3-ϕ AC circuits

* Introduction:

→ Alternating voltages and current known till now are 1-ϕ voltage and current as it has only one armature wdg (for alternator).
→ But if the armature wdgs are increased then it is known as poly-phase alternator (Poly - many, phase - wdgs or ckts)
→ Accordingly, alternator with one wdg is called 1-ϕ alternator and with two wdgs in armature is called a 2-ϕ alternator. Same way for 3-ϕ.
→ Hence the no. of phases depends on the no. of wdgs.
→ These wdgs are displaced from each other by equal angles which may be obtained from the following relation except for 2-ϕ alternator where two wdgs are displaced by 90° elect.

$$\text{Phase angles} = \frac{360°}{\text{no. of phases or arm. wdgs}}$$

→ For 3-ϕ,

$$\text{phase angles} = \frac{360}{3} = 120° \text{ (elect)}$$

→ 3-ϕ are most common although for certain special applications, greater no. of phases found to be economical.
→ All modern generators are practically 3-ϕ.
→ A polyphase s/m is said to be symmetrical when the various voltages are equal in magnitude and are displaced from one another by equal angles.

* Advantages of 3-ϕ system:

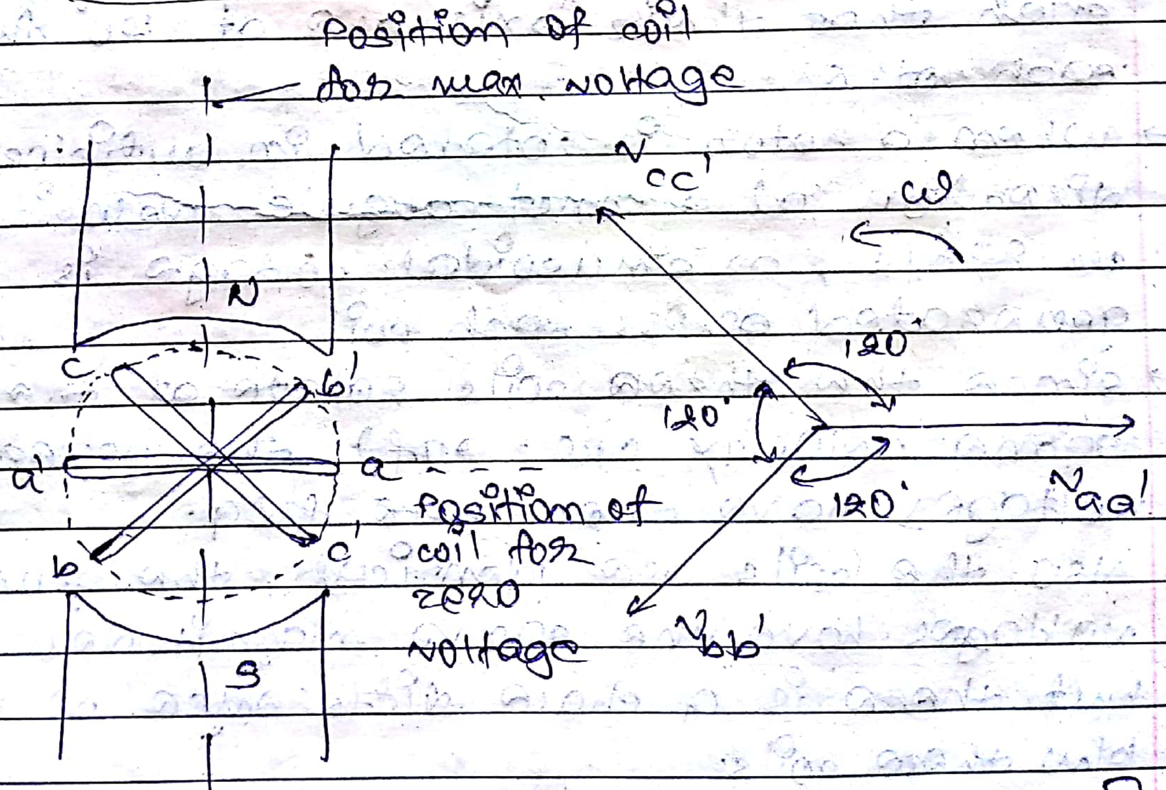
→ 3-ϕ s/m are in common use for generations, x'mission, distribution + utilization.
→ 6-ϕ, + 12-ϕ s/m are used in poly phase rectifiers.
→ 3-ϕ s/m are preferred over 1-ϕ s/m for following reasons:

1. 3-ϕ s/m produce uniform torque whereas 1-ϕ torque pulsates.
2. 3-ϕ s/m produces a rotating magnetic field. so, 3-ϕ motors are simpler in construction, smaller in size, better operating characteristics.
3. To transmit the same amount of power over a fixed distance at a given voltage 3-ϕ s/m requires only 3/4th weight of copper compared to required in 1-ϕ s/m.
4. voltage regulation is better.
5. The domestic power and industrial or commercial power can be supplied from the same source.
6. 3-ϕ motors are self starting.
7. A 3-ϕ m/c gives more o/p than a 1-ϕ m/c of the same size.

