Series circuits:

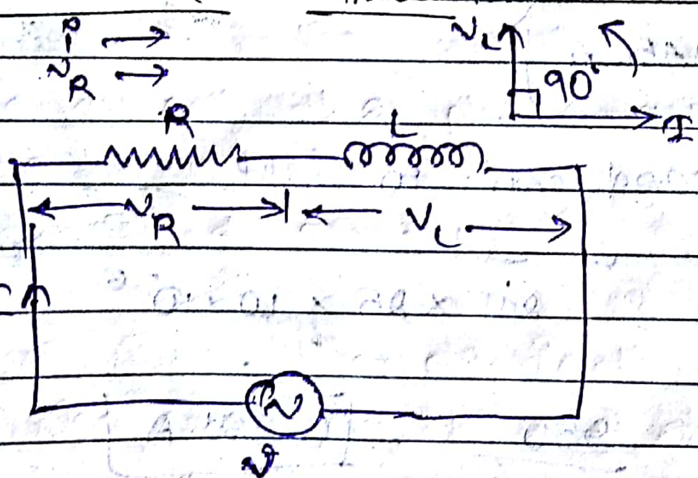
* Introduction:

→ In the previous sections, a detailed discussion has been dealt with the effects produced by each of the three fundamental elements R, L & C when a sinusoidal emf is applied.

→ However, practical ckt normally comprise of the above elements connected in combination. For example, a coil can be represented by its resistance & inductance connected in series. Thus coil can be considered as a series combination of the fundamental elements R & L.

→ We will consider various series combinations for analysis of ac circuits.

* R-L series circuit:



[Fig.36 R-L series circuit]

* Addition & Subtraction of vectors:

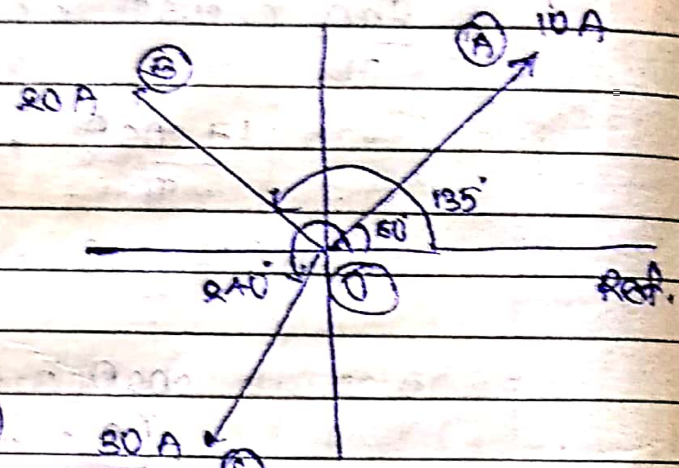
(1) Addition:

→ Suppose we are given the following three alternating current & it is required to find the equation of resultant current.

$i_1 = 10 \sin(\omega t + \pi/3)$

$i_2 = 20 \sin(\omega t + 3\pi/4)$

$i_3 = 30 \sin(\omega t + 4\pi/3)$



[Fig.18 vector of i_1, i_2 & i_3]

→ The current can be written in vector forms as under.

