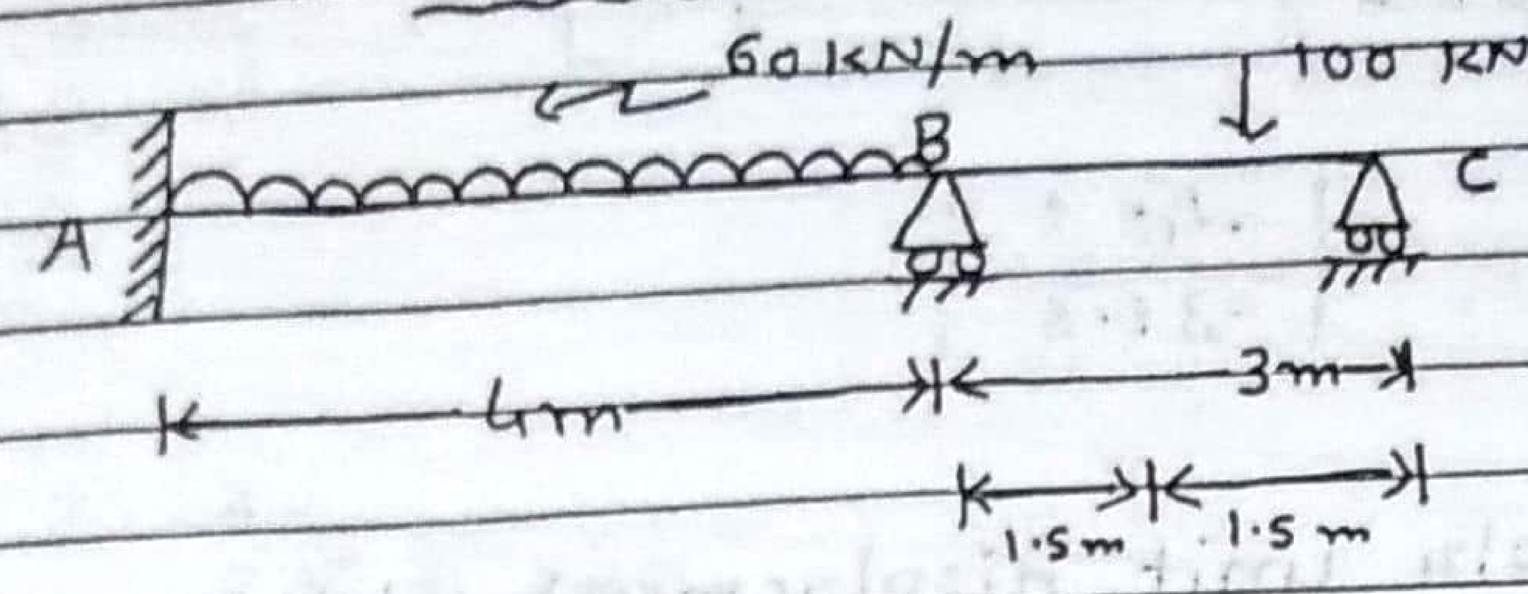
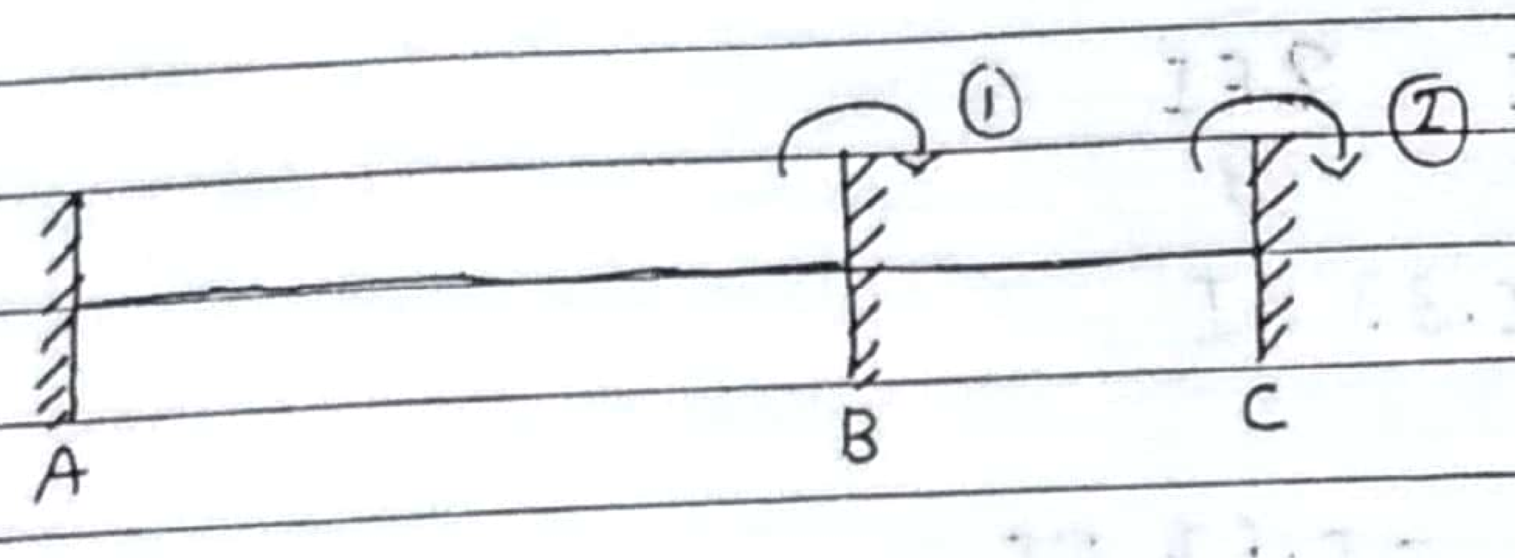
Advance structure Analysis

1.



-> Step:-1 Find KI

$KI = 2 \quad [\theta_B, \theta_C]$



-> Step:-2 Find Fixed end moment:-

$MF_{AB} = \frac{-wl^2}{12} = -80 \text{ kN.m}$

$MF_{BA} = \frac{wl^2}{12} = 80 \text{ kN.m}$

$MF_{BC} = \frac{-wl}{8} = -37.5 \text{ kN.m}$

$MF_{CB} = \frac{wl}{8} = 37.5 \text{ kN.m}$

$P_1L = MF_{AB} + MF_{BC}$

$= -80 + -37.5$

$= 42.5 \text{ kN.m.}$

$P_2L = MF_{BA} + MF_{CB}$

$= 37.5 \text{ kN.m.}$

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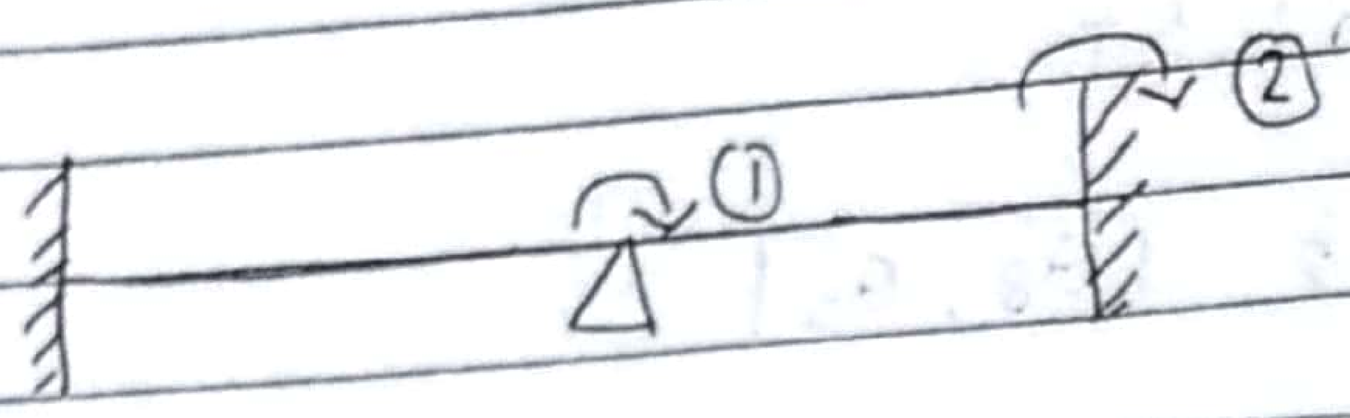
$\therefore [P_L] = \begin{bmatrix} 42.5 \\ 37.5 \end{bmatrix}$ $[P] = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$[P - P_L] = \begin{bmatrix} -42.5 \\ -37.5 \end{bmatrix}$

→ step: 3 Apply unit displacment at (1).



$K_{11} = \frac{4EI}{4} + \frac{2EI}{3}$

$= 2.33\ EI$

$K_{21} = \frac{2EI}{3} = 0.67\ EI$

$K_{12} = \frac{2EI}{3} = 0.67\ EI$

→ Apply unit displacment at (2.)

$K_{22} = \frac{2EI}{3} = 1.33\ EI.$

$K = EI \begin{bmatrix} 2.33 & 0.67 \\ 0.67 & 1.33 \end{bmatrix}$

$[K]^{-1} = \frac{1}{EI\,(2.65)} \begin{bmatrix} 1.33 & -0.67 \\ -0.67 & 2.33 \end{bmatrix}$

Using stiffness eq^n:-

$[\Delta] = [K]^{-1} \{[P - P_L]\}$

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Pg:-3

$$= \frac{1}{2.65\,EI}\begin{bmatrix}1.33 & -0.67\\ -0.67 & 2.33\end{bmatrix}\begin{bmatrix}-42.5\\ -37.5\end{bmatrix}$$

$$\theta_B = \frac{-11.85}{EI}.$$

$$\theta_C = \frac{-22.23}{EI}$$

→ Step:-4 Bending moment:-

$$M_{AB} = -80 + \frac{2EI}{4}\left[\frac{11.85}{EI}\right] \qquad \left[\therefore M_{FAB} + \frac{2EI}{L}\left[2\theta_A + \theta_B - \frac{3\delta}{L}\right]\right]$$

$$= -80 + (-0.5)(11.85)$$

$$= -85.92 \text{ kN.m.}$$

$$M_{BA} = 80 + \frac{2EI}{4}\left[\frac{2\times 11.85}{EI}\right]$$

$$= 80 + (0.5)(23.7)$$

$$= 91.85 \text{ kN.m.}$$

$$M_{BC} = -37.5 + \frac{2EI}{3}\left[\frac{(2\times(-11.85)(-22.23)}{EI}\right]$$

$$= -37.5 + 0.66\left[26.5\right]$$

$$= -68.12 \text{ kN.m.}$$

$$M_{CB} : 37.5 + \frac{2EI}{3}\left[\frac{(2\times -22.23) - 11.85}{EI}\right]$$

$$= 37.5 + 0.66(-56.31)$$

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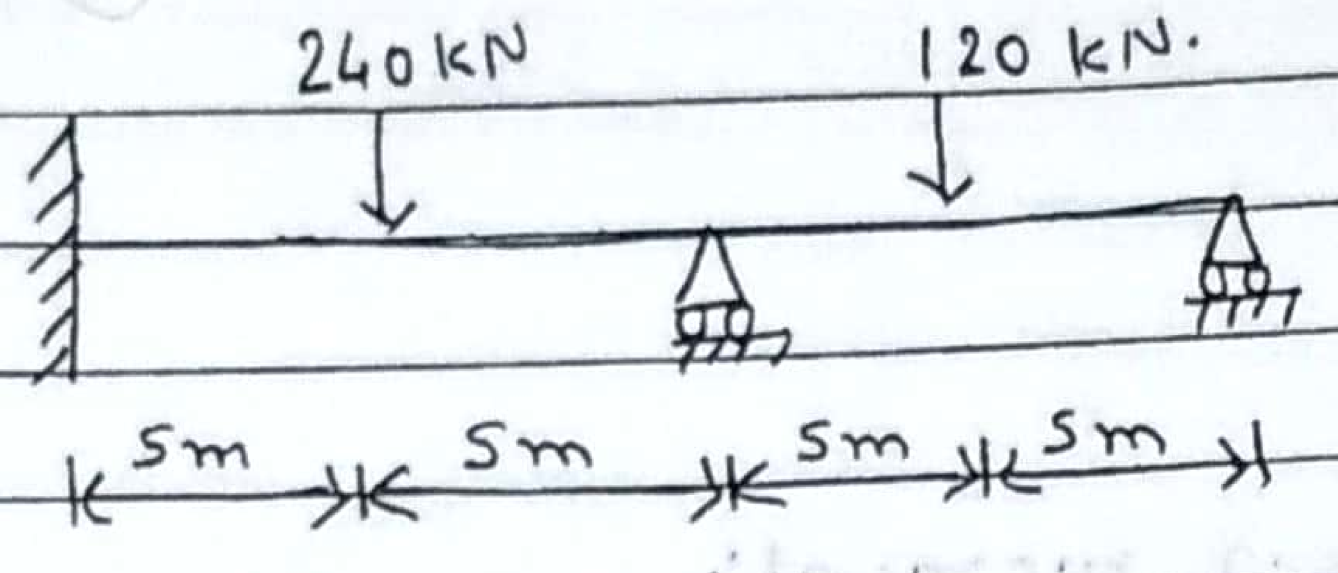
Beams

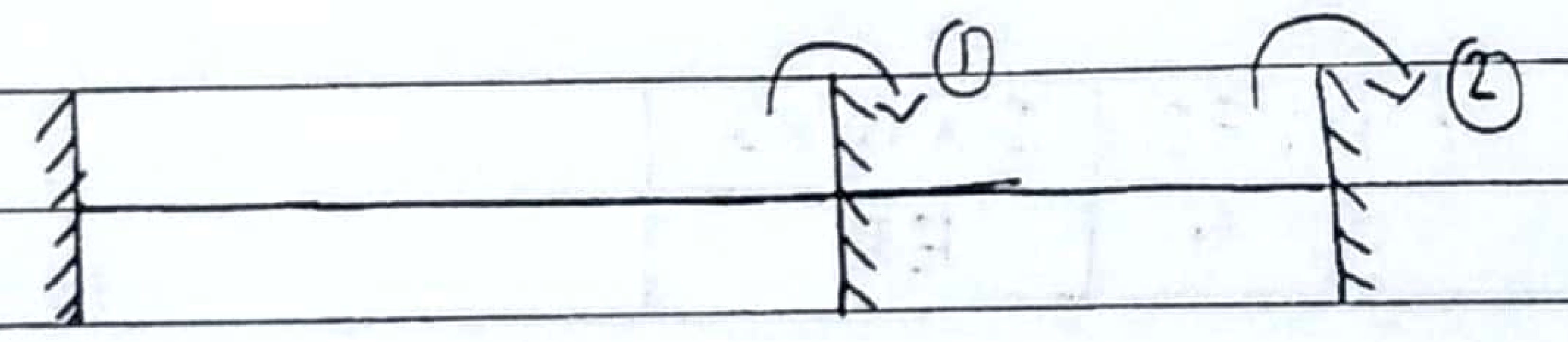
2. Analysis the beam by stiffness method.