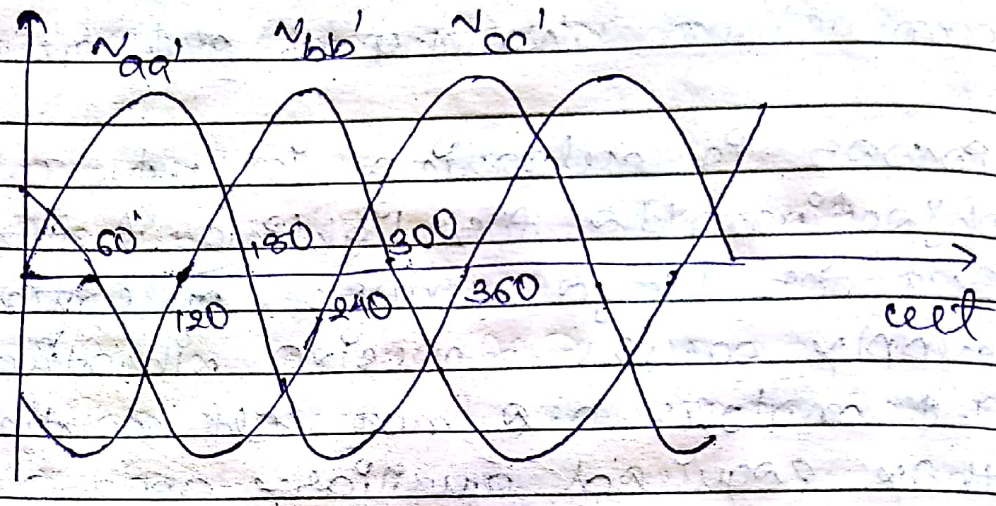
* comparison b/w 1-ϕ & 3-ϕ supply s/m:

→ Power is pulsating in 1-ϕ. This is not objectionable for lighting & for small motors. But for large motors, pulsating power supply causes excessive vibrations.
→ 1-ϕ motors are not self starting, hence they required auxiliary apparatus for self starting. This is not required in 3-ϕ.
→ Power factor in 1-ϕ motor is less.
→ For same size, 3-ϕ motor can generate more o/p.
→ In 3-ϕ x'mission, the weight of copper wire is $3/4^{th}$ of 1-ϕ.

* Generation of 3-ϕ supply:



[Fig. 58 Generation of 3-ϕ supply]



→ When 3 identical coils (or armature winding) are placed with their axis at 120° apart from each other and rotated in a uniform magnetic field a sinusoidal voltage is generated across each coil.

→ Fig. 58 shows a 3-ϕ 2 pole generator.

→ It has sets of coil aa', bb' and cc' symmetrically mounted on a rotor such that their axis are at 120° from each other.

→ When a rotor is rotated in anticlockwise direction at a const. angular velocity ω rad/s, a sinusoidal voltage is generated across each coil.

→ Since the three coils rotate at the same velocity ($\omega = 2\pi f$), the generated voltages have the same freq.

→ Also the coils are identical, the generated voltages have the same magnitudes, but there is a phase difference of 120° b/w these coils.

→ The generated voltages in the coils are given by,

$$e_a = E_m \sin \omega t$$
$$e_b = E_m \sin(\omega t - 120^\circ)$$
$$e_c = E_m \sin(\omega t - 240^\circ)$$

* Phase sequence:

→ The sequence in which the voltages in the 3 phases reach their max. positive value is called the phase sequence or phase order.
→ From a ~~3-φ~~ phase diagram of 3-φ s/m, it is clear that the voltage of coil 'a' attains max. value first and then in coil 'b' and 'c'.
→ So, the phase sequence is a → b → c.

* Balanced supply and Balanced Load:

→ A 3-φ s/m is said to be balanced if
1. voltages in the ~~3-φ~~ 3 phases are equal in magnitude + displaced in phase from one another by 120°
2. currents in the 3 phase are equal in magnitude and differ in phase from one another by 120°
3. load connected across the three phases are identical i.e. all the loads per phase have the same magnitude + power factor.

Star or wye connection:

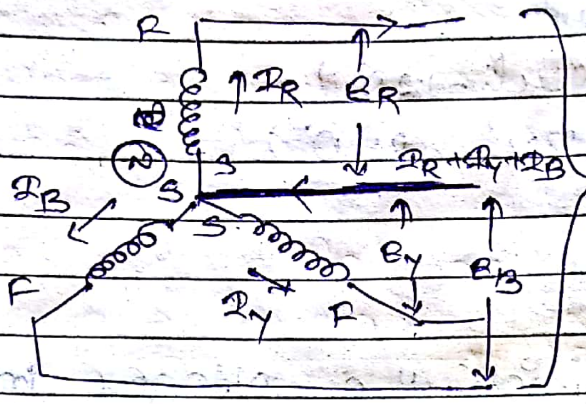


Fig. 59

star connection

→ In this method, the interconnection of the similar ends of three coils (starting or finishing end) are joined together at point (N) as shown in fig. 59.

→ The point N is known as star point or neutral point.

→ The 3 conductors meeting at a junction N are replaced by a single conductor known as neutral conductor.

→ Such an interconnection s/m is known as 3-ϕ, 4-wire s/m.

→ If this 3-ϕ voltage s/m is applied to a balanced load system then the neutral wire will be carrying three currents equal in magnitude but are 120° out of phase with each other.