$i_1 = 10 \sin(\omega t + 60°) = 10\angle 60° = OA$

$i_2 = 20 \sin(\omega t + 135°) = 20\angle 135° = OB$

$i_3 = 30 \sin(\omega t + 240°) = 30\angle 240° = OC$

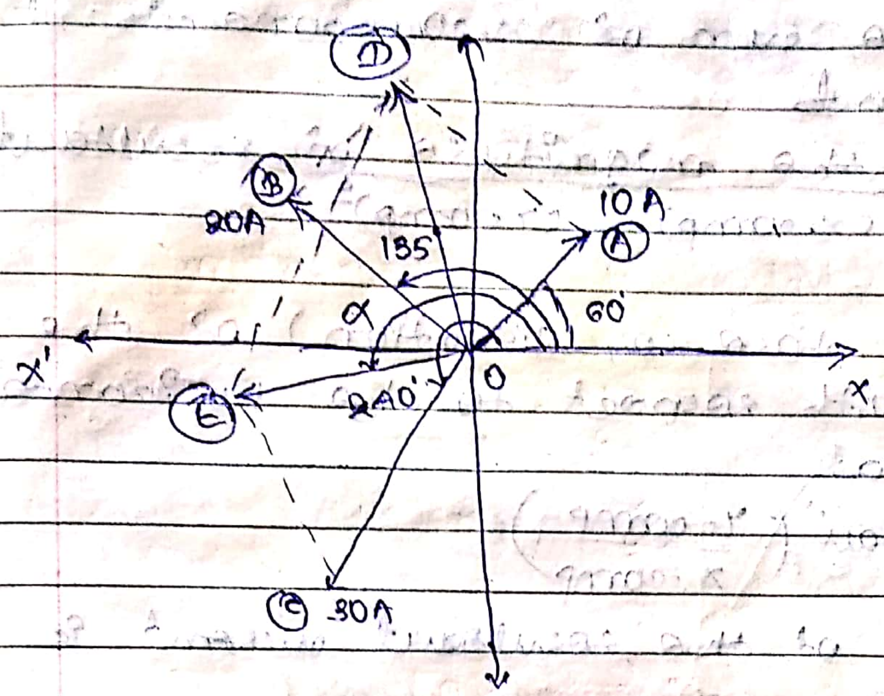
→ Now, the resultant as addition of the three currents can be obtained by any of the following methods:

(A) By compounding according to parallelogram law:

steps:

(1) Obtain the resultant of vector OA & OB as OD.

(2) Obtain the resultant of OD & OC as OE which is the resultant of vectors OA, OB & OC.

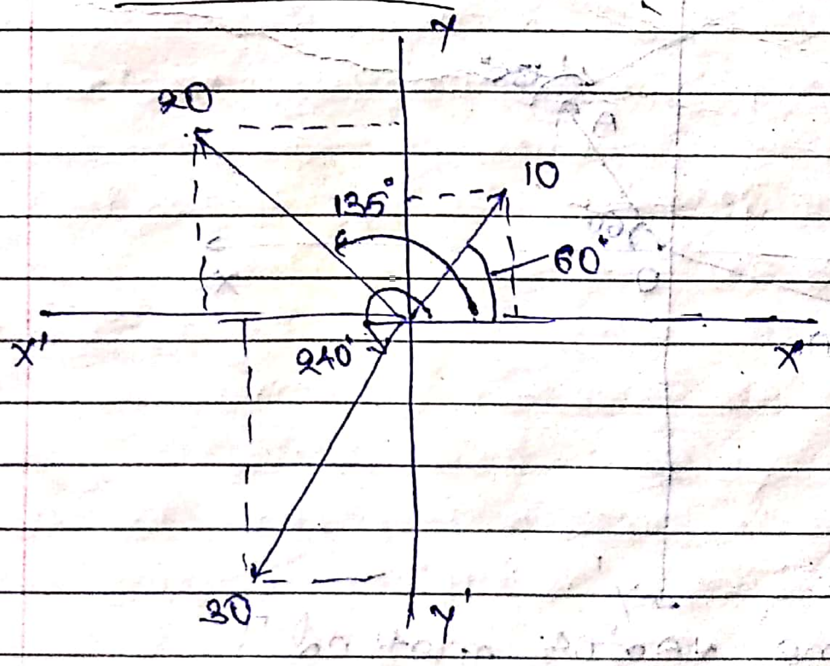


[Fig. 19 compounding of vector]

(3) Measure α which gives its phase with horizontal axis.

(4) The equation of resultant current is
$i_R = I_m \sin(\omega t + \alpha)$

(B) By resolving the various vector into their x-components & y-components



[Fig. 20 resolving vector]