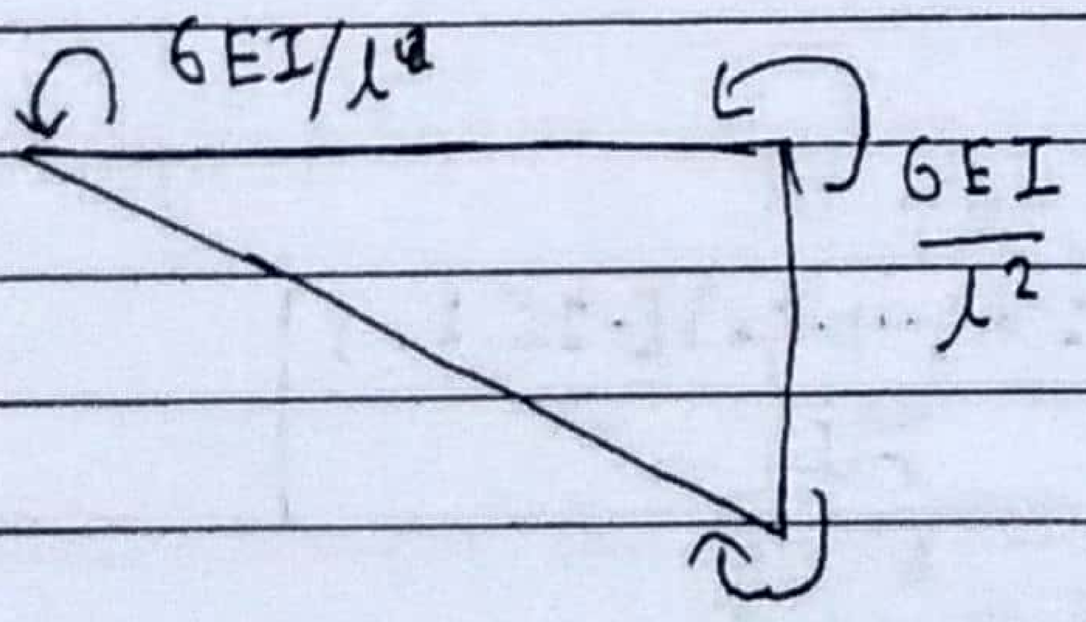
If downward Suttlement of supports B and C in kN.m are $\frac{2000}{EI}$ and $\frac{1000}{EI}$ respectively.

Step:-1 Find KI.

$KI = 2 \; [\theta_B , \theta_C]$



Step:-2 Fixed end moments:-



$\delta = \frac{2000}{EI}$

$MF_{AB} = \frac{-Wl}{8} - \frac{6EI \cdot \delta}{L^2}$

$= \frac{-240 \times 10}{8} - \frac{6EI\,(2000)}{(10)^2 \times EI}$

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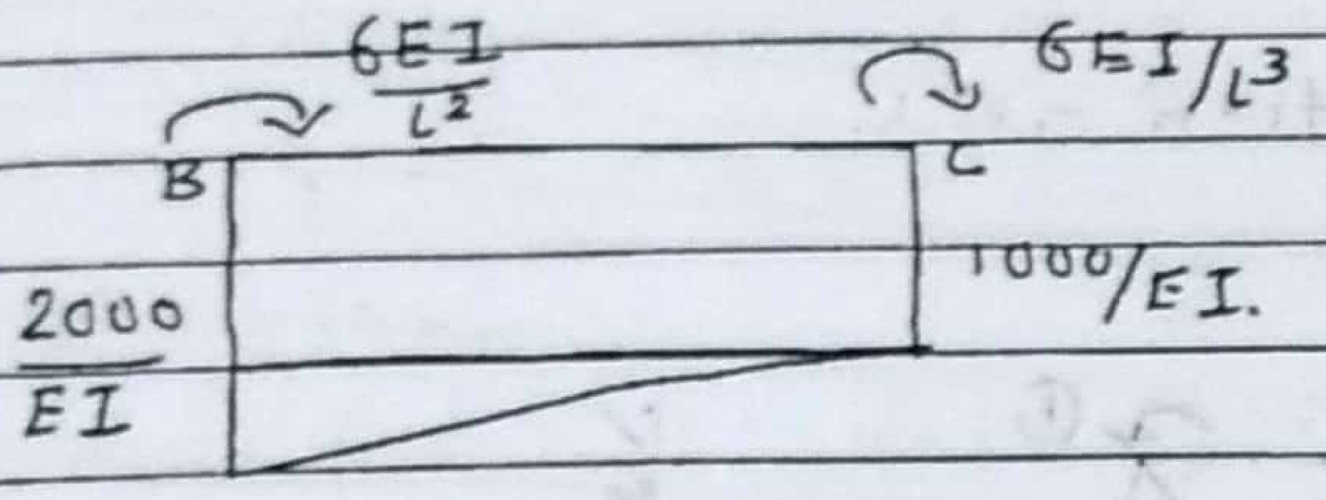
$$MFBA = \frac{wl}{8} - \frac{6EI.\delta}{L^2}$$

$$= 180 \text{ kN.m.}$$

– Member BC.



$$\delta = \frac{1000}{EI.}$$

$$MFBC = -\frac{wl}{8} + \frac{6EI\delta}{L^2}$$

$$= \frac{-120 \times 10}{8} + \frac{6EI \times 1000}{(10)^2 \times EI}$$

$$= -150 + 60$$

$$= -90 \text{ kN.m.}$$

$$MFCB = \frac{wl}{8} + \frac{6EI\delta}{L^2}$$

$$= \frac{120 \times 10}{8} + \frac{6EI \times 1000}{(10)^2 \times EI}$$

$$= 150 + 60$$

$$= 210 \text{ kN.m.}$$

$$P_{1L} = MFBA + MFBC \qquad P_{2L} = 210 \text{ kN.m}$$

$$= 180 - 90$$

$$= 90 \text{ kN.m.}$$

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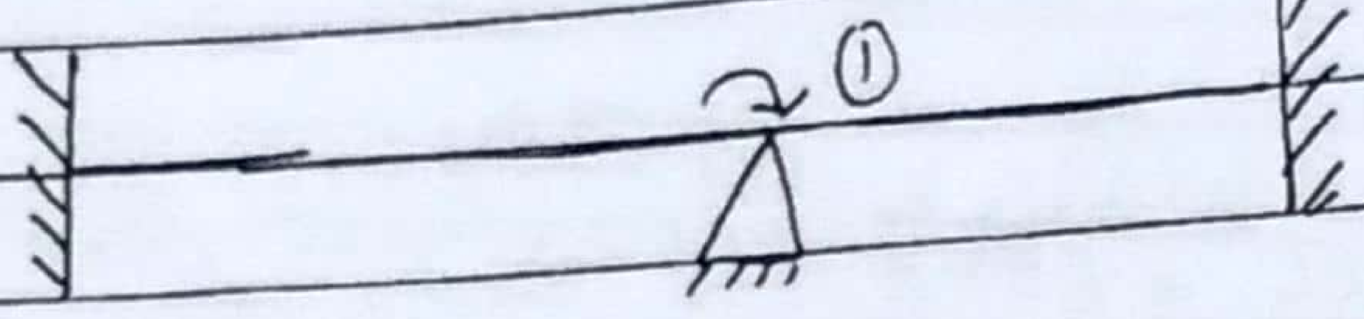
$$[P_L] = \begin{bmatrix} 90 \\ 210 \end{bmatrix}, \quad [P] = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

$$[P - P_L] = \begin{bmatrix} -90 \\ -210 \end{bmatrix}$$

Step:-3 Apply unit rotation at 1.



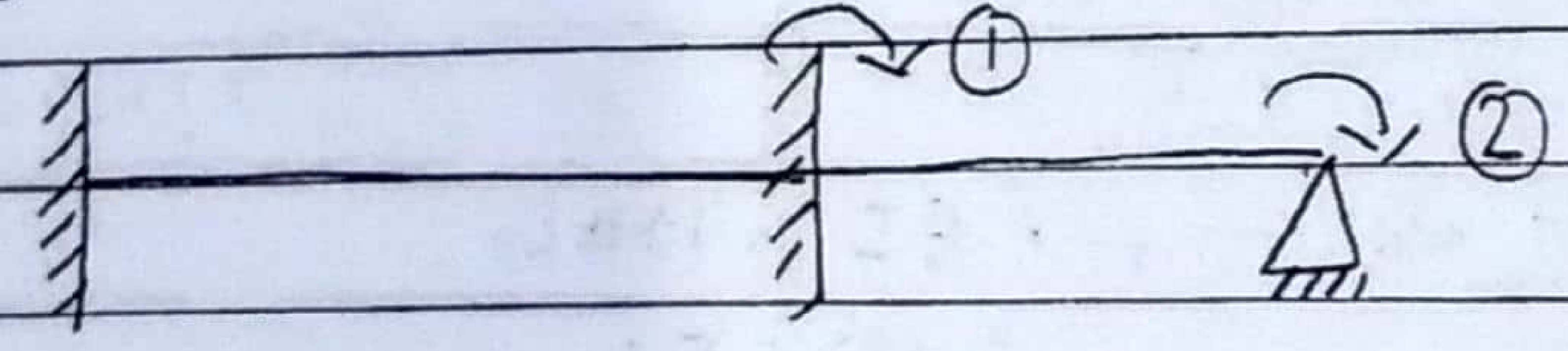
$$K_{11} = \frac{4EI}{L} + \frac{4EI}{L}$$

$$= 0.8\,EI.$$

$$K_{21} = \frac{2EI}{L} \qquad K_{12} = \frac{2EI}{L}$$

$$= 0.2\,EI. \qquad = 0.2\,EI.$$

Apply unit rotation at 2:



$$K_{22} = \frac{4EI}{L} = 0.4\,EI.$$

$$[\mu] = EI \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.4 \end{bmatrix}$$

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Using stiffness eqn:-

$$[\Delta] = [\mu][(P-P_L)]$$

$$= \frac{1}{EI(0.28)} \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.8 \end{bmatrix} \begin{bmatrix} \cdot 90 \\ -210 \end{bmatrix}$$

$$= \frac{1}{0.28\,EI} \begin{bmatrix} 6 \\ -150 \end{bmatrix}$$

$$[\Delta] = \frac{1}{EI} \begin{bmatrix} 21.42 \\ -535.71 \end{bmatrix}$$

$$\theta_B = \frac{21.42}{EI}, \quad \theta_C = \frac{-535.71}{EI}.$$

step:-4 Bending moment:-

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left[2\theta_A + \theta_B - 3\delta/L\right] + \frac{2EI}{10}$$

$$= -420 + \frac{2EI}{10}\left[2(0) + \frac{21.42}{EI}\right]$$

$$= -415.71 \text{ kN.m.}$$

$$M_{BA} = M_{FAB} + \frac{2EI}{L}\left[2\theta_B + \theta_A - 3\delta/L\right]$$

$$= 180 + \frac{2EI}{10}\left[2\left(\frac{21.42}{EI}\right) + 0\right]$$

$$= 188.57 \text{ kN.m.}$$

$$M_{BC} = M_{FBC} + \frac{2EI}{L}\left[2\theta_B + \theta_C - 3\delta/L\right]$$

$$= -90 + \frac{2EI}{10}\left[\frac{2(21.42)}{EI} - \frac{535.71}{EI} + 0\right]$$

$$= -188.57 \text{ kN m}$$

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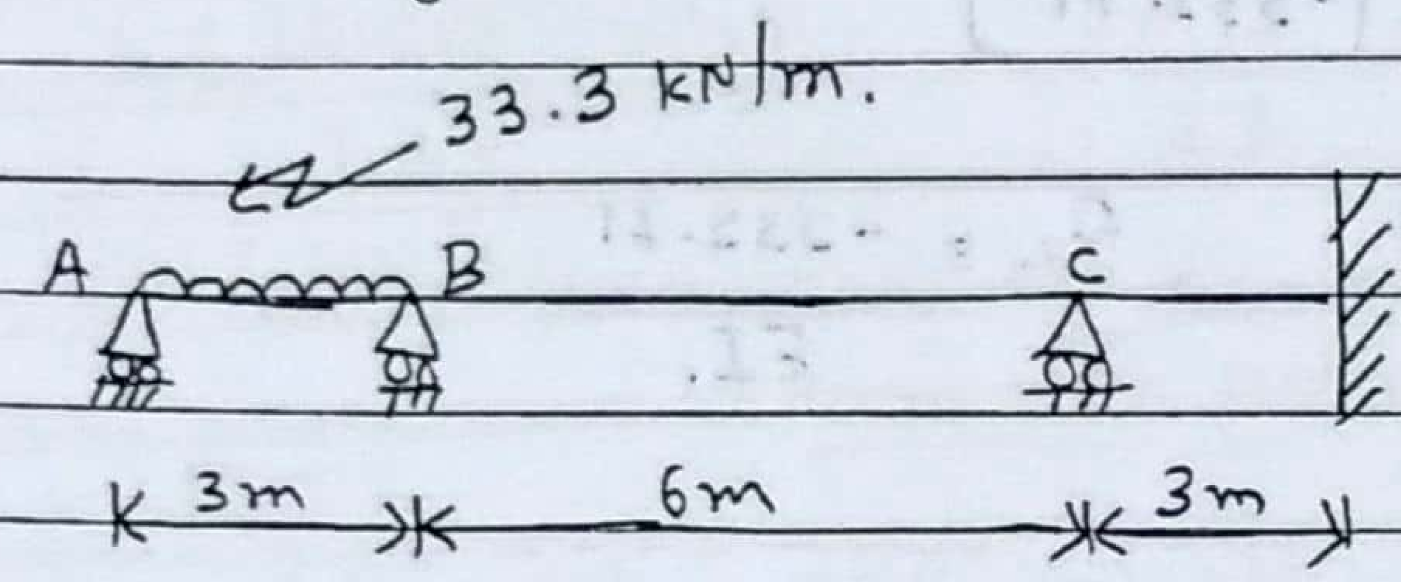
$$M_{CB} = M_{fCB} + \frac{2EI}{L}\left[2\theta_C + \theta_B - 3\delta/L\right]$$

$$= 210 + \frac{2EI}{10}\left[\frac{2(-535.71)}{EI} + \frac{21.42}{EI} + 0\right]$$

$$= 210 - 210$$

$$= 0.$$

3. Support C and a Continous beam as a downward settlement of 30 mm Calculate support reaction at D using displacement method.