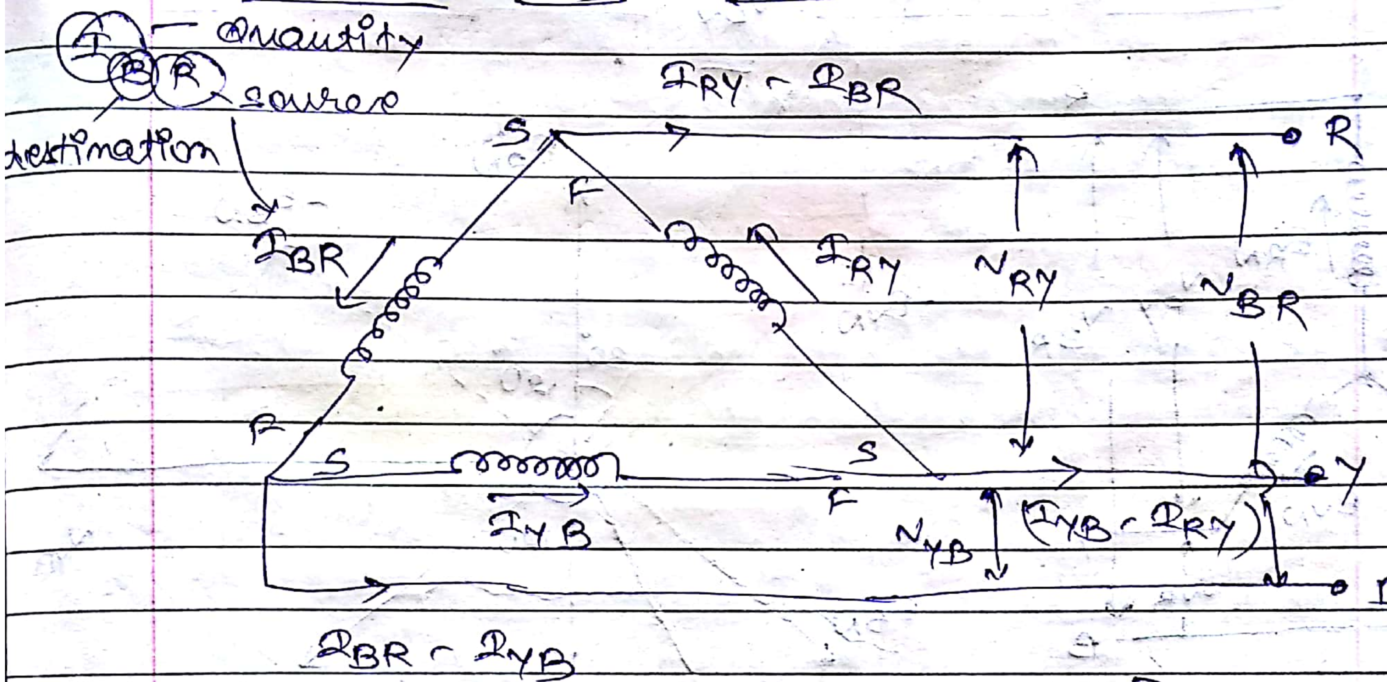
→ Hence, the vector sum is zero.

i.e. $I_R + I_Y + I_B = 0$ (vectorially)

→ The voltage induced in each wdg is called the phase voltage (V_{ph}) and current in each wdg is known as phase current (I_{ph}).

→ However, the voltage available between any pairs of terminals is called line voltage (V_L) and current flowing in each line is called line current (I_L).

* Delta or Mesh connection:



[Fig. 60 Delta connection]

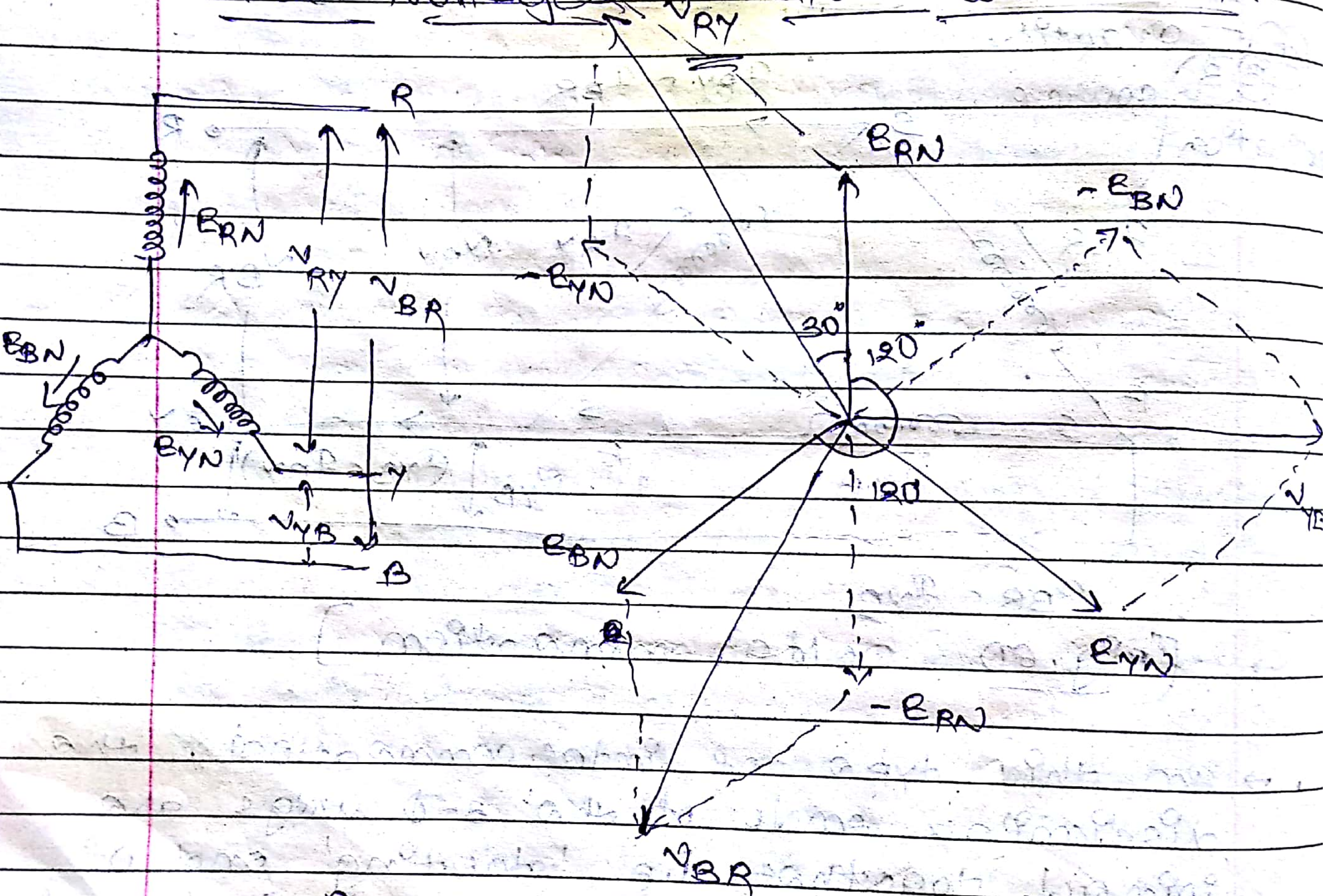
→ In this type of interconnection, the dissimilar ends of the 3-Φ wdgs are joined together i.e, 'starting' end of one phase is joined to 'finishing' end of the other phase and so on as shown in fig. 60.

