

Unit - II

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Zeeeman Effect :

- Importance
- development of quantum theory
- illustrates phenomenon of space quantization, which refers to the angular momentum L of the atom, assuming only a set of discrete orientations w.r.t. external magnetic field B .

Spin angular momentum S

$$\mu = -\frac{\mu_0}{h} (L + 2S) \quad \text{--- (1)}$$

whereas the total angular momentum J is simply the vector sum

$$J = L + S.$$

Factor of 2 appearing in the expression for magnetic dipole moment means that the magnetic dipole moment vector is not, in general, collinear with the total angular momentum¹.

If, however, the total electron spin couples to zero i.e. $S=0$, then the expression reduces to that in eqⁿ (1)

Historically the case with zero spin was the first one discovered by Zeeman. It has to come to be referred to as "normal" Zeeman splitting, as opposed to "anomalous" Zeeman splitting, which is the case of non-zero spin.

— Assume we are dealing with regular Zeeman splitting.

$$(A) \text{ so } J = L$$

assume magnetic field is weak because if field is strong then it destroys the coupling between L & S

— fields are considered as "weak" if they are about 0.8 Tesla and lower, & "strong" if > 1.5 .

Torque on the atom due to the external field is

$$\tau = \mu \times B$$

$$= -\frac{\mu_0}{h} L \times B$$

Perpendicular to both L & B

Causes the tip of the orbital angular momentum vector L to precess in a circular orbit about B

Interaction energy betⁿ. atom's magnetic dipole moment and the field is,

$$\Delta E = -\mu \cdot B = + \frac{\mu_0}{h} L \cdot B \quad \text{--- (2)}$$

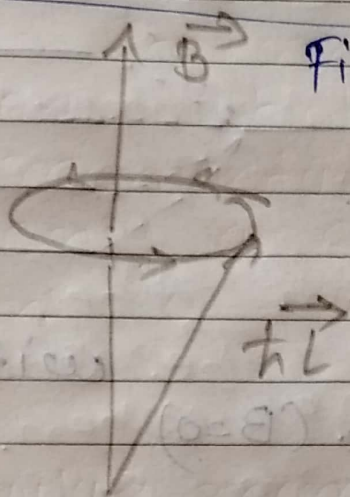


Fig: Precession of the orbital angular momentum L about the direction of the magnetic field B .

Let $B = B_z$

$$\Delta E = \frac{\mu_0}{h} B L_z$$

$L_z =$ Projection of the L on z axis

according to quantum mechanics

$$L_z = m_l h$$

where $m_l = -l, -l+1, \dots, +l-1, +l$

Hence, we obtain for the energy levels

$$\Delta E = \mu_0 B m_l \quad \text{--- (3)}$$

Therefore, when an atom is placed in a magnetic field, the energy level with principal quantum number n and orbital angular momentum indexed by azimuthal quantum