$EI = 5600\ kN/m^2$

- $KI = 3\ \boxed{\theta_A, \theta_B, \theta_C}$

- Fixed end moment:-

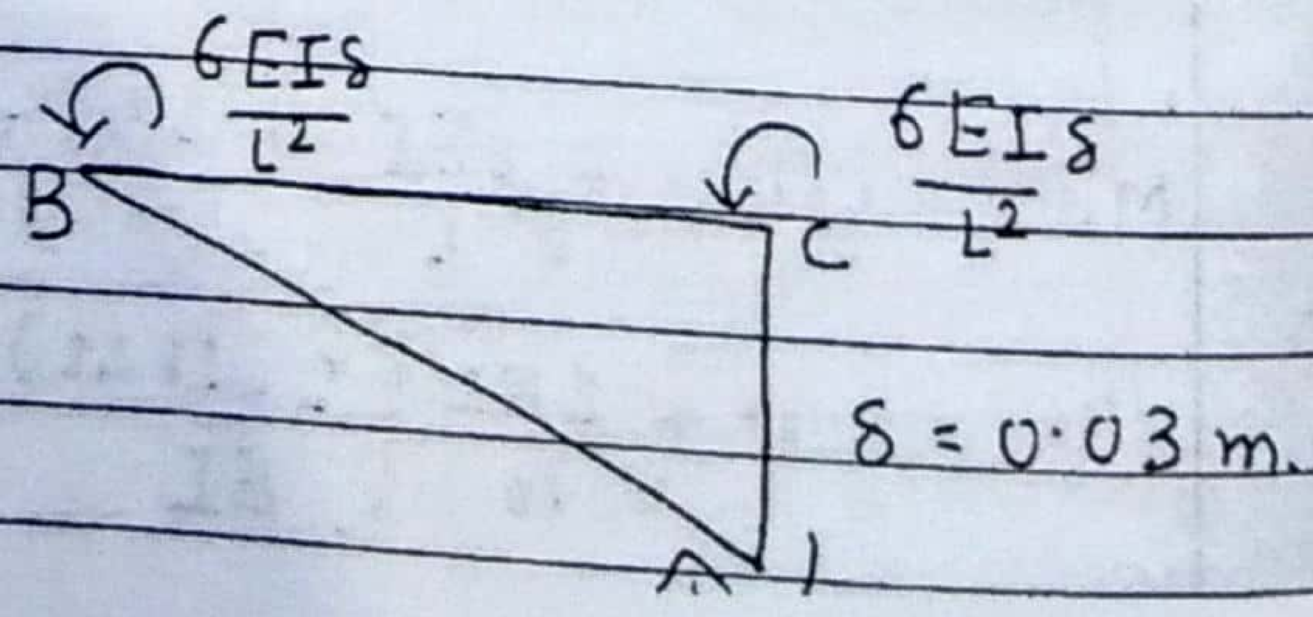
$$MFAB = \frac{-Wl^2}{12} = \frac{-33.3 \times (3)^2}{12}$$

$$= -25\ kN.m$$

$$MFBA = \frac{Wl^2}{12} = \frac{33.3 \times (3)^2}{12}$$

$$= 25\ kN.m$$

Member BC



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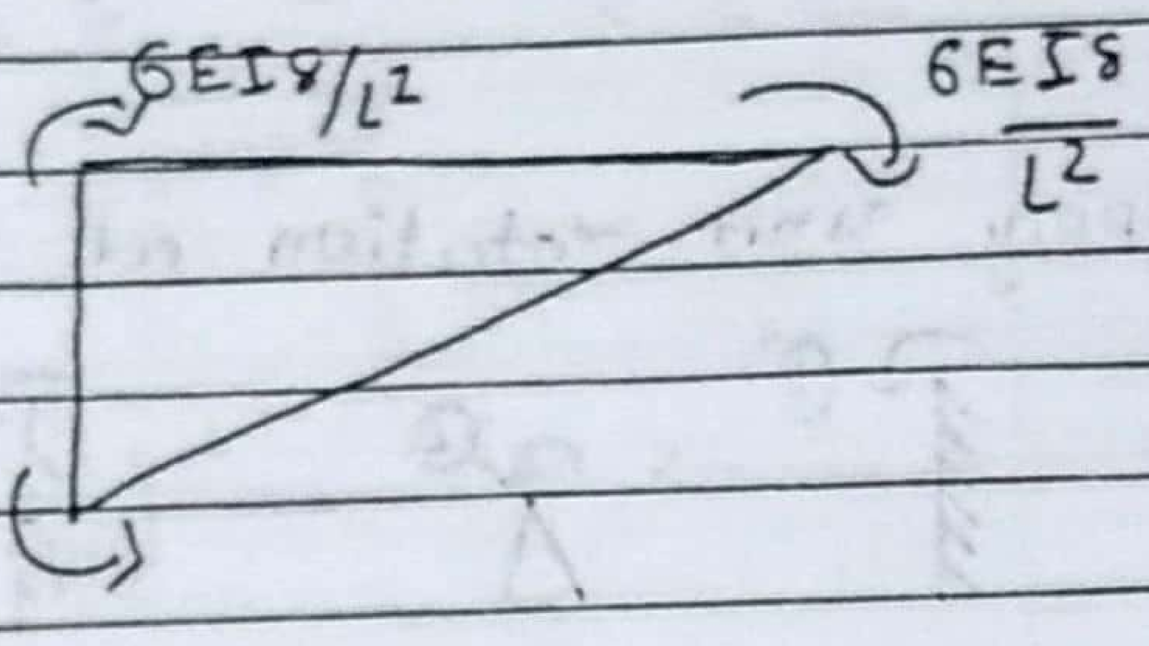
$$MF_{BC} = \frac{-6EI\delta}{L^2}$$

$$= \frac{-6 \times 5600 \times 0.03}{(6)^2}$$

$$= -28 \text{ kN.m.}$$

$$MF_{CB} = 28 \text{ kN.m.}$$

Member CD.



$$MF_{CD} = \frac{6EI\delta}{L^2}$$

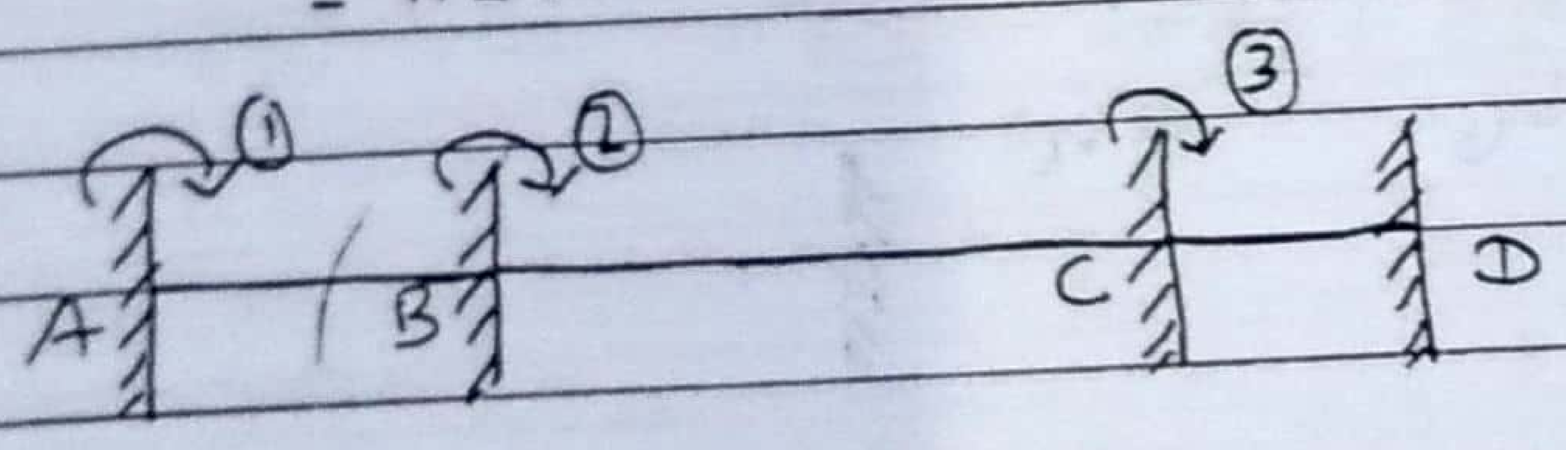
$$= \frac{6 \times 5600 \times 0.03}{9}$$

$$= -112 \text{ kN.m.}$$

$$MF_{DC} = \frac{6EI\delta}{L^2}$$

$$= \frac{6 \times 5600 \times 0.03}{9}$$

$$= 112 \text{ kN.m.}$$

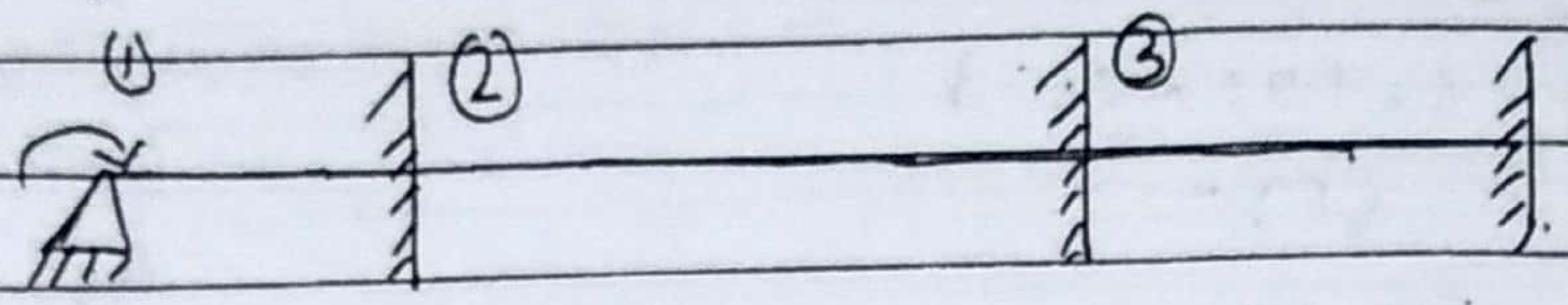


$P_{1L} = -25$, $P_{2L} = -3$, $P_{3L} = 84$.

$$[P_L] = \begin{bmatrix} -25 \\ -3 \\ +84 \end{bmatrix} \qquad [P] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[P - P_L] = \begin{bmatrix} 25 \\ 3 \\ -84 \end{bmatrix}$$

Apply unit rotation at (1)

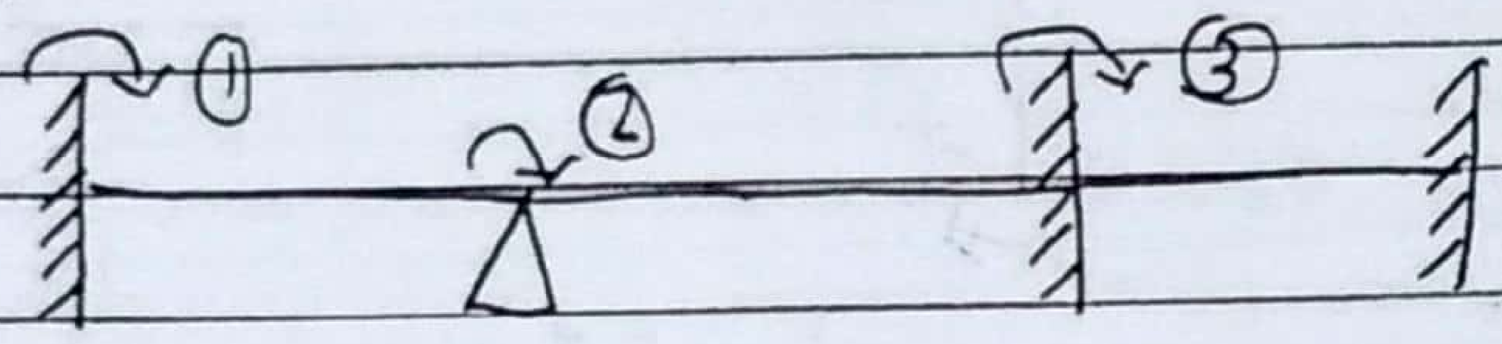


$K_{11} = \frac{4EI}{3}$, $k_{21} = \frac{2EI}{3}$, $k_{31} = 0.$

$\frac{EI}{3}\begin{bmatrix}4 & 2 & 0\\ 2 & 6 & 1\\ 0 & 1 & 6\end{bmatrix}$

$K_{13} = 0$

Apply unit rotation at (2) ~~and~~ ~~(3)~~).