→ Three leads are taken out from the junctions & outward ~~junctions~~ directions are taken as +ve.

→ As this is a balanced s/m, the sum of the three voltages around the closed path is zero.

→ Hence, no current can flow around the mesh when terminals are open.

* Relationship b/w line voltages and phase voltages in star connection:



[Fig. 61 star connection]

→ Fig. 61 shows a balanced 3-φ star connected s/m.

→ E_{RN}, E_{YN}, E_{BN} = rms values of emfs (phase voltages) generated in 3-φ

Kaushal Patel

→ As it is balanced s/m, the magnitude of all emf will be same but differ in phase to one another by 120°

→ So, $E_{RN} = E_{YN} = E_{BN} = E_{ph}$

$E_{RN} = E_{ph} \angle 0°$

$E_{YN} = E_{ph} \angle -120°$

$E_{BN} = E_{ph} \angle -240°$

→ It is clear from the fig. that voltage b/w any two line terminals is the phasor difference b/w voltages of three terminals w.r.t. to neutral points.

→ Voltage b/w line R and Y,

$V_{RY} = E_{RN} + E_{NY}$ (phasor sum)

$= E_{RN} - E_{YN}$

→ Similarly,

$V_{YB} = E_{YN} + E_{NB}$

$= E_{YN} - E_{BN}$

& $V_{BR} = E_{BN} + E_{NR}$

$= E_{BN} - E_{RN}$

→ To find V_{RY}, take the phasor sum of E_{RN} & E_{YN}. Draw reverse ~~Ey~~ E_{YN}. So, the difference of angle b/w phasors are 60°

→ Thus, two phasors E_{RN} & $-E_{YN}$ are equal in magnitude (E_{ph}) and are 60° apart.

→ Hence, $V_{RY} = 2 E_{ph} \cos\left(\frac{60°}{2}\right)$ (Parallelogram law)

$= 2 E_{ph} \cos 30°$

$= 2\ E_{ph} \times \frac{\sqrt{3}}{2}$

$\boxed{V_{RY} = \sqrt{3}\ E_{ph}}$

→ Similarly,

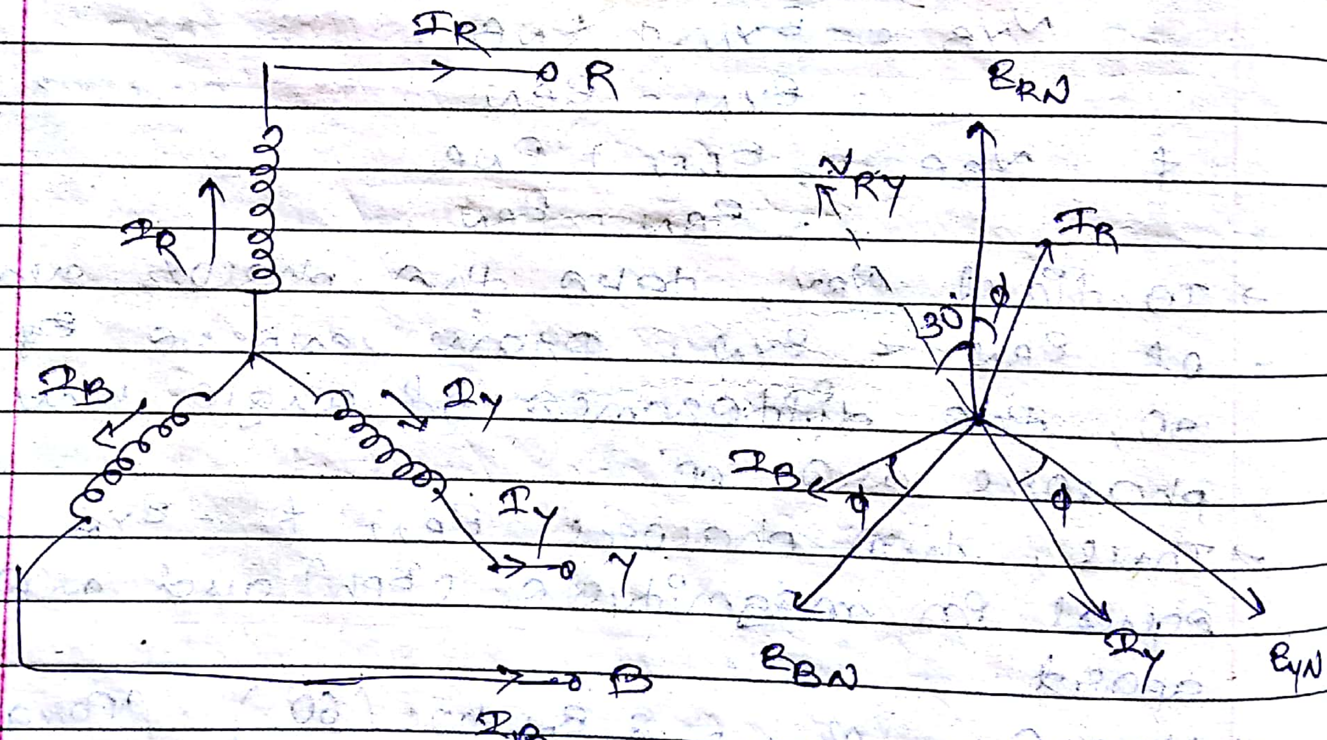
$\boxed{\begin{array}{l} V_{YB} = \sqrt{3}\ E_{ph} \\ V_{BR} = \sqrt{3}\ E_{ph} \end{array}}$

→ Now, $V_{RY} = V_{YB} = V_{BR} =$ line voltage (say V_L)

