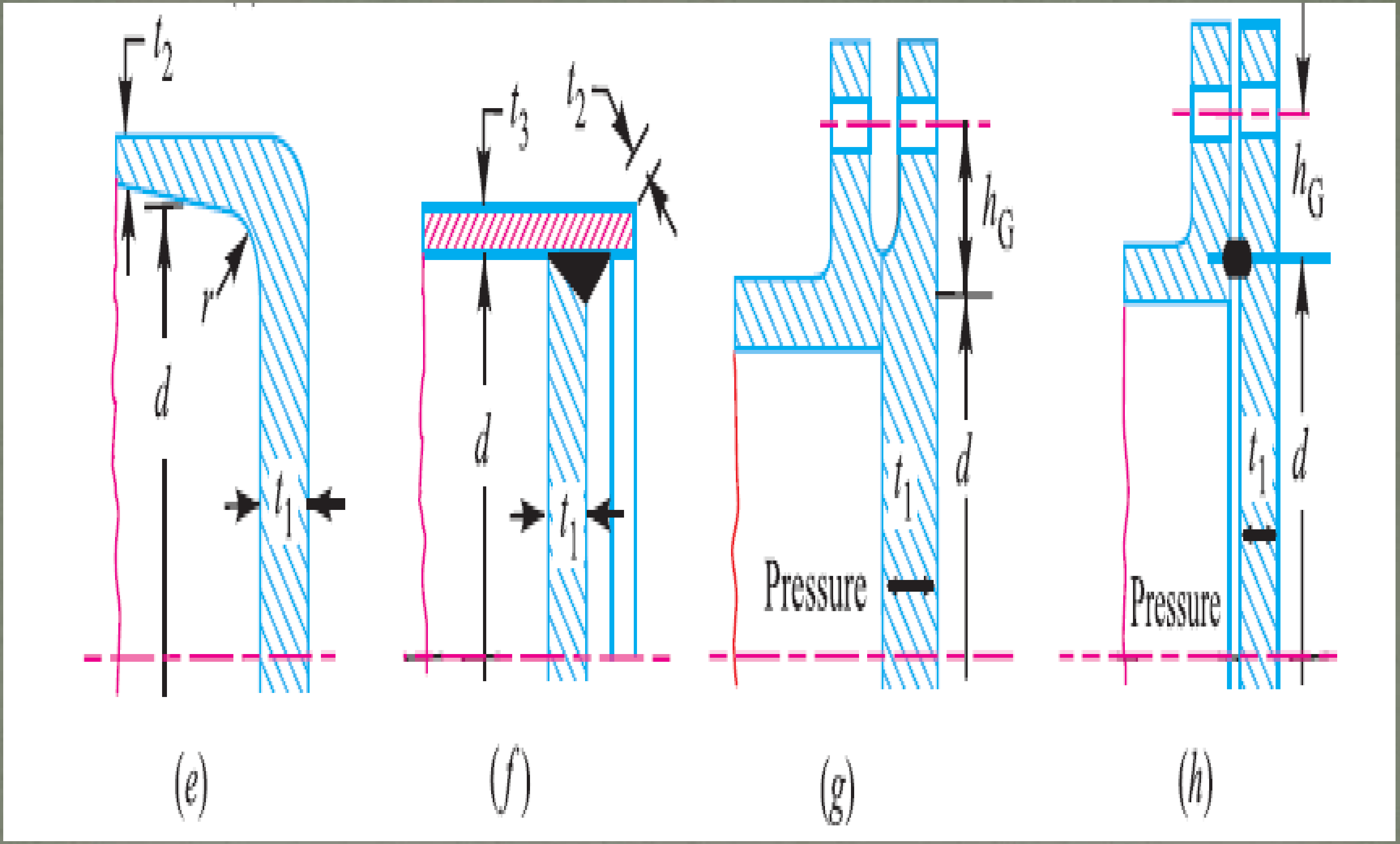
$$t_1 = \frac{p\left(d^2 + 4 \times \dfrac{d^2}{4}\right)}{16\,\sigma_t \times \dfrac{d}{2}} = \frac{p.d}{4\,\sigma_t} \qquad \text{...(Same as for thin spherical shells)}$$

7. *Unstayed flat plate with uniformly distributed load.* The minimum thickness (t_1) of an unstayed steel flat head or cover plate is given by

$$t_1 = d\sqrt{\frac{k.p}{\sigma_t}}$$



(a) (b) (c) (d)



t2
t3
t2
r
d
t1
hG
Pressure
(e)
(f)
(g)
(h)

S.No.	*Particulars of plate connection*	*Value of 'k'*
1.	Plate riveted or bolted rigidly to the shell flange, as shown in Fig. 7.13 (*a*).	0.162
2.	Integral flat head as shown in Fig. 7.13 (*b*), $d \leq 600$ mm, $t_1 \geq 0.05\ d$.	0.162
3.	Flanged plate attached to the shell by a lap joint as shown in Fig. 7.13 (*c*), $r \geq 3t_1$.	0.30
4.	Plate butt welded as shown in Fig. 7.13 (*d*), $r \geq 3\ t_2$	0.25
5.	Integral forged plate as shown in Fig. 7.13 (*e*), $r \geq 3\ t_2$	0.25
6.	Plate fusion welded with fillet weld as shown in Fig. 7.13 (*f*) , $t_2 \geq 1.25\ t_3$.	0.50
7.	Bolts tend to dish the plate as shown in Fig. 7.13 (*g*) and (*h*).	$0.3 + \dfrac{1.04\ W.h_G}{H.d}$, W = Total bolt load, and H = Total load on area bounded by the outside diameter of the gasket.