$K_{12} = \frac{2EI}{3}$

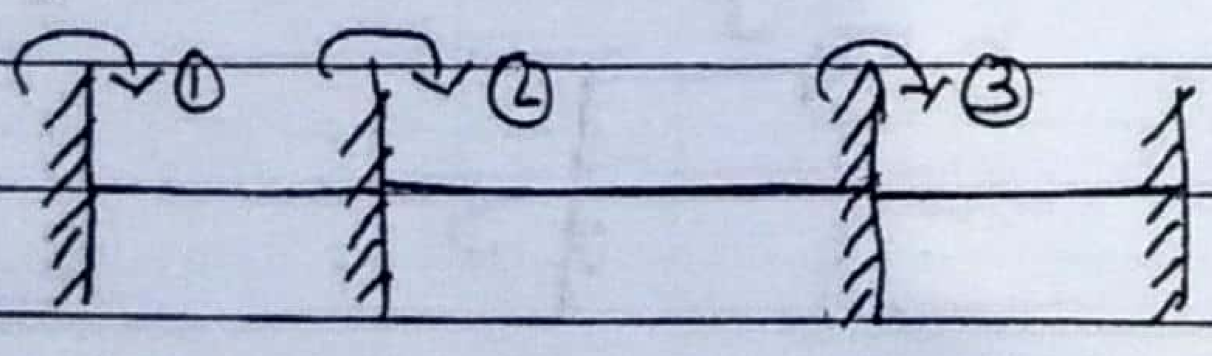
$K_{23} = 1$

$K_{22} = \frac{4EI}{3} + \frac{2EI}{6}$

$= \frac{6EI}{3}$

$K_{32} = \frac{2EI}{6} = \frac{EI}{3}$

Apply unit rotation at (3)



$K_{31} = 0.$ $K_{33} = \frac{4EI}{L} + \frac{2EI}{L}$

$$[\mu] = \frac{EI}{3}\begin{bmatrix} 4 & 2 & 0 \\ 2 & 6 & 1 \\ 0 & 1 & 6 \end{bmatrix}$$

$$[\mu]^{-1} = \frac{3}{EI[116]}\begin{bmatrix} 35 & -12 & 0 \\ -12 & 24 & -4 \\ 0 & -4 & 20 \end{bmatrix}$$

- Using stiffness eqn:-

$$[\Delta] = [\mu]^{-1}[(P - P_L)]$$

$$= \frac{3}{EI(116)}\begin{bmatrix} 35 & -12 & 0 \\ -12 & 24 & -4 \\ 0 & -4 & 20 \end{bmatrix}\begin{bmatrix} 25 \\ 3 \\ -84 \end{bmatrix}$$

$$= \frac{1}{EI}\begin{bmatrix} 21.69 \\ 2.79 \\ -43.76 \end{bmatrix}$$

$$\theta_A = \frac{21.69}{EI}, \quad \theta_B = \frac{2.79}{EI}, \quad \theta_C = \frac{-43.76}{EI}.$$

- Bending moment:

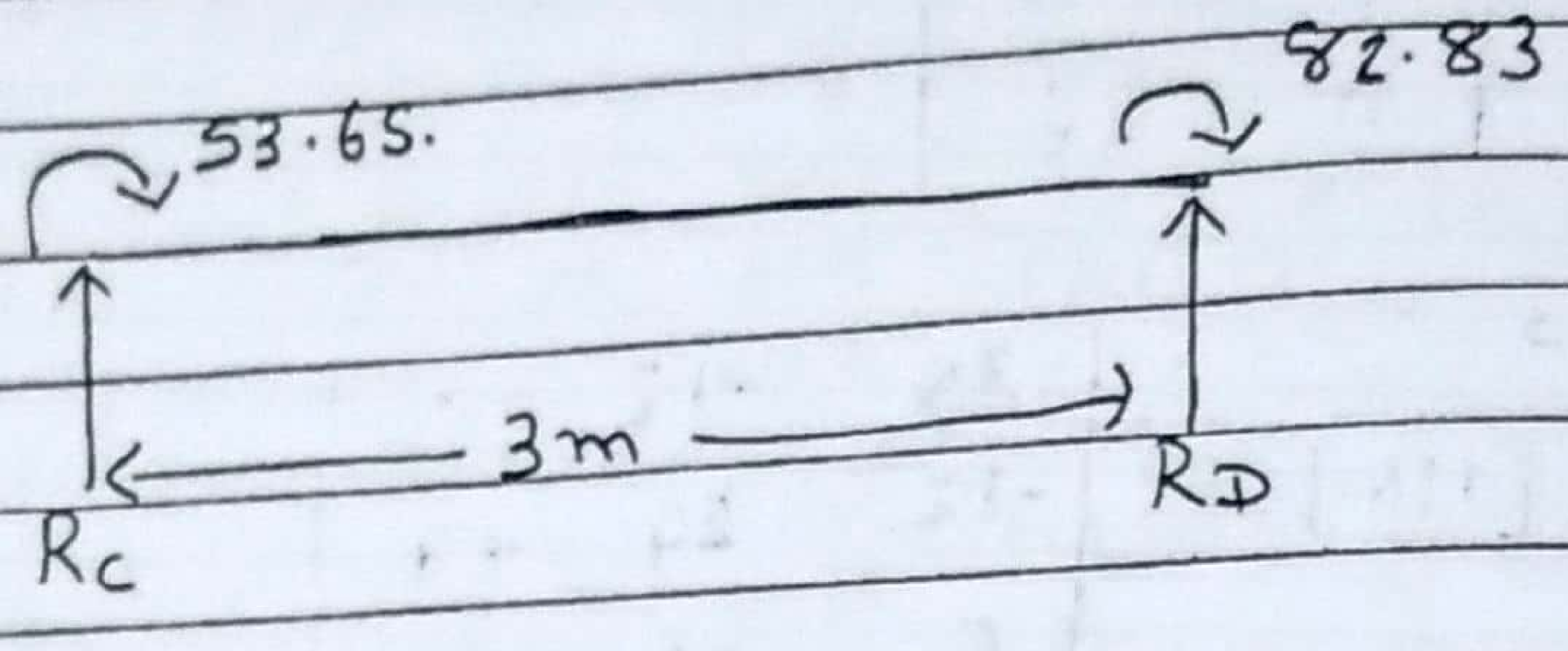
$$M_{CD} = MF_{CD} + \frac{2EI}{L}\left[2\theta_C + \theta_D - 3\delta/L\right]$$

$$= 112 + \frac{2EI}{3}\left[\frac{-2 \times 43.76}{EI} + 0\right]$$

$$= 53.65 \text{ kN.m.}$$

$$M_{DC} = Mf_{DC} + \frac{2EI}{L}\left[2\theta_D + \theta_C - 3\delta/L\right]$$

- Reaction at D.

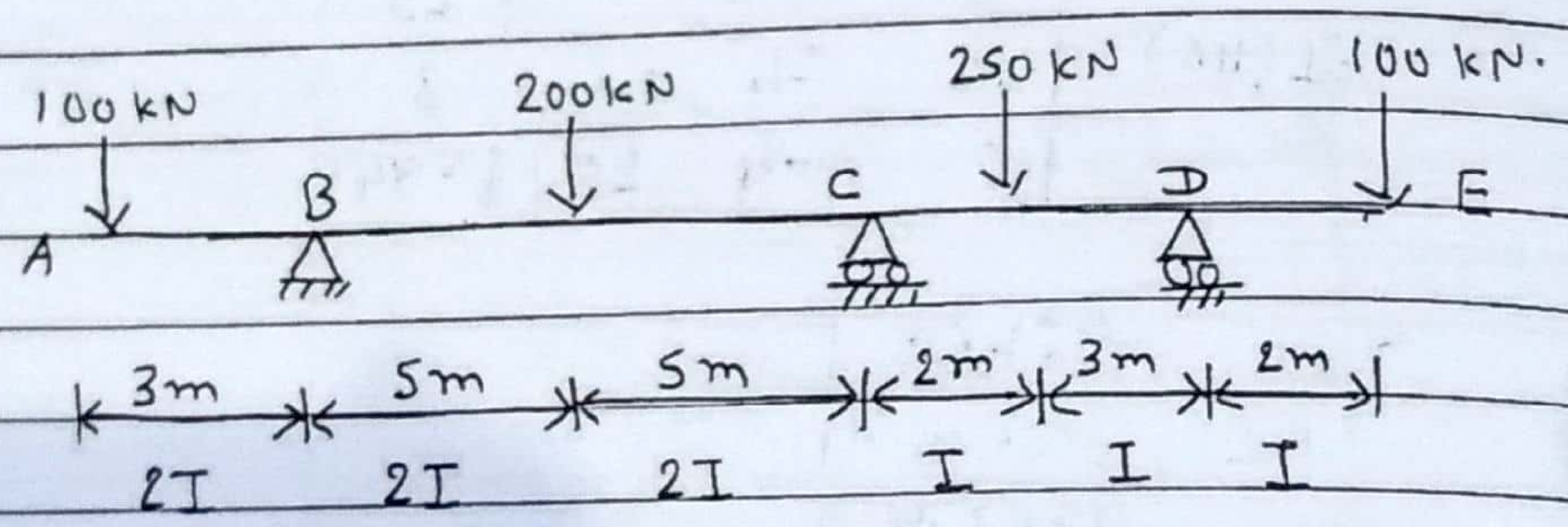


Taking moment at C,

$\therefore R_D \times 3 = 53.65 + 82.83.$

$\boxed{\therefore R_D = 45.49 \text{ kN}}$

4.



Find out $\theta_A, \theta_B, \theta_C$.

Step:-1 $KI = 3, \boxed{\theta_A, \theta_B, \theta_C}$

Step:-2 Fixed end moment:-

For member AB:-

13

Apply unit rotation at (1).

$$k_{11} = \frac{4E(2I)}{10} = \frac{4EI}{5} = 0.8\,EI$$

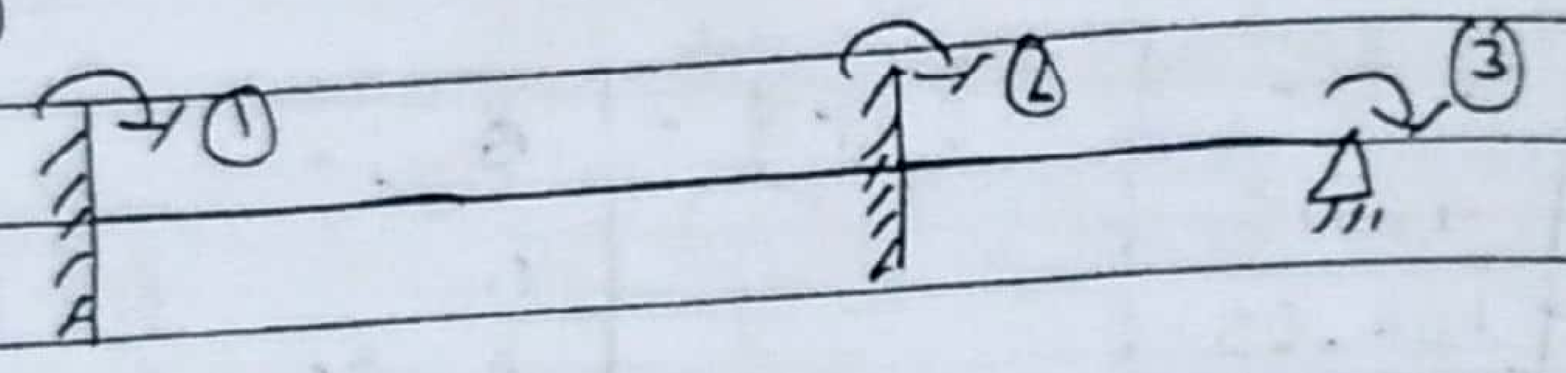
$$K_{22} = \frac{4E(2I)}{10} + \frac{4EI}{5}$$

$$= \frac{8EI}{5}$$

$$= 1.6\,EI.$$

$$K_{32} = \frac{2EI}{5} = 0.4\,EI.$$

- Apply unit rotation at (3).



$$K_{31} = 0.$$

$$K_{32} = \frac{2EI}{5} = 0.4\,EI$$

$$K_{33} = \frac{4EI}{5} = 0.8\,EI.$$

$$[K] = \frac{EI}{5}\begin{bmatrix} 4 & 2 & 0 \\ 2 & 4 & 2 \\ 0 & 2 & 4 \end{bmatrix}$$

$$[K]^{-1} = \frac{5}{EI[4(12) - 2(8)]}\begin{bmatrix} 12 & -8 & 0 \\ -8 & 16 & -8 \\ 0 & -8 & 12 \end{bmatrix}$$

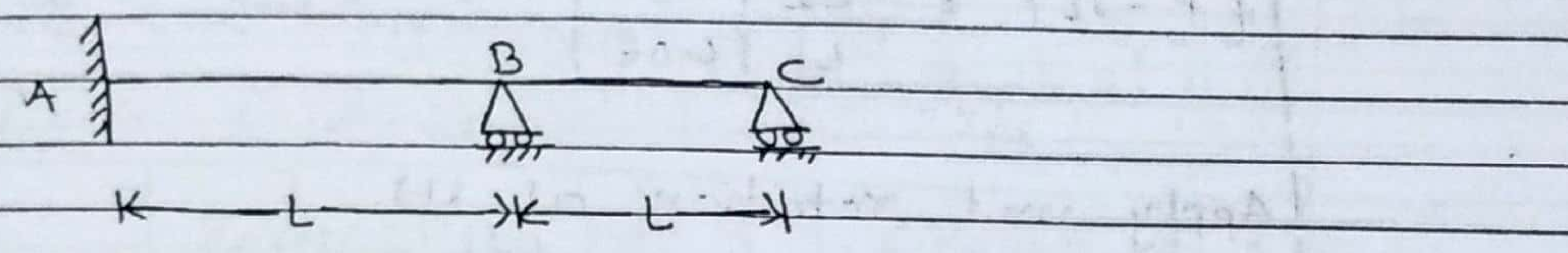
$$= 5 \quad [12 \;\; -8 \;\; 0$$

$[\Delta] = [k]^{-1} \, [(P - P_L)]$

$$= \frac{5}{32EI}\begin{bmatrix} 12 & -8 & 0 \\ -8 & 16 & -8 \\ 0 & -8 & 12 \end{bmatrix}\begin{bmatrix} -50 \\ -93.75 \\ 43.75 \end{bmatrix}$$

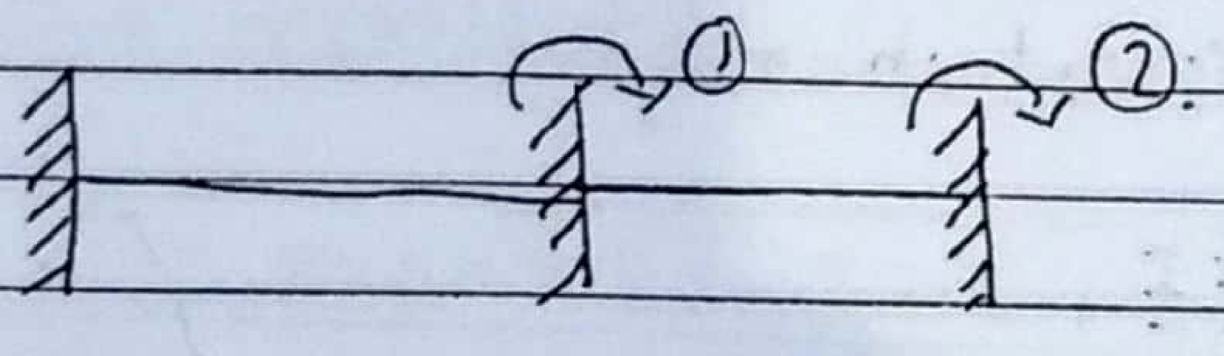
$$= \frac{1}{EI}\begin{bmatrix} 23.44 \\ -226.56 \\ 199.21 \end{bmatrix}$$

5.