→ So, in star connection,

$\boxed{V_L = \sqrt{3}\ E_{ph}}$

* Relationship between line currents and phase currents:



[Fig. 62 line currents & phase currents in star connection]

→ It is seen from fig. 82, that each line is in series with its individual phase winding. Assuming balanced system,
current in line R = I_R
current in line Y = I_Y
current in line B = I_B
∴ $I_R = I_Y = I_B$ = line current I_L

→ similarly,
current in phase winding R = I_R
current in phase winding Y = I_Y
current in phase winding B = I_B

→ currents in phase winding,
$I_R = I_Y = I_B$ = phase current I_{ph}

→ As phase windings and lines are in series, so same current will flow through them.
→ so,
$I_L = I_{ph}$
→ Hence, in balanced system,
(1) Line current I_L = Phase current I_{ph}
(2) All currents are equal in magnitude and displaced from each other by 120°.
(3) The angle b/w line current and line voltage is $30° \pm \phi$.
+ for lagging p.f.
− for leading p.f.

* Expression of power in star connection

→ Total power $P = 3 \times$ power in each phase

$= 3 \times E_{ph} I_{ph} \cos\phi$

$= 3 E_{ph} I_{ph} \cos\phi$

→ Now, for star connection,

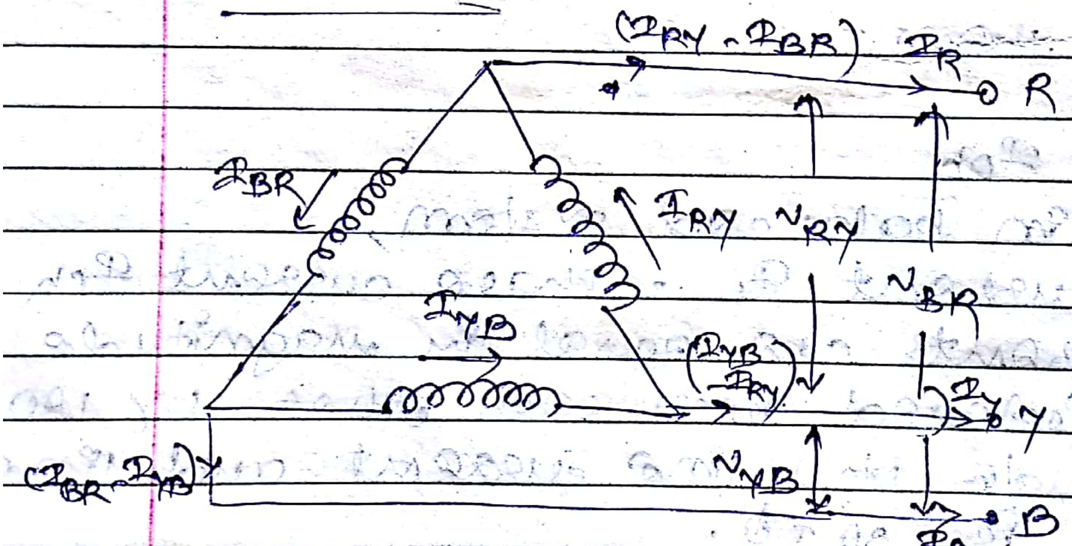
$E_{ph} = \frac{V_L}{\sqrt{3}}$ & $I_{ph} = I_L$

So,

$P = 3 \frac{V_L}{\sqrt{3}} I_L \cos\phi$

$$\boxed{P = \sqrt{3} V_L I_L \cos\phi}$$

* Relationship between line voltages & phase voltages in balanced delta connection.



[Fig. 63 Delta connection]

→ As due to absence of neutral point, phase voltage will be equal to line voltage.

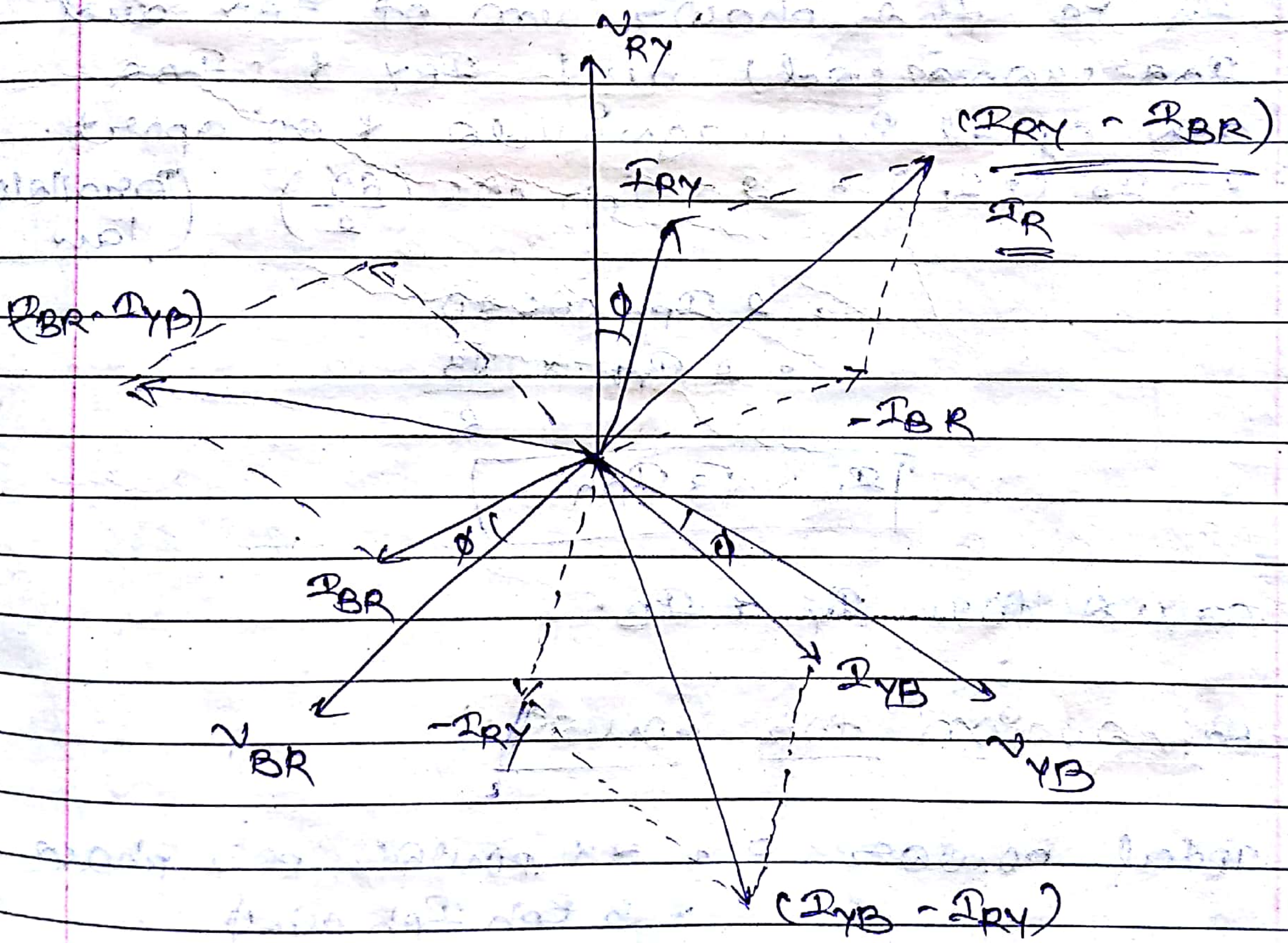
→ And due to balanced system, all line voltages are equal in magnitude and displaced from each other at 120° angle.