Steps: (1) Determine sum of x-components & y-components

(2) Determine the magnitude of resultant

$I_m = \sqrt{(x\text{-comp})^2 + (y\text{-comp})^2}$

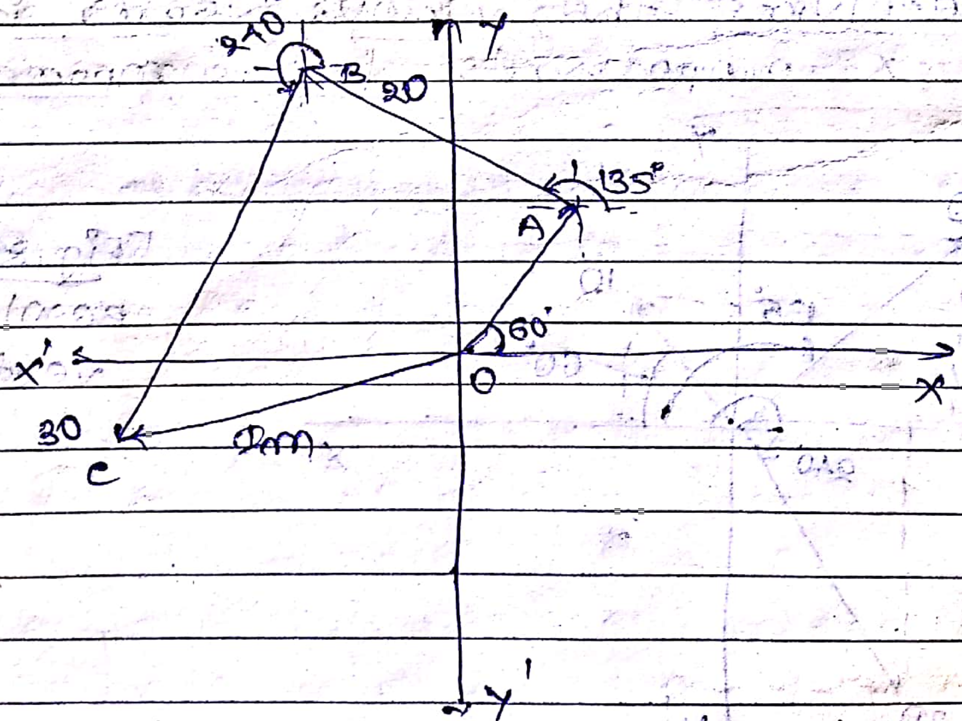
(3) Determine phase (or direction) of the resultant with respect to the horizontal (or x-axis) as

$$\alpha = \tan^{-1}\left(\frac{y\text{-comp}}{x\text{-comp}}\right)$$

(4) The equation of the resultant current is given by,

$i = i_R = I_m \sin(\omega t + \alpha)$

(c) By laying the various vector end-on-end at their proper phase angles and then measuring the closing vector:

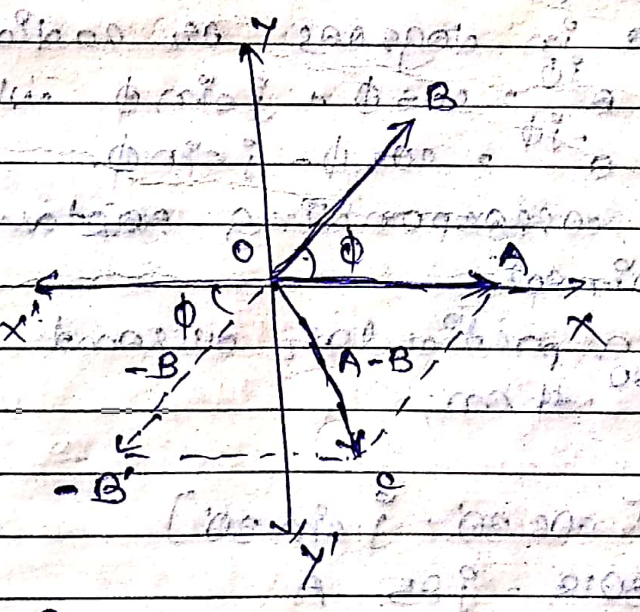


[Fig. 2) closing vector method]

steps: (1) Draw phasor OA at angle 60° with x-axis.
(2) At A, draw the vector AB at an angle 135° with x-axis.
(3) At B, draw the vector BC at angle 240° with x-axis
(4) Join C with origin to get the resultant vector OC. Its inclination with x-axis is given by α.
(5) Here, the equation of resultant current is given by, $i_R = I_m \sin(\omega t + \alpha)$ where $I_m = OC$

(2) Subtraction of vectors:

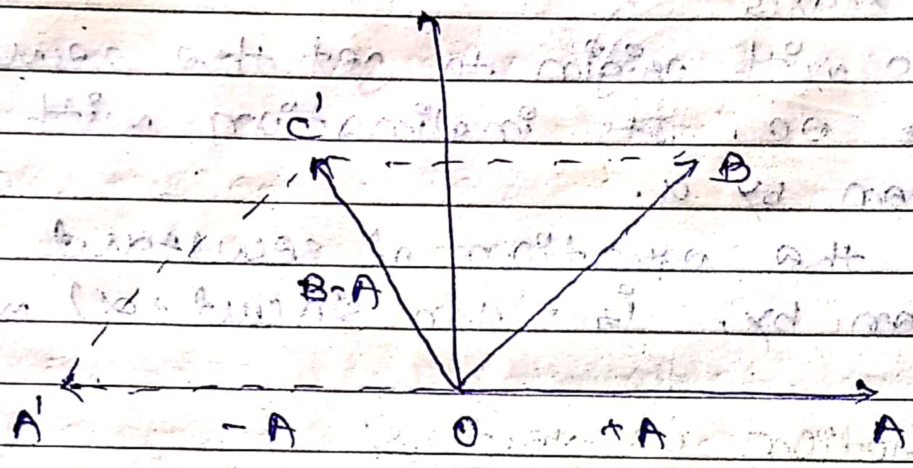
→ If difference of two vectors is required, one of the vector is reversed and this reversed vector is then compounded with the other vector as usual.



[Fig.22 subtraction of vectors]

→ Suppose two vector OA & OB are given as shown in fig.
→ Suppose, it is required to subtract vector OB from OA.
→ So, reverse the vector OB as shown in fig. and obtain the resultant of vector OA & OB' according to parallelogram law.

→ The vector difference (OA − OB) is given by the vector OC.
→ Similarly, vector difference (OB − OA) can be obtained as OC' as shown in fig. 23.



[Fig. 23]

* Another way of complex no. Representation:

→ In complex number analysis, the ~~analysis~~ number may be expressed as $|z| e^{j\phi}$ where ϕ may be in degrees or radians. Mathematically, $e^{j\phi} = \cos\phi + j\sin\phi$ while $e^{-j\phi} = \cos\phi - j\sin\phi$.
Using these eq's. corresponding rectangular form can be obtained.
→ For example, if a particular current is given as $50\, e^{-j30}$ then,

$$50\, e^{-j30} = 50\,[\cos 30° - j \sin 30°]$$
$$= 43.3012 - j25\ A$$

While particular voltage is given by $150\, e^{+j100}$ is

$$150\, e^{+j100} = 150\,[\cos 100° + j\sin 100°]$$
$$= -26.047 + j\,147.721\ V$$

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→ If ϕ is given in radians, sin & cos must be calculated in radian mode.
→ Infact, $|z| e^{\pm j\phi}$ can be directly expressed in polar form as $|z| \angle \pm\phi$ where ϕ may be expressed ~~as~~ ~~do~~ in degrees or radians

$$\therefore 50 e^{-j30} = 50 \angle -30^\circ$$

while $150 e^{+j100} = 150 \angle 100^\circ$

→ This can be cross checked by using rectangular to polar conversion.

$$|z| e^{\pm j\phi} = |z| \angle \pm\phi$$

Exa. Two current $I_1 = 10 e^{j50^\circ}$ and $I_2 = 5 e^{-j100^\circ}$ flow in a 1ϕ AC ckt.
Estimate (1) $I_1 + I_2$ (2) $I_1 - I_2$ & (3) I_1 / I_2 in complex form.