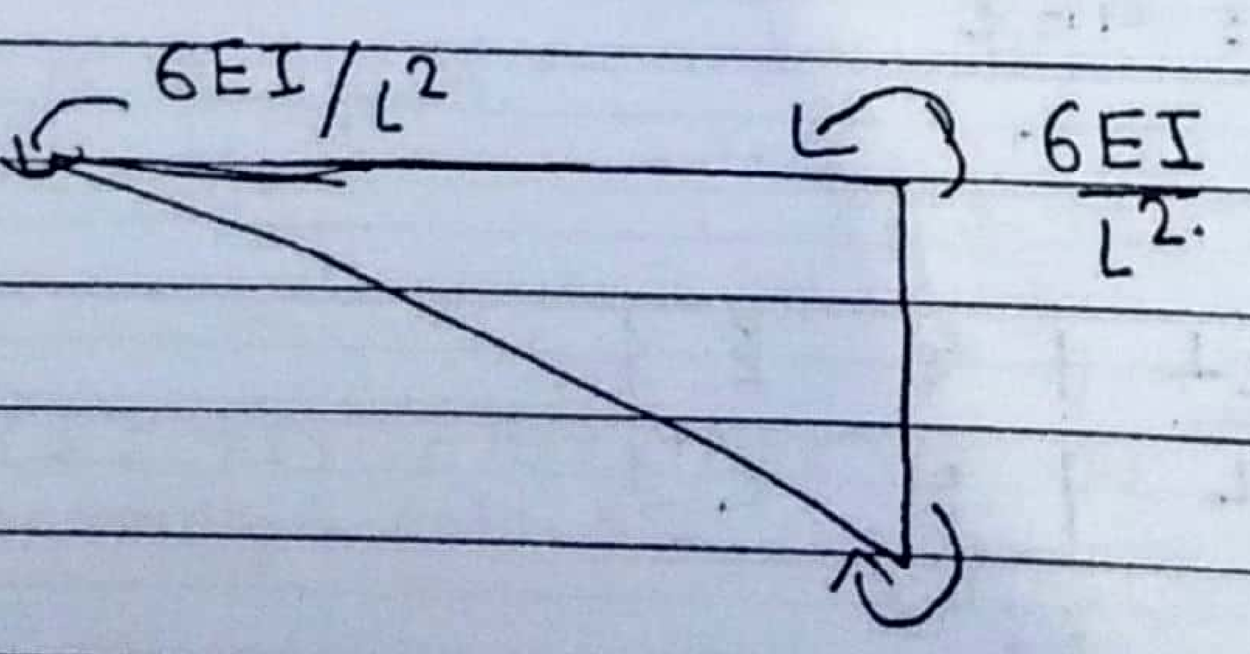
A Permits an anti-clockwise rotation $\theta = 0.004$ rad.
B Settles downwards by a distance $\Delta = L/100$
C upward reaction at support $81EI/1750\,L^2$.

Step:-1 K.I = 2 $[\theta_B, \theta_C]$



Step:-2 Fixed end moment:-

Span AB



$$MF_{AB} = MF_{BA} = -\frac{6EI}{L^2}(0.01 \times L)$$

$= -0.06\,EI$

Span BC:-

$$MF_{BC} = MF_{BC} = \frac{0.06EI}{L}$$

$$P_{1L} = 0, \quad P_{2L} = \frac{0.06EI}{L}$$

$$[P_L] = \frac{EI}{L}\begin{bmatrix} 0 \\ 0.06 \end{bmatrix}, \quad P = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$[P - P_L] = -\frac{EI}{L}\begin{bmatrix} 0 \\ 0.06 \end{bmatrix}$$

$$\frac{EI}{L}\begin{bmatrix} 8 & 2 \\ 2 & 4 \end{bmatrix}$$

$$\Delta^{-1} = \frac{1}{28EI}\begin{bmatrix} +4 & -2 \\ -2 & +8 \end{bmatrix}$$

Apply unit rotation at (1).

$$\frac{2EI}{L}\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$$

$$k_{11} = \frac{4EI}{L} + \frac{4EI}{L}$$

$$= \frac{8EI}{L}$$

$$K_{22} = \frac{4EI}{L}$$

$$K_{21} = \frac{2EI}{L}$$

Apply unit rotation at (2)

$$K_{12} = \frac{2EI}{L}$$

using stiffness eqn:-

$$\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \frac{L}{28EI} \begin{bmatrix} 4 & -2 \\ -2 & 8 \end{bmatrix} \frac{EI}{L} \begin{bmatrix} 0 \\ 0.06 \end{bmatrix}$$

$$\theta_B = 0.0048$$

$$\theta_C = -0.017.$$

Step:-3 Beanding moment:-

$$M_{BC} = MF_{BC} + \frac{2EI}{L} \left[2\theta_B + \theta_C - 3\delta/L \right]$$

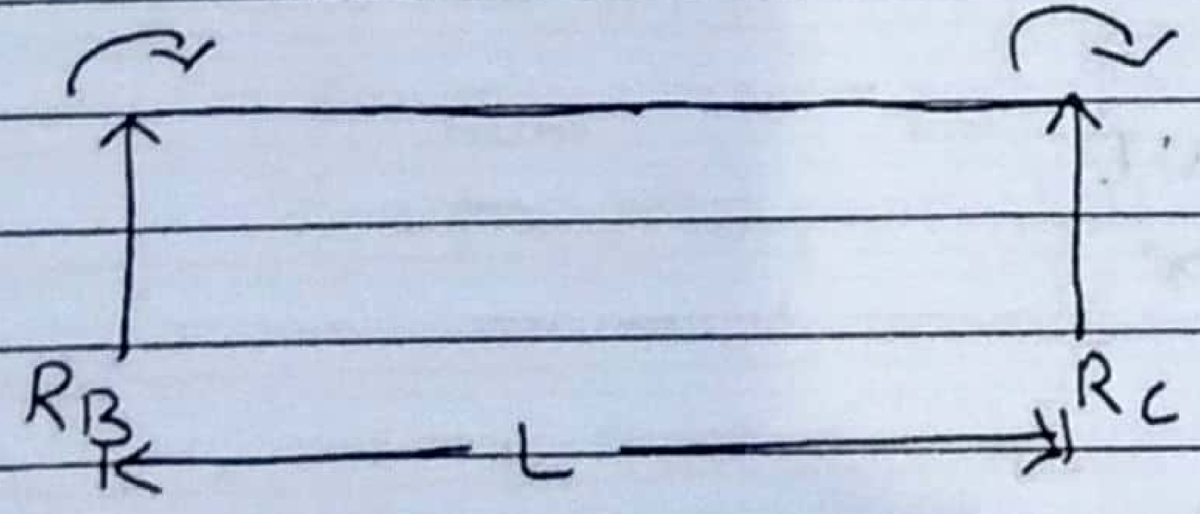
$$= \frac{0.06EI}{L} + \frac{2EI}{L} \left[2(0.0048) + (-0.017) \right]$$

$$= \frac{0.0452EI}{L}$$

$$M_{CB} = \frac{0.06EI}{L} + \frac{2EI}{L} \left[2(-0.017) + 0.0048 \right]$$

$$= 0.0016 \frac{EI}{L}$$

$$= 0.002 \frac{EI}{L}.$$



Taking moment at RB.

$$R_C \times L = \frac{0.0452EI}{L} + \frac{0.002EI}{L}$$

$$R_C = \frac{0.0472EI}{12}$$

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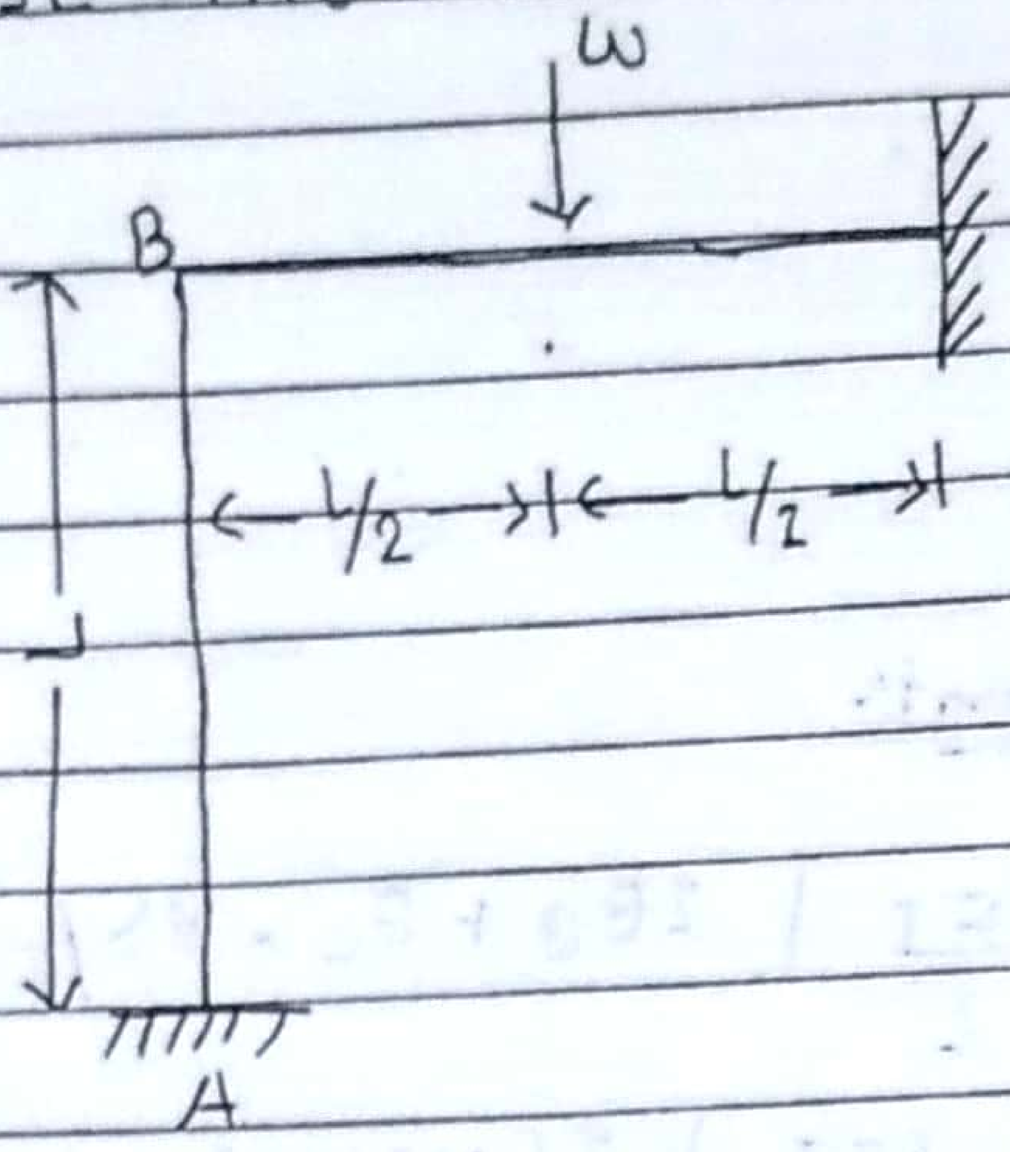
Frame

1. Analysis of rigid jointed frame (Non-sway).
analyise the structure by stiffness method.



- K·I = 1 [θ_B].

- Fixed end moment.

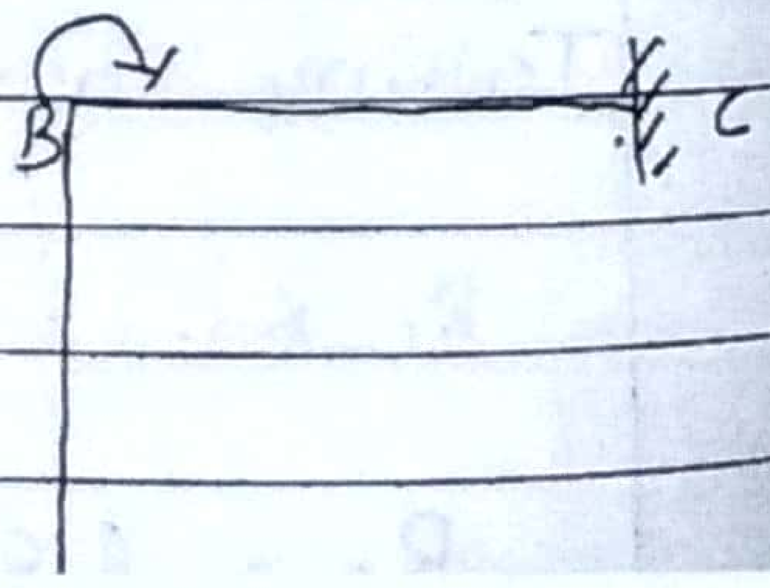
$M_{FAB} = M_{FBA} = 0.$

$M_{FBC} = -\frac{wl}{8}$, $M_{FCB} = +\frac{wl}{8}$

$[P_{1L}] = -\frac{wl}{8}$, $[P] = 0.$

$[P - P_L] = +\frac{wl}{8}$

- Apply unit rotation at (1).