number of will split into $2l+1$
Sub levels

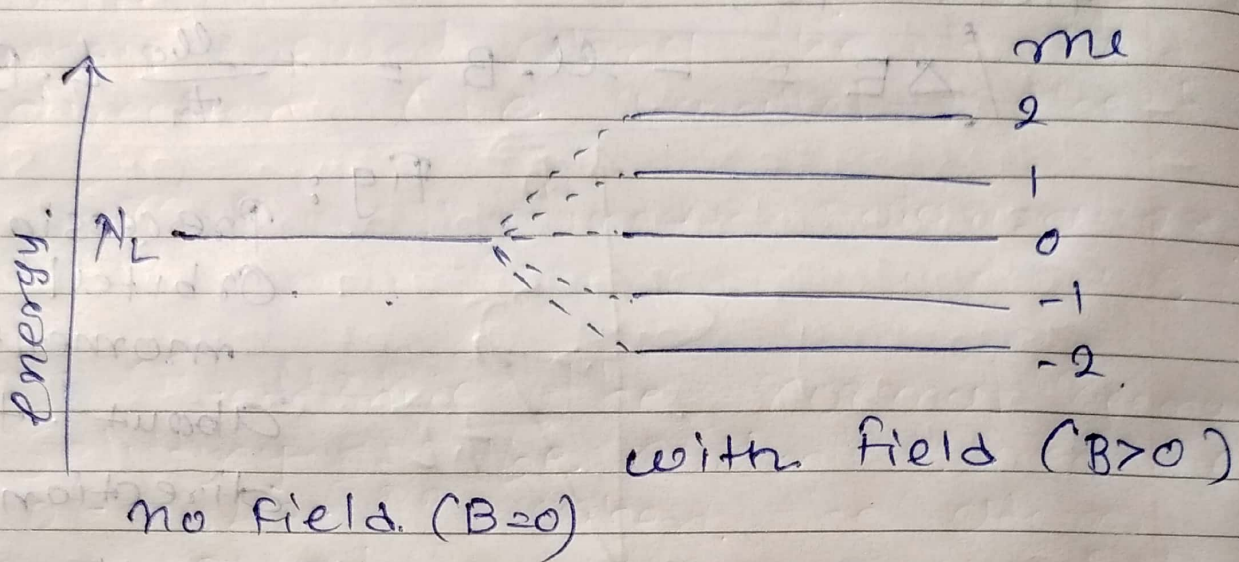


Fig 2: energy level splitting corresponding to different projections of orbital angular momentum onto the axis of the magnetic field.

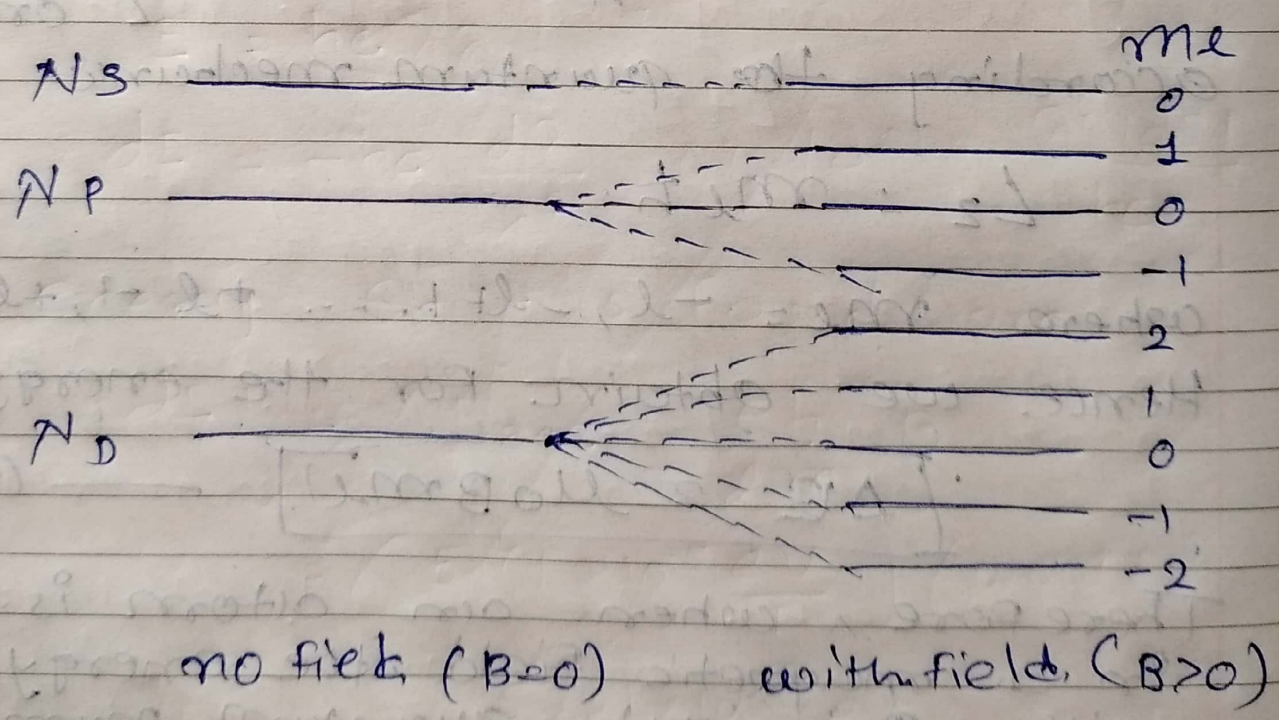
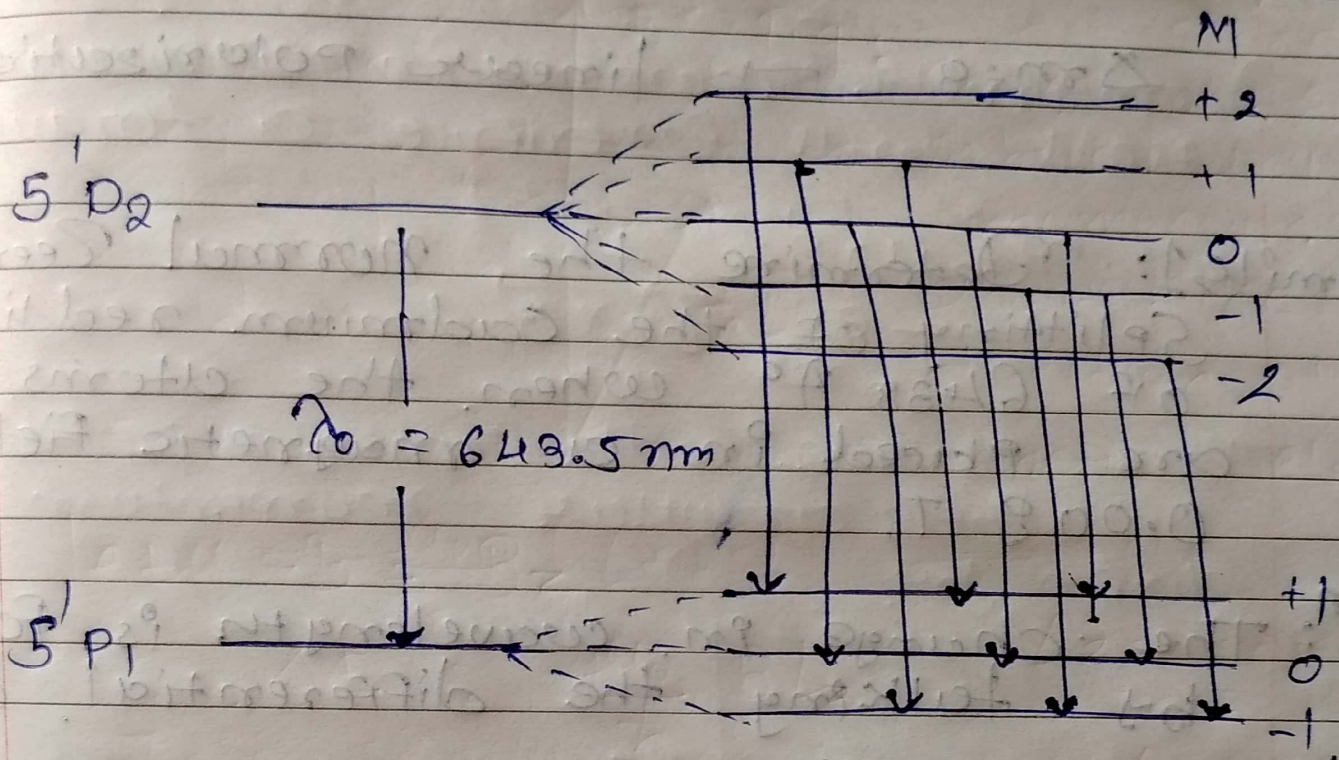