Ans $I_1 = 10 [\cos 50^\circ + j \sin 50^\circ]$
$= 6.4278 + j\, 7.66$ A
$= 10 \angle 50^\circ$ A
$I_2 = 5 [\cos 100^\circ - j \sin 100^\circ]$
$= -0.8682 - j\, 4.924$
$= 5 \angle -100^\circ$ A

(1) $I_1 + I_2 = [6.4278 + j\,7.66] + [-0.8682 - j\,4.924]$
$= 5.5596 + j\, 2.736$ A
$= 6.196 \angle 26.2^\circ$ A

(2) $I_1 - I_2 = [6.4278 + j\,7.66] - [-0.8682 - j\,4.924]$

$= 7.296 + j\,12.584$ A

$= 14.546 \angle 60^\circ$ A

(3) $I_1 / I_2 = \dfrac{10 \angle 50^\circ}{5 \angle -100^\circ} = \dfrac{10}{5} \angle 50^\circ + 100^\circ$

$= 2 \angle +150^\circ$ A

* Polar system: $r \angle \pm \phi$
* Rectangular system: $x + jy$

* convert polar to rectangular system
$x = r\cos\phi \quad , \quad y = r\sin\phi$

* convert rectangular to polar system
$r = \sqrt{x^2 + y^2}$
$\phi = \tan^{-1}\left(\dfrac{y}{x}\right)$

* Multiplication & Division of phasors:

→ Addition & subtraction of phasors is discussed, which is to be carried out using rectangular form of phasors. But the rectangular form is not suitable to perform multiplication & division of phasors. Hence multiplication & division must be performed using polar form of phasors.

→ Let P and Q be the two phasors such that,
$P = x_1 + jy_1$ and $Q = x_2 + jy_2$

→ To obtain the multiplication $P \times Q$, both must be expressed in polar form.

$P = r_1 \angle \phi_1$ and $Q = r_2 \angle \phi_2$

then $P \times Q = [r_1 \angle \phi_1] \times [r_2 \angle \phi_2]$

$= [r_1 \times r_2] \angle \phi_1 + \phi_2$

note: In multiplication of complex number in polar form, the magnitudes get multiplied while their angles get added.

→ The result then can be expressed back to rectangular form, if required.

→ Now, consider the division of the phasors P and Q.

$$\frac{P}{Q} = \frac{r_1 \angle \phi_1}{r_2 \angle \phi_2} = \left|\frac{r_1}{r_2}\right| \angle \phi_1 - \phi_2$$

note: In division of complex numbers in polar form, the magnitudes get divides while their angles get ~~substract~~ subtracted.