$K_{11} = \frac{4EI}{L} + \frac{4EI}{L}$

$= \frac{8EI}{L}$

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$[K] = \frac{8EI}{L}$ $\quad [K]^{-1} = \frac{L}{8EI}$

using stiffness eqn:

$[\Delta] = [k]^{-1} [P - P_L]$

$= \frac{L}{8EI} \left[\frac{wl}{8} \right]$

$\theta_B = \frac{wl^2}{64EI}$

- Final moment:

$M_{AB} = M_{fAB} + \frac{2EI}{L} [2\theta_A + \theta_B - 3\delta/L]$

$= 0 + \frac{2EI}{L} [0 + wL^2/64EI]$

$= \frac{wl}{32}$

$M_{BA} = 0 + \frac{2EI}{L} \left[\frac{2 \times wl^2}{64EI} \right]$

$= \frac{wl}{16}$

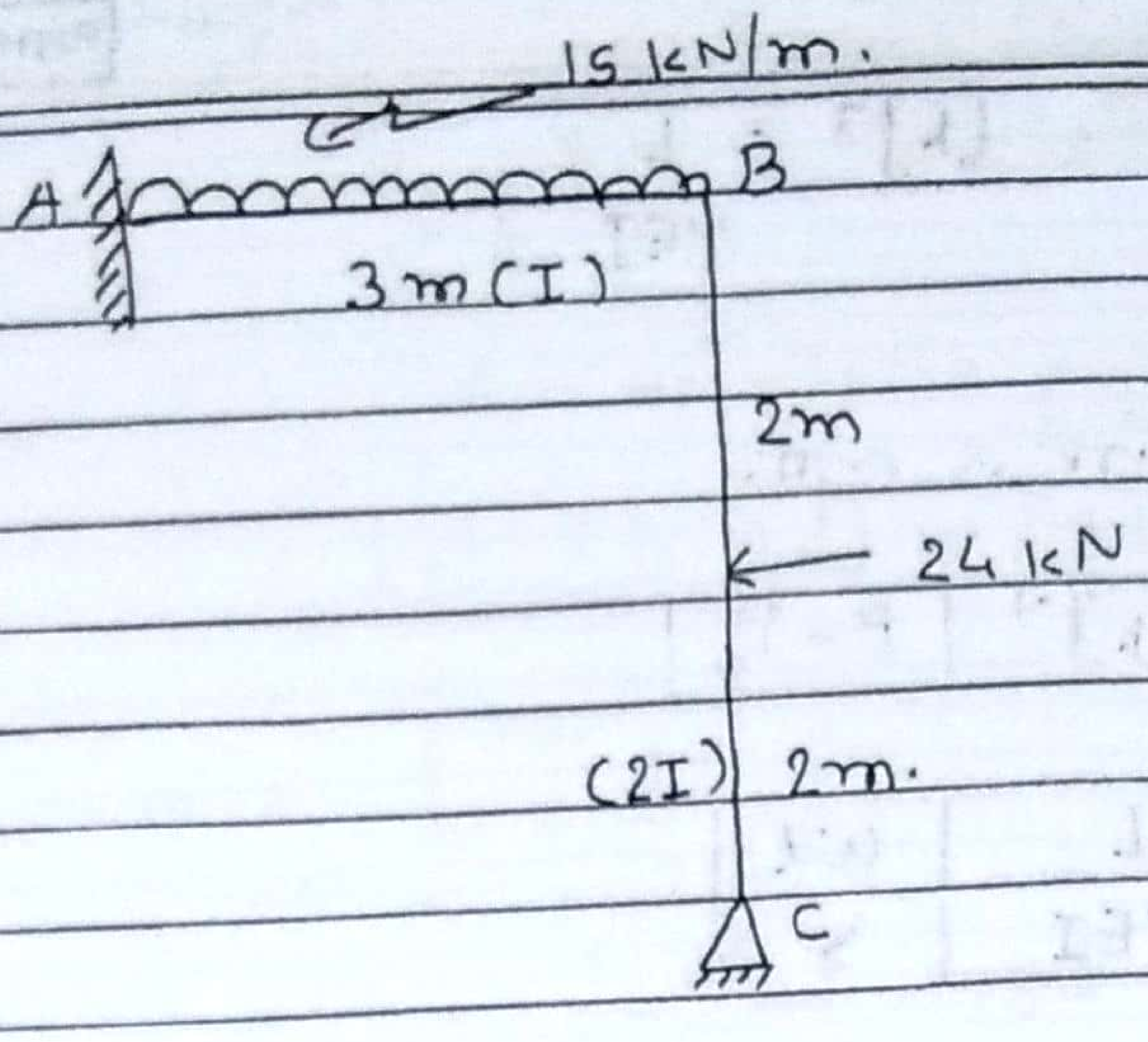
$M_{BC} = \frac{-wl}{8} + \frac{2EI}{L} \left[\frac{2 \times wl^2}{64EI} \right]$

$= \frac{-wl}{16}$

$M_{CB} = \frac{wl}{8} + \frac{2EI}{l} \left[\frac{wl^2}{64EI} \right]$

$= 5wL$

2.



K.I = 2 $[\theta_B, \theta_C]$

- Fixed end moment:

$MF_{AB} = \frac{-wl^2}{12}$

$= \frac{-15 \times (3)^2}{12}$

$= -11.25$ kN.m.

$MF_{BA} = \frac{wl^2}{12}$

$= \frac{15 \times (3)^2}{12}$

$= 11.25$ kN.m.

$MF_{BC} = -\frac{wl}{8} = \frac{-24 \times 4}{8} = 12$ kN.m.

$MF_{CB} = \frac{wl}{8} = \frac{24 \times 4}{8} = 12$ kN.m.

$P_{1L} = 11.25 - 12 = -0.75$ $\begin{bmatrix} -0.75 \\ 12 \end{bmatrix}$

$P_{2L} = 12.$

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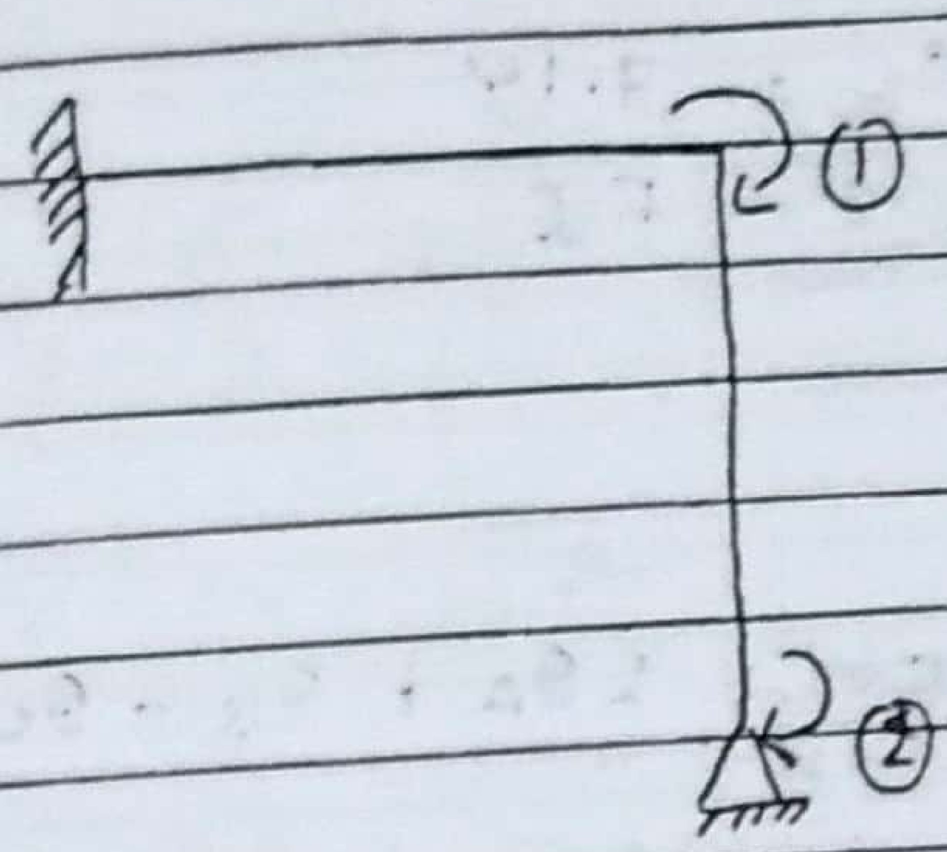
$P = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$[P - P_L] = \begin{bmatrix} 0.75 \\ -12 \end{bmatrix}$

- Applying unit rotation at (1).



$K_{11} = \dfrac{4EI}{3} + \dfrac{4E(2I)}{4}$

$= \dfrac{10EI}{3}$

$K_{21} = \dfrac{2E(2I)}{4}$

$= EI$

- Applying unit rotation at (2).

$K_{12} = EI \qquad K_{22} = \dfrac{4E(2I)}{4} = 2EI$

$[K] = \begin{bmatrix} 3.33 & 1 \\ 1 & 2 \end{bmatrix}$

$[K]^{-1} = \dfrac{1}{5.67EI} \begin{bmatrix} 2 & -1 \\ -1 & 3.33 \end{bmatrix}$

- Using stiffness eq^n!

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$[\Delta] = [K]^{-1}[(P-P_L)]$

$$= \frac{1}{5.67EI}\begin{bmatrix} 2 & -1 \\ -1 & 3.33 \end{bmatrix}\begin{bmatrix} 0.75 \\ -12 \end{bmatrix}$$

$$= \frac{1}{EI}\begin{bmatrix} 2.38 \\ -7.18 \end{bmatrix}$$

$\theta_B = \frac{2.38}{EI}$, $\theta_C = -\frac{7.18}{EI}$

- Bending moment:

$M_{AB} = -11.25 + \frac{2EI}{3}\left[2\theta_A + \theta_B - \theta_C\right]$

$= -11.25 + \frac{2EI}{3}\left[2\theta_A + 2.38/EI + 0\right]$

$= -9.66$ kN.m.

$M_{BA} = 11.25 + \frac{2EI}{3}\left[\frac{2(2.38)}{EI} + 0\right]$

$= 14.41$ kN.m.

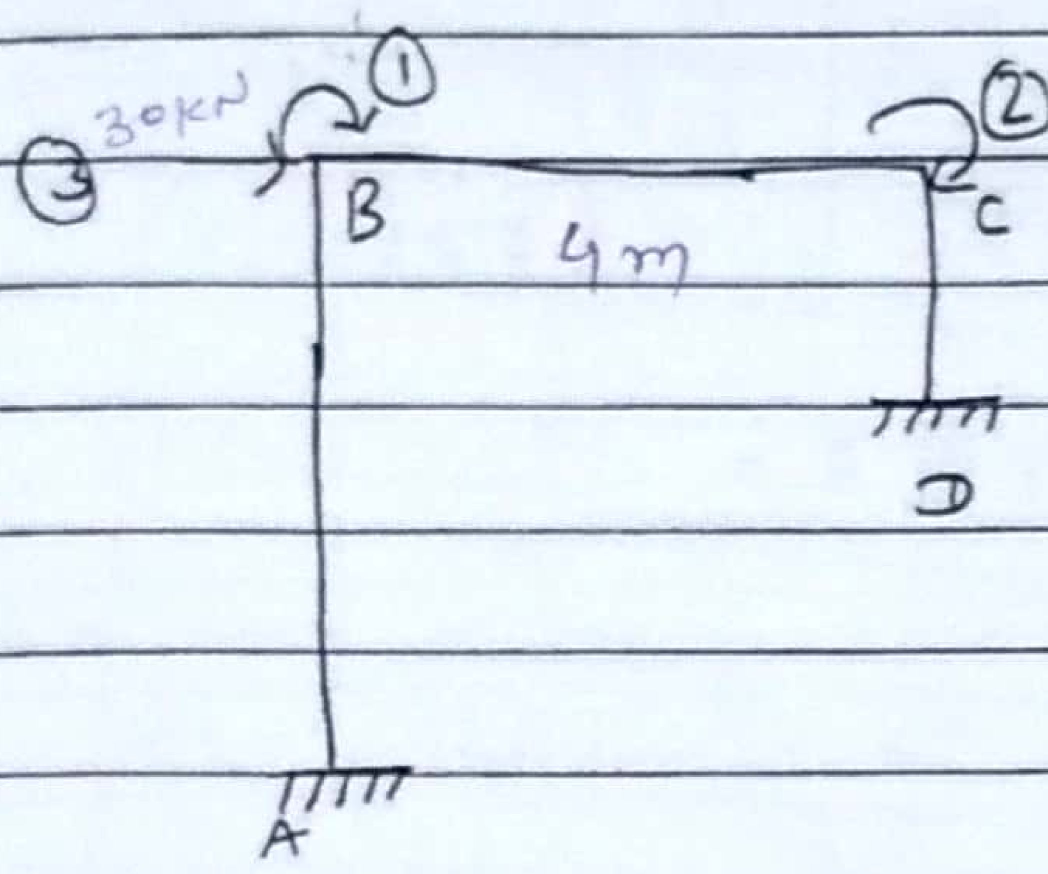
$M_{CB} = 12 + \frac{2E(2I)}{4}\left[2\frac{(-7.18)}{EI} + \frac{2.38}{EI}\right]$

$= 0.02$ kN.m.

$M_{BC} = -14.41$ kN.m

* Analysis of frame with Sway:

→ K.I = 3 $[\theta_B, \theta_C, \delta)$



→ Fixed end moment :-

$M_{FAB} = 0$. $M_{FBA} = 0$.

$M_{FBC} = -\frac{Wl^2}{8} = -\frac{30 \times 4}{8} = -40$ kN.m.

$M_{FCD} = 0$.