Fig 3: energy level splitting for different orbitals. (i.e. diffⁿ angular momentum)

By convention, orbital angular momentum states of the atom are given letter labels. S ($l=0$), P ($l=1$), D ($l=2$), F ($l=3$), G ($l=4$), H ect.



No magnetic field.

In Magnetic field.

Fig 4: allowed energy level transitions of Cadmium (Cd)

we have three allowed electric dipole transitions with magnetic field on. they have frequencies

$$\nu = \nu_0$$

$$\nu = \nu_0 + \mu_0 B$$

$$\nu = \nu_0 - \mu_0 B$$

magnitudes)

last two, with $\Delta m = +1$ and -1

Correspond to circularly polarized light when viewed along the B. field direction i.e. parallel to the field.

$\Delta m = 0$ \rightarrow linear polarization

Example 1: Determine the normal Zeeman splitting of the cadmium red line of 6488 \AA when the atoms are placed in a magnetic field of 0.009 T .

Solⁿ: The change in wavelength is found by taking the differential of

$$E = hc / \lambda$$

$$dE = -hc \frac{d\lambda}{\lambda^2} \quad \text{or} \quad |d\lambda| = \frac{\lambda^2 |dE|}{hc}$$

Energy shift is found from

$$|dE| = \Delta E_{zee} = \frac{eh}{2m} B$$

$$= \left(5.79 \times 10^{-5} \frac{\text{eV}}{\text{T}} \right) (0.009 \text{ T})$$

$$= 5.21 \times 10^{-7} \text{ eV}$$

$$\text{given } |d\lambda| = \frac{\lambda^2 |dE|}{hc}$$

$$= \frac{(6438 \text{ \AA})^2 (5.21 \times 10^{-7} \text{ eV})}{12.4 \times 10^3 \text{ eV \AA}}$$

$$\Delta \lambda = 1.74 \times 10^{-3} \text{ \AA}$$

Example 2: what magnetic flux density B is required to observe the normal Zeeman effect if a spectrometer can resolve spectral lines separated by 0.5 \AA at 500 \AA .