→ So,

$V_{RY} = V_{YB} = V_{BR} = V_L$ line voltage

and $V_{ph} = V_L$.

* Relationship between line current & phase current:



(Fig. 64 phasor diagram of delta connection)

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→ Since, the s/m is balanced; all the three phase currents are equal in magnitude and displaced from each other by angle 120°

→ So,

$I_{RY} = I_{YB} = I_{BR} = I_{ph}$

→ For each line, the current is the difference of two currents.

So,

$I_R = I_{RY} - I_{BR}$

$I_Y = I_{YB} - I_{RY}$

$I_B = I_{BR} - I_{YB}$

→ I_R is the phasor sum of I_{RY} and I_{BR} (reversed) and I_{RY} & $-I_{BR}$ are equal in magnitude & 60° apart.

$\therefore I_R (= I_L) = 2 \times I_{ph} \cos\left(\frac{60°}{2}\right)$ (Parallelogram law)

$= 2 I_{ph} \cos 30°$

$= 2 I_{ph} \times \frac{\sqrt{3}}{2}$

$$\boxed{I_L = \sqrt{3} I_{ph}}$$

→ same for I_Y & I_B.

* Expression for power:

→ Total power $P = 3 \times$ power per phase

$= 3 V_{ph} I_{ph} \cos\phi$

→ For delta connection,

$$V_L = E_{ph}$$

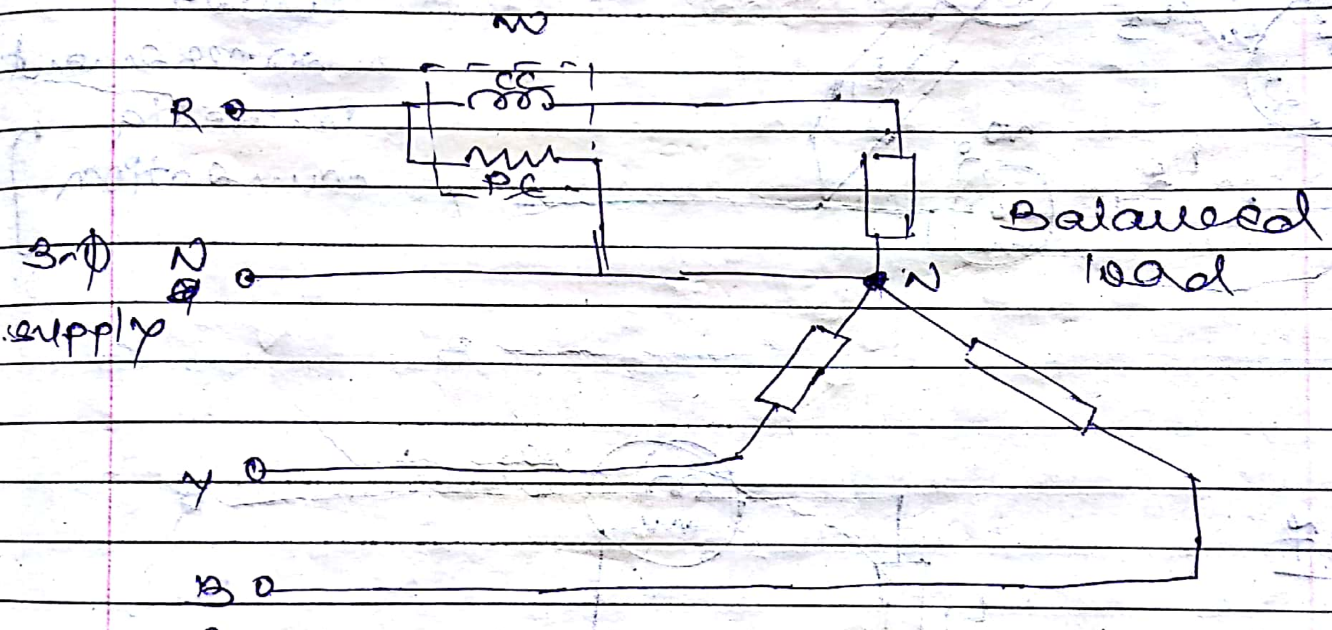
$$I_{ph} = \frac{I_L}{\sqrt{3}}$$

→ so, $P = 3 \times V_L \times \frac{I_L}{\sqrt{3}} \times \cos\phi$

$$= \sqrt{3} \cdot V_L I_L \cos\phi$$

* Measurement of power in 3-ϕ circuit:

① One wattmeter method:



[Fig. 85 Power measurement using a single wattmeter.]