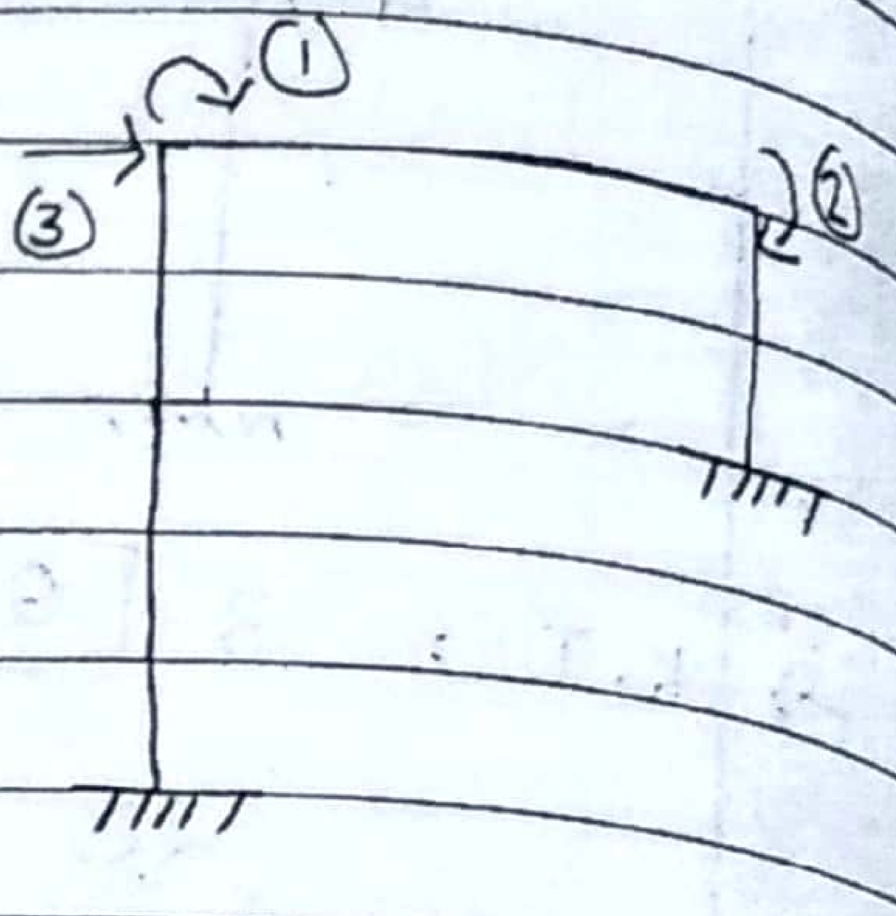
$M_{FDC} = \frac{WL}{8} = \frac{30 \times 4}{8} = 40$ kN.m.

$P_{1L} = 0 + (-40) = -40$ $P_{3L} = 0$.

$$[P_L] = \begin{bmatrix} -40 \\ 40 \\ 0 \end{bmatrix} \qquad P = \begin{bmatrix} 0 \\ 0 \\ 50 \end{bmatrix}$$

$$[P - P_L] = \begin{bmatrix} 40 \\ -40 \\ 50 \end{bmatrix}$$

- Applying unit rotation at (1).



$$K_{11} = \frac{4EI}{4} + \frac{4EI}{4}$$

$$= 2EI.$$

$$K_{21} = \frac{2EI}{4} = 0.5\,EI.$$

$$K_{12} = 0.5\,EI.$$

$$K_{31} = \frac{-6EI}{L^2}$$

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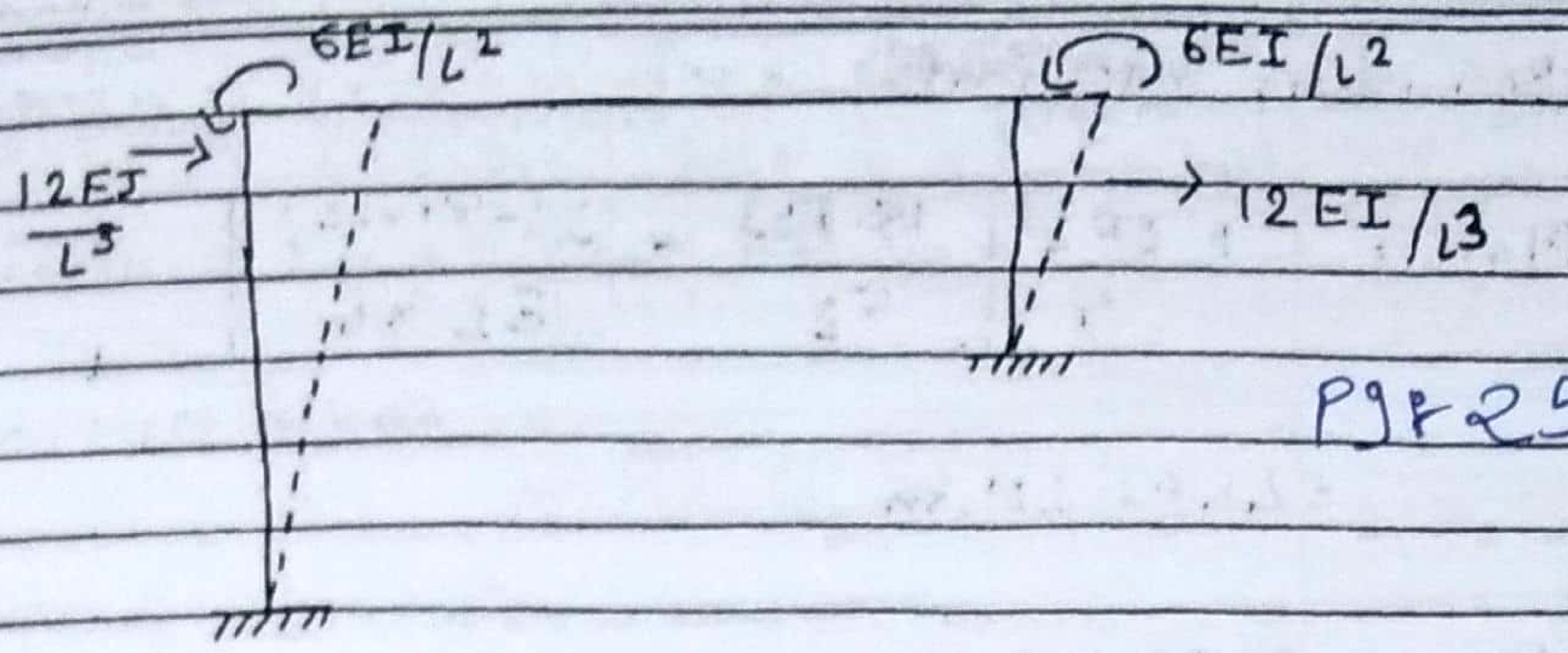

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$k_{13} = \frac{-6EI}{L^2} = -0.375\,EI.$

$k_{23} = \frac{-6EI}{(2)^2} = -1.5\,EI.$

$k_{33} = \frac{12EI}{L^3} + \frac{12EI}{L^3}$

$= 0.375\,EI.$

$$[k] = EI\begin{bmatrix} 2 & 0.5 & -0.375 \\ 0.5 & 3 & -1.5 \\ -0.375 & -1.5 & 0.375 \end{bmatrix}$$

$$[k]^{-1} = \frac{1}{EI}\begin{bmatrix} 0.5106 & -0.17 & -0.17 \\ -0.17 & -0.276 & -1.276 \\ -0.17 & -1.276 & -2.609 \end{bmatrix}$$

Using stiffness eqn:-

$$[\Delta] = [k]^{-1}[(P-P_L)]$$

$$= EI\begin{bmatrix} 0.5106 & -0.17 & -0.17 \\ -0.17 & -0.276 & -1.276 \\ -0.17 & -1.276 & -2.609 \end{bmatrix}\begin{bmatrix} 40 \\ -40 \\ 50 \end{bmatrix}$$

→ Beanding moment:-

$$M_{AB} = 0 + \frac{2EI}{4}\left[\frac{18.723}{EI} - \frac{3(-86.24)}{EI \times 4}\right]$$

$= 41.93$ kN.m

$M_{BA} = 51.28$ kN.m.

$$M_{BC} = -40 + \frac{2EI}{4}\left[\frac{2(18.723)}{EI} + \frac{(-59.57)}{EI}\right]$$

$= -11.06$ kN.m.

$$M_{CB} = 40 + \frac{2EI}{4}\left[\frac{2(-59.57)}{EI} + \frac{18.729}{EI}\right]$$

$= -10.20$ kN.m.

$$M_{CD} = 0 + \frac{2EI}{2}\left[\frac{2(-59.57)}{EI} + \frac{3(-89.24)}{EI}\right]$$

$= 10.20$ kN.m.

$$M_{DC} = 0 + \frac{2EI}{4}\left[\frac{-59.57}{EI} - \frac{3(-89.84)}{2EI}\right]$$

$= 69.79$ kN.m

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* Stiffness method steps for beam analysis :

Step :- 1 Find KI.

Step :- 2 Find Fixed end moment.

Step :- 3 $[P - P_L]$

Step :- 4 Applied unit rotation at each K.I.

Step :- 5 Find $[K]^{-1}$

Step :- 6 Using stiffness equation.

Step :- 7 Final moment.

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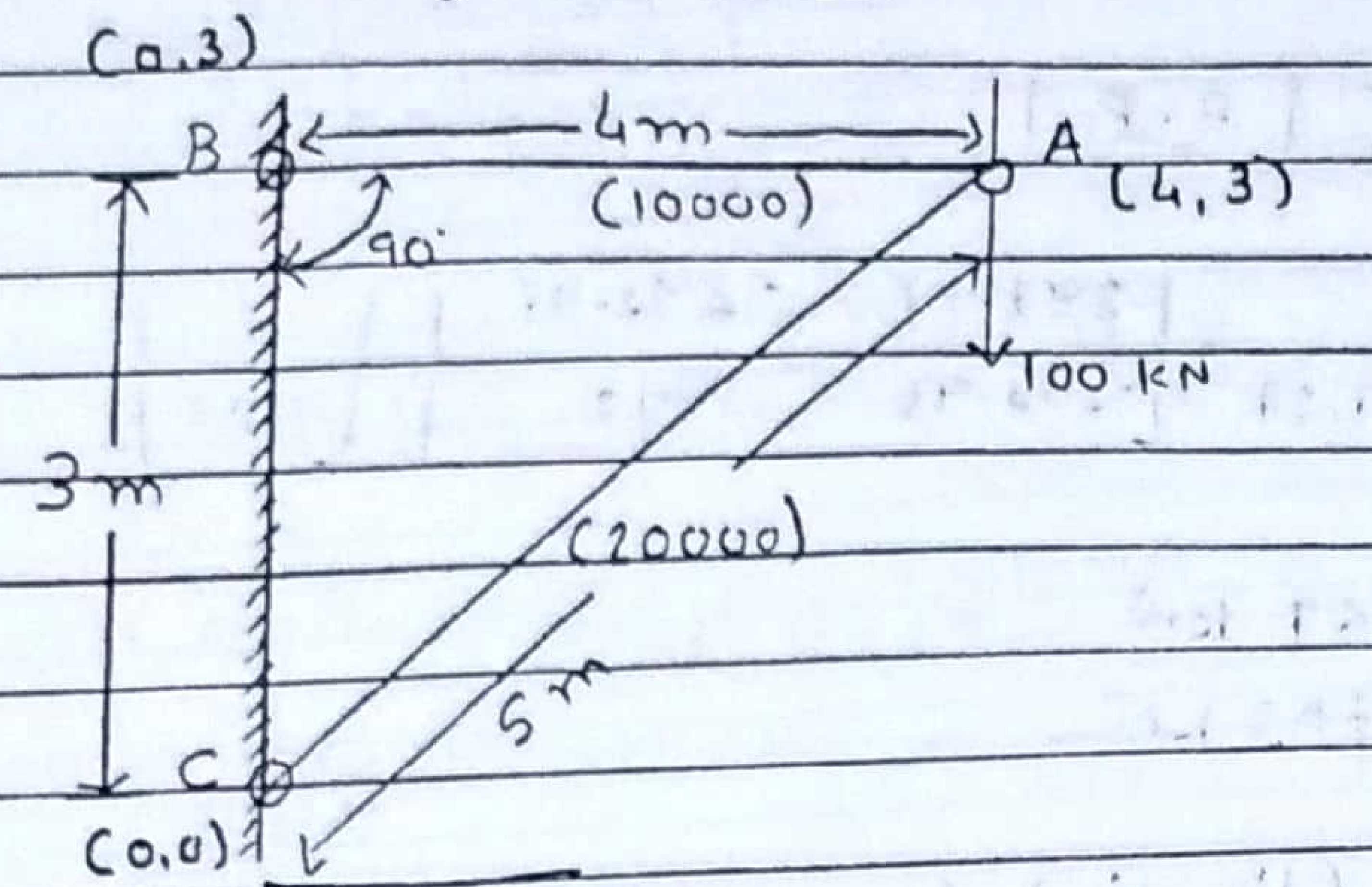
Advance structure Analysis

Pin-jointed Frame

1\.