$$\frac{\Delta \lambda}{\lambda} = \frac{\Delta E}{hc/\lambda} = \frac{(e\hbar/2m) B}{hc/\lambda}$$

giving

$$B = \frac{\Delta \lambda}{\lambda} \left(\frac{hc}{\lambda} \right) \left(\frac{2m}{e\hbar} \right) = \left(\frac{0.5 \text{ \AA}}{5000 \text{ \AA}} \right)$$

$$\sim \left(\frac{12.4 \times 10^3 \text{ eV \AA}}{5000 \text{ \AA}} \right)$$

$$\boxed{B = 4.28 \text{ T}}$$

Example: 3

Calculate the freqⁿ. at which an electron's orbital magnetic moment μ precesses in a magnetic field B .

A magnetic moment μ in a magnetic field will experience a torque τ ,

$$\tau = \mu \times B$$

$$= -\frac{e}{2m} \vec{L} \times \vec{B}$$

This torque will cause a change in the angular momentum given by.

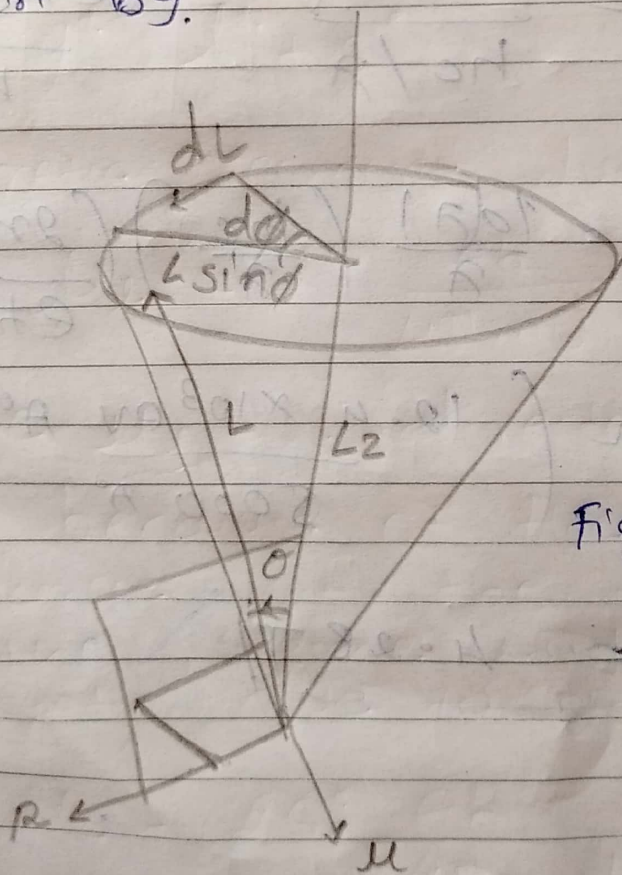


Fig: 1

$$\dot{\varphi} = \frac{d\vec{L}}{dt} = -\frac{e}{2m} \vec{L} \times \vec{B}$$

Change in L , dL , is perpendicular to both L and B . as shown in fig 1

resulting in a Precession of L about the direction of B . From fig. it is seen that

$$d\phi = \frac{|dL|}{L \sin\alpha}$$

which gives

$$\begin{aligned} \omega_p &= \frac{d\phi}{dt} = \frac{\left| \frac{dL}{dt} \right|}{L \sin\alpha} \\ &= \frac{e}{2m} \frac{LB \sin\alpha}{L} \end{aligned}$$

$$= \frac{e}{2m} B$$

This is known as Larmor Precession

ω_p is equal to the frequency difference observed in the Normal Zeeman effect.

Vector Model and Anomalous Zeeman Effect

- Anomalous Zeeman effect finds its explanation when we introduce spin of the electron in vector atom model with the introduction of electron spin, we say that l^* & s^* vectors.

$$l^* = \sqrt{l(l+1)}$$

$$s^* = \sqrt{s(s+1)}$$

Process around the result vector g^* .
(Total angular momentum vector)

$$g^* = l^* + s^*$$

magnetic moment due to orbital motion

$$\mu_l = l^* \frac{eh}{4\pi m_e}$$

where μ_l is directed opposite to l^* because of the negative charge of the electron.
In the same way magnetic moment due to spin is given by

$$\mu_s = 2s^* \frac{eh}{4\pi m_e}$$

which is directed opposite to s^* due to the reason stated above