→ condition: Balanced load should be there.

→ The wattmeter is connected in any one phase and the reading is taken.

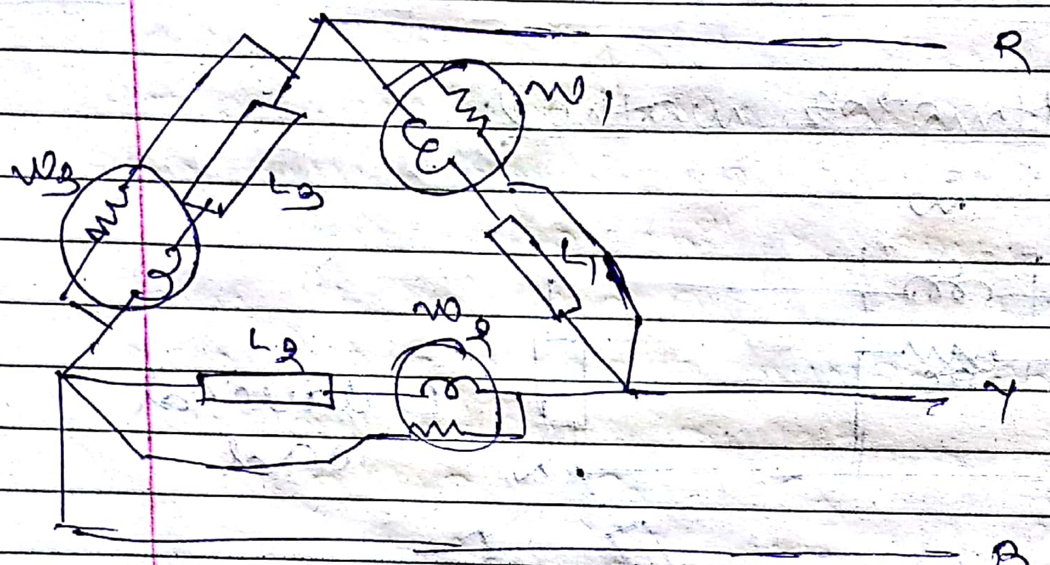
→ Then this reading is multiplied with

no. of phases to find total power in the circuit.

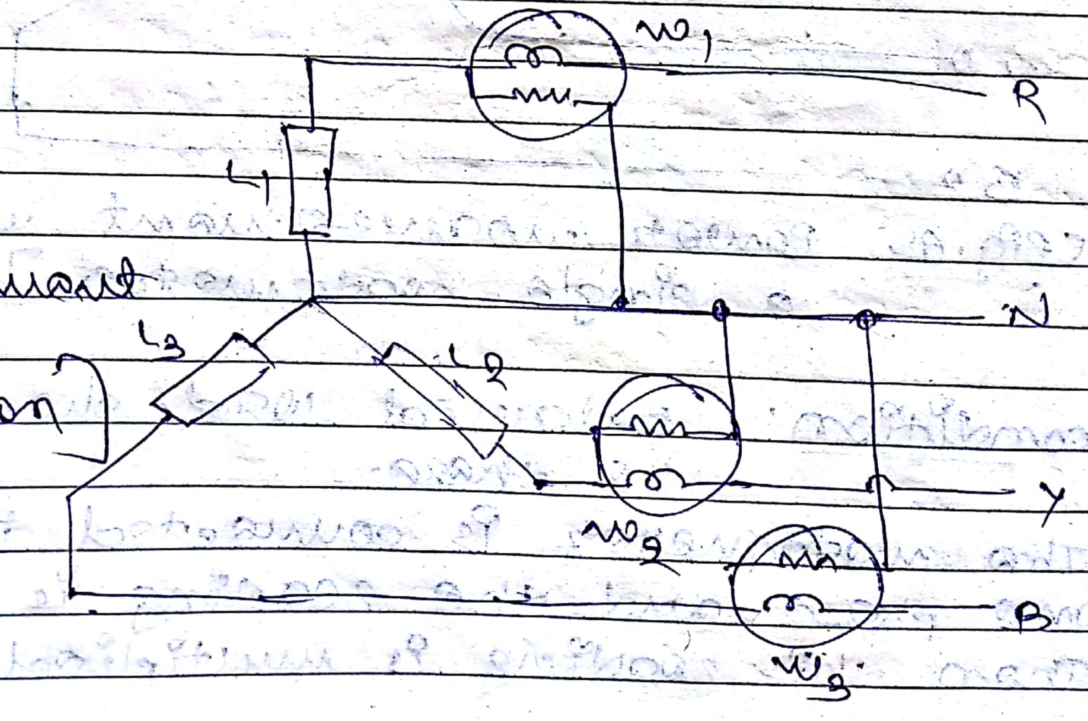
→ so,

total power $P = 3 \times$ power per phase
$= 3 \times$ wattmeter reading

② Three wattmeter method!