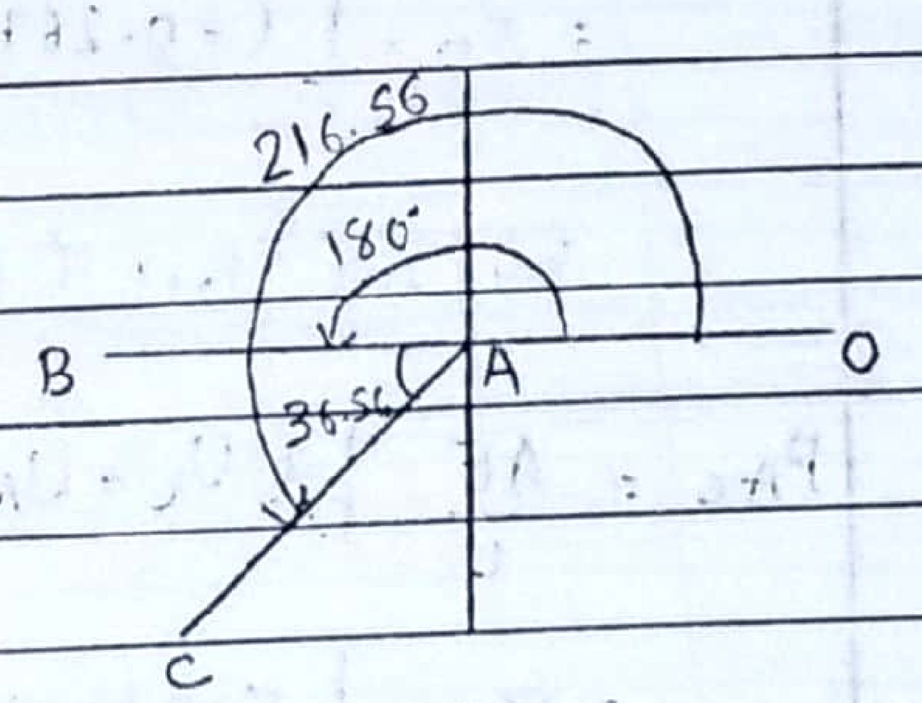
E = 200 kN/m²

$\therefore\ L_{AB} = \sqrt{(4)^2 + (0)^2} = 4\,m$

$L_{BC} = 3\,m$

$L_{AC} = \sqrt{(-4)^2 + (-3)^2}$

$= 5\,m$



$\left[P_L\right] = \begin{bmatrix} 0 \\ -100 \end{bmatrix}$

$\left[P - P_L\right] = \begin{bmatrix} 0 \\ -100 \end{bmatrix}$

$\sin\theta = \frac{3}{5}$ $\therefore\ \theta = 36.56.$

Member	θ	$AE/L\ \cos^2\theta$	$AE/L\ \sin\theta.\cos\theta$	$AE/L\ \sin^2\theta$
AB	180°	500	0	0
AC	216.56°	512	383.96	287.86
		1012	383.96	287.86

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$\therefore [k]^{-1} = \frac{1}{143909.27} \begin{bmatrix} 287.86 & -383.96 \\ -383.96 & 1012 \end{bmatrix}$

$[\Delta] = [k]^{-1} [P - P_L]$

$= \frac{1}{143909.27} \begin{bmatrix} 287.86 & -383.96 \\ -383.96 & 1012 \end{bmatrix} \begin{bmatrix} 0 \\ -100 \end{bmatrix}$

$U_A = 0.267$ kN

$U_B = -0.703$ kN

$P_{AB} = \frac{AE}{L} \left[(U_B - U_A) . \cos\theta + (U_B - U_A) . \sin\theta \right]$

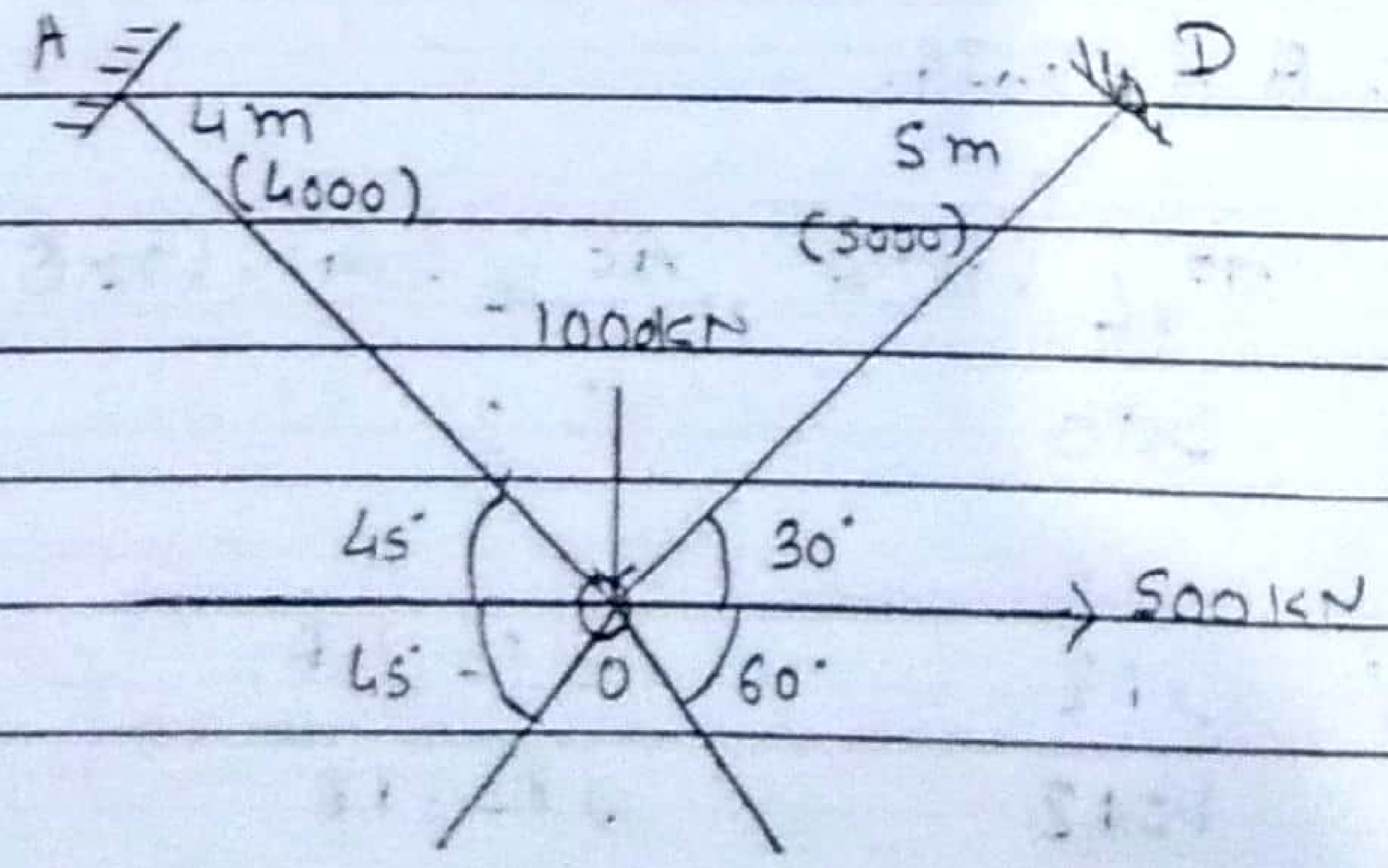
$= 500 \left[(-0.267) \cos 180^\circ + 0 \right]$

$= 133.5$ kN (T)

$P_{AC} = \frac{AE}{L} \left[(U_C - U_A) \cos\theta + (U_C - U_A) \sin\theta \right]$

$= 800 \left[0.213 + (-0.423) \right]$

$= -166.48$ kN (C)

2. 

$E = 200$ kN/mm^2

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$P = \begin{bmatrix} 500 \\ -1000 \end{bmatrix}$

Member	$\frac{AE}{L}$	θ	$\cos^2\theta$	$\sin\theta.\cos\theta$	$\sin^2\theta$
OD	200	30°	150	86.60	50.
OA	200	135°	100	-100	100
OB	266.67	225°	133.33	133.33	133.33
OC	250	300°	62.5	-108.25	187.5
			445.83	11.68	470.83

$$[k] = \begin{bmatrix} 445.83 & 11.68 \\ 11.68 & 470.83 \end{bmatrix}$$

$$[k]^{-1} = \frac{1}{209780.42} \begin{bmatrix} 470.83 & -11.68 \\ -11.68 & 445.83 \end{bmatrix}$$

$$[\Delta] = [k]^{-1} [P - P_L]$$

$$\begin{bmatrix} U_0 \\ U_0 \end{bmatrix} = \begin{bmatrix} 1.18 \\ -2.25 \end{bmatrix}$$

$$P_{OD} = \frac{AE}{L} \left[(U_D - U_0) \cos\theta + (U_D - U_0) \sin\theta \right]$$

$$= 200 \left[1.18 \cos 30° + 2.15 \sin 30° \right]$$

$$= 10.62 \text{ kN (T)}$$

$$P_{OA} = \frac{AE}{L} \left[(U_A - U_0) \cos\theta + (U_A - U_0) \sin\theta \right]$$

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3)

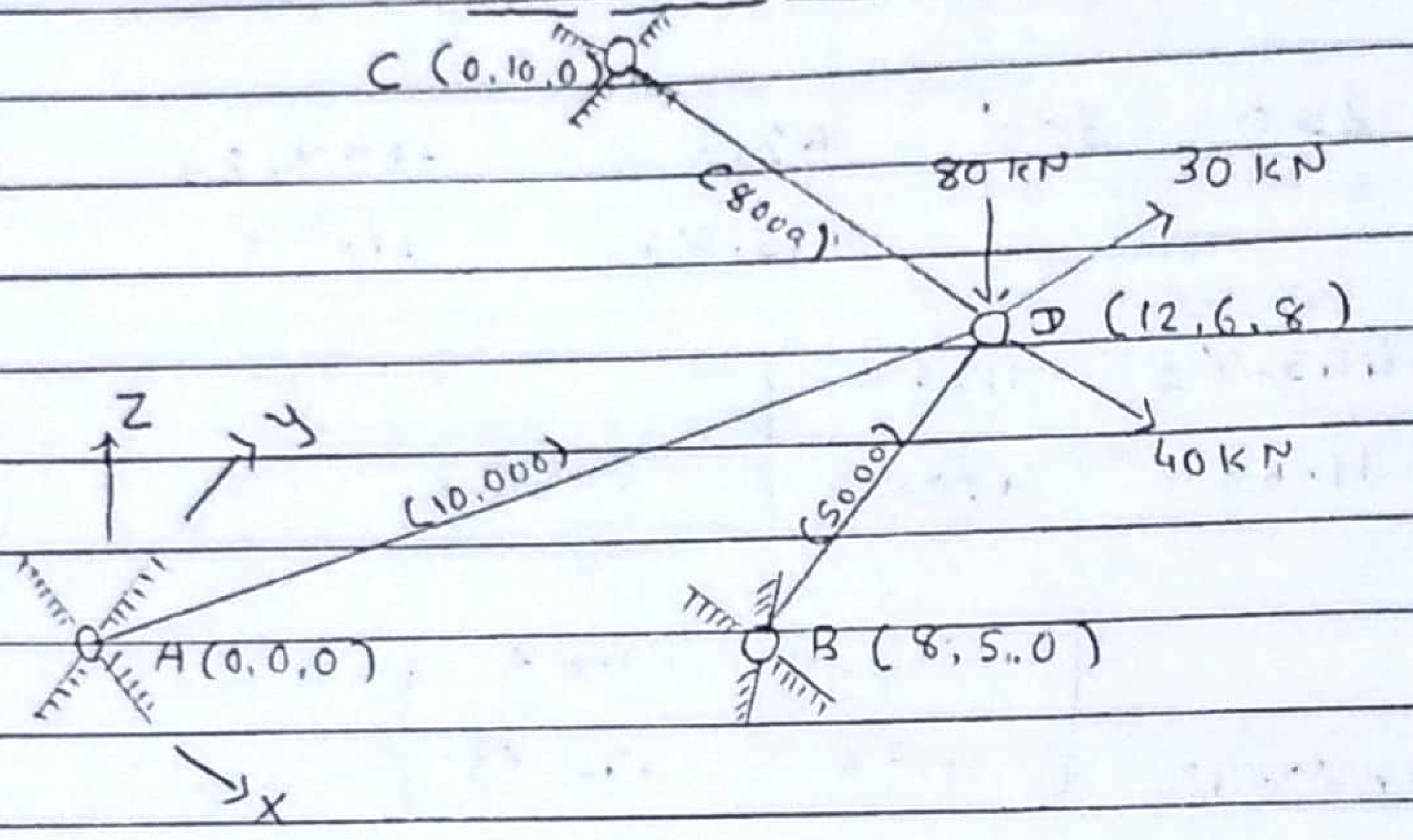
$$P_{OB} : \frac{AE}{L}\left[(U_B - U_0)\cos\theta \ (U_B - U_0)\sin\theta\right]$$

$$= -182.90 \text{ kN (c)}$$

$$P_{OC} : \frac{AE}{L}\left[(U_C - U_0)\cos\theta \ (U_C - U_0)\sin\theta\right]$$

$$= -612.49 \text{ kN (c)}.$$

Pin Joint Spaced Frame



$$L_{DA} = \sqrt{(12)^2+(6)^2+(8)^2} = 15.62 \text{ m}$$

$$L_{DB} = \sqrt{(12-8)^2+(6-5)^2+(8-0)^2} = 9 \text{ m}$$

$$L_{DC} = \sqrt{(12)^2+(6-10)^2+(8)^2} = 14.96 \text{ m}$$

Members	L(m)	A (mm^2)	C_x	C_y	C_z
DA	15.62	10,000	-0.76	-0.38	-0.51
DB	9	5000	-0.44	-0.11	-0.89
DC	14.96	8000	-0.8	+0.26	-0.54

— Distance of DA = $C_x = \frac{x_A - x_D}{L_{AD}} = -0.76$

$= (0-12)/15.62$

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$P_1 = 40$, $P_2 = 30$, $P_3 = 80$.

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$[\Delta] = [K]^{-1} [P]$

$$\begin{bmatrix} \Delta_{1D} \\ \Delta_{2D} \\ \Delta_{3D} \end{bmatrix} = \begin{bmatrix} 3.03 \\ 1.51 \\ -3.52 \end{bmatrix}$$

$$S_{DA} = -\frac{AE}{L} \left[(\Delta_{1D} . C_x) + (\Delta_{2D} . C_y) + (\Delta_{3D} . C_z) \right]$$

$= 140.54$ kN

$$S_{DB} = -\frac{AE}{L} \left[(\Delta_{1D} . C_z) + (\Delta_{2D} . C_y) + (\Delta_{3D} . C_z) \right]$$

$= -180$ kN

$$S_{DC} = -\frac{AE}{L} \left[(\Delta_{1D} . C_x) + (\Delta_{2D} . C_y) + (\Delta_{3D} \, C_z) \right]$$

$= 14.97$ kN