[Fig. 66 Power measurement in delta connection]



[Fig. 67 Power measurement in star connection]

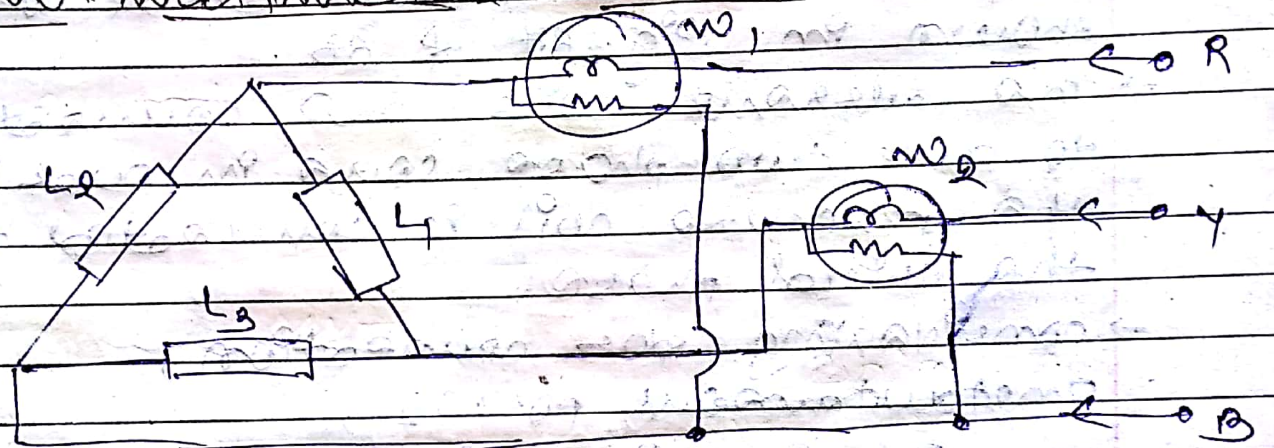
→ This method can be applicable to balanced and unbalanced s/m both.

→ The total power,

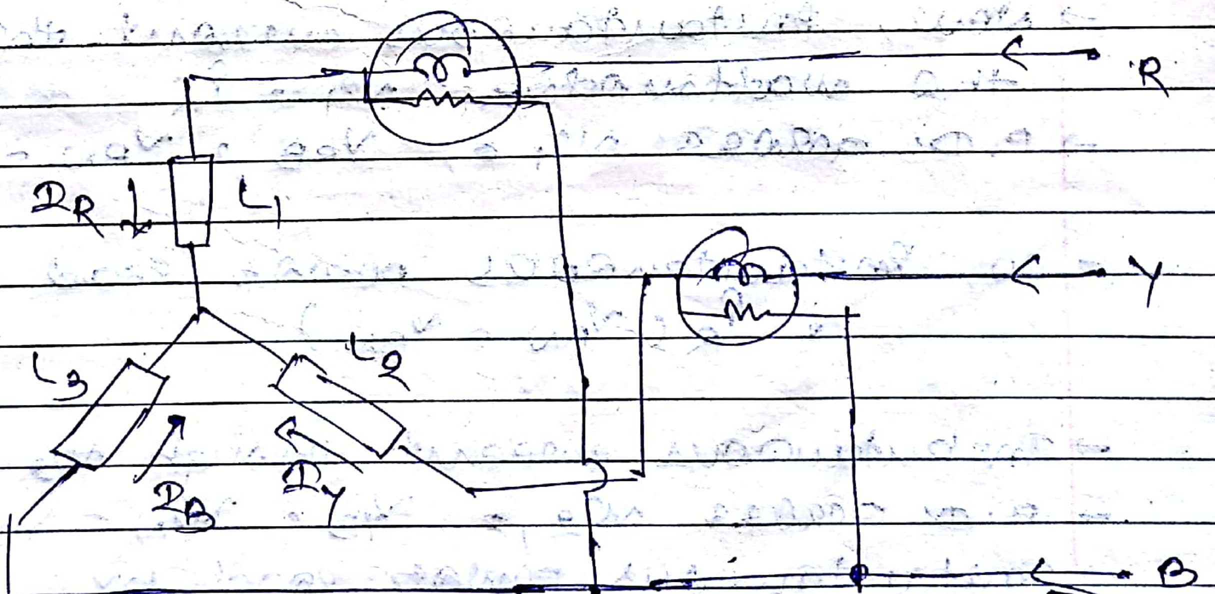
$P = W_1 + W_2 + W_3$ (algebraic sum)

→ If neutral is not provided in star then the potential (pressure) coil is been grounded.

(3) Two wattmeter method:



[Fig. 67 2 wattmeter method for delta connected load]



[Fig. 68 2 wattmeter method for star connected load]

→ This method of power measurement can be applied to 3-phase, 3-wire s/m (balanced/unbalanced, star/delta) and 3-phase, 4-wire s/m (balanced). (not in unbalanced as $I_1 + I_2 + I_3 \neq 0$ in unbalanced load, so power differs)

→ The connections of two wattmeters for star & delta connected load is shown in fig. 67 & 68.
→ The current coils are connected to any two phase (line in each) and the pressure coil is connected to the third phase.
→ considering star connection, instantaneous power,

$$P = P_1 + P_2 + P_3$$
$$= v_{RN} i_R + v_{YN} i_Y + v_{BN} i_B$$

→ Now, instantaneous current through the wattmeter $W_1 = i_R$
→ P.D. across W_1 = $v_{RB} = v_{RN} - v_{BN}$

→ so, instantaneous power read by W_1
$$= i_R (v_{RN} - v_{BN})$$

→ Instantaneous current through $W_2 = i_Y$
→ P.D. across W_2 = $v_{YB} = v_{YN} - v_{BN}$
→ Instantaneous power read by
$$W_2 = i_Y (v_{YN} - v_{BN})$$

Kaushal Patel

→ so, $W_1 + W_2 = i_R (v_{RN} - v_{BN}) + i_Y (v_{YN} - v_{BN})$

$= i_R v_{RN} - i_R v_{BN} + i_Y v_{YN} - i_Y v_{BN}$

$= i_R v_{RN} + i_Y v_{YN} - (i_R v_{BN} + i_Y v_{BN})$

$= i_R v_{RN} + i_Y v_{YN} - (i_R + i_Y) v_{BN}.$

→ Now, $i_R + i_Y + i_B = 0$ (KCL)

$\therefore i_R + i_Y = -i_B$

→ so, $W_1 + W_2 = i_R v_{RN} + i_Y v_{YN} + i_B v_{BN}$

$= P_1 + P_2 + P_3$