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Unit -2

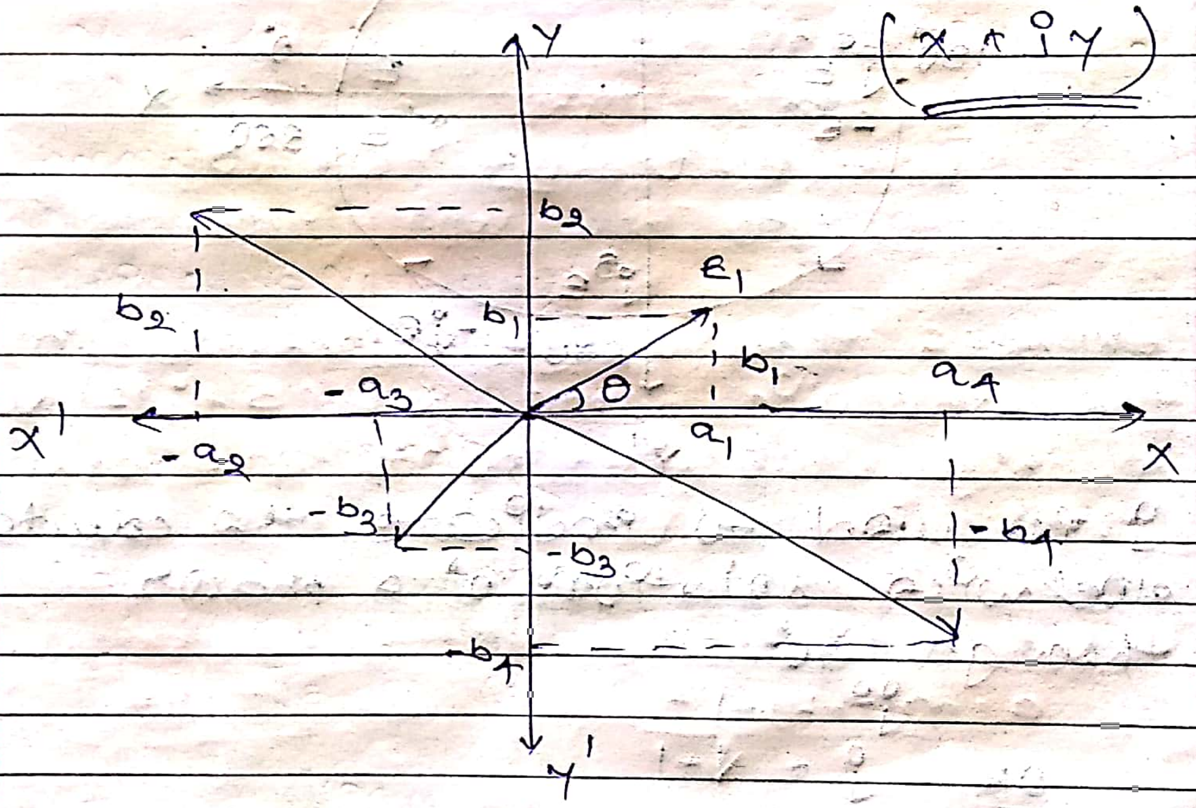
Complex Algebra

* 4 ways of representation of vector

1. Symbolic notation or Rectangular or cartesian form
2. Trigonometric form
3. Exponential form
4. Polar form

(1) Symbolic or Rectangular :

$(x + iy)$

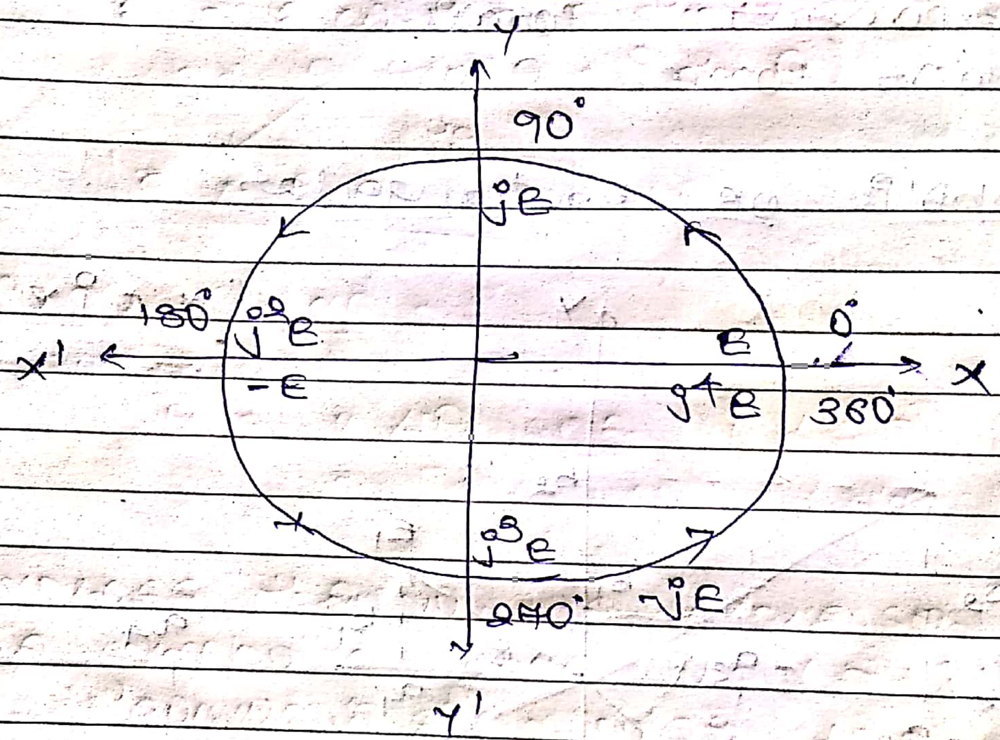


$OE_1 = a_1 + jb_1$ —— (1)
$OE_2 = -a_2 + jb_2$ —— (2)
$OE_3 = -a_3 - jb_3$ —— (3)
$OE_4 = a_4 - jb_4$ —— (4)

→ length of vector = $\sqrt{a^2+b^2}$

→ Angle with +ve x-axis = $\theta = \tan^{-1}\left(\frac{b_1}{a_1}\right)$

* Significance of j-operator:



→ j is used to indicate the counter clockwise rotation of a phasor through 90°.

$$j^2 = -1$$

OR $j = \sqrt{-1}$

$\therefore j^2E = -E$

and $\frac{1}{j} = \frac{j}{j^2} = \frac{j}{-1} = -j$

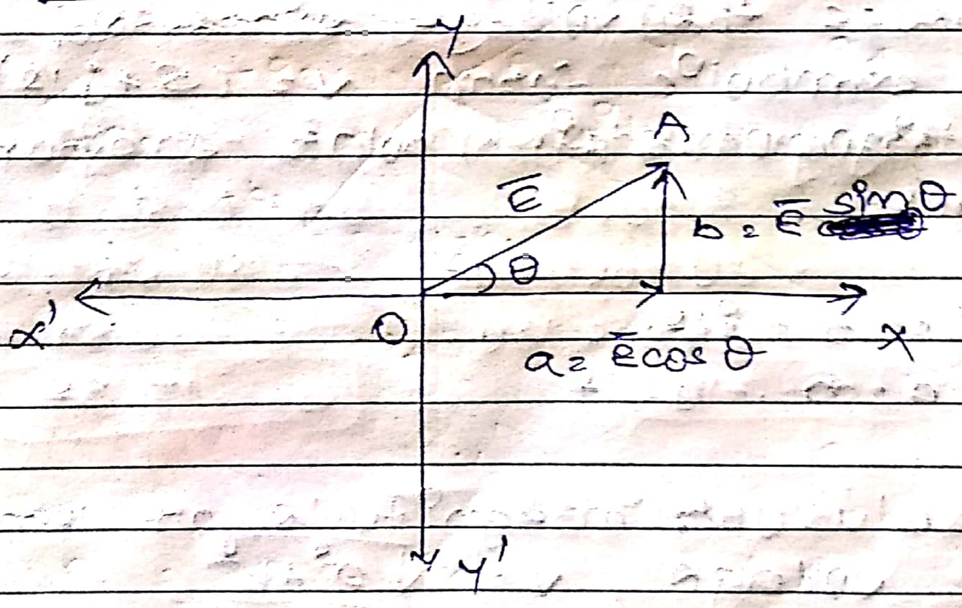
→ complex no. $\bar{A} = a + jb$

~~[struck-out word]~~ $Re(\bar{A}) = a$ (Real)
$Im(\bar{A}) = jb$ (Imaginary)

→ $(a+jb) + (a-jb) = 2a$ = active comp
$(a+jb) - (a-jb) = j2b$ = Reactive comp.

$$(a+jb) \times (a-jb) = a^2 - (jb)^2$$
$$= a^2 - j^2 b^2$$
$$= a^2 + b^2$$
= active comp.

✗ Trigonometric form of phasor representation:



$$\bar{E} = E\cos\theta + j E\sin\theta \qquad (a + jb)$$

* **Exponential form:**

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$\bar{E} = E\cos\theta \pm jE\sin\theta$$
$$= E(\cos\theta \pm j\sin\theta)$$
$$\boxed{\bar{E} = E e^{\pm j\theta}}$$

* **Polar form:**

$$\bar{E} = E\angle\theta \quad \text{OR} \quad E\angle\pm\theta$$

Ex.1 Express the voltage expressed in the symbolic form $v = (5 + j12)$ volts in trigonometric, polar & exponential forms.

A. $v = 5 + j12$
$a = 5, \ b = 12$

modulus or magnitude or length of voltage, $v = \sqrt{a^2 + b^2}$
$= \sqrt{5^2 + 12^2}$
$= 13 \text{ V}$

phase angle $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

$= \tan^{-1}\left(\frac{12}{5}\right)$

$= 67.38^\circ$

$= 1.176$ rad

(1) Trigonometric form

$\bar{V} = V(\cos\theta + j\sin\theta)$

$= 13(\cos 67.38 + j\sin 67.38)$ V

(2) Polar form $\bar{V} = V\angle\theta$

$= 13\angle 67.38^\circ$ V

(3) Exponential form $\bar{V} = Ve^{+j\theta}$

$= 13e^{j1.176}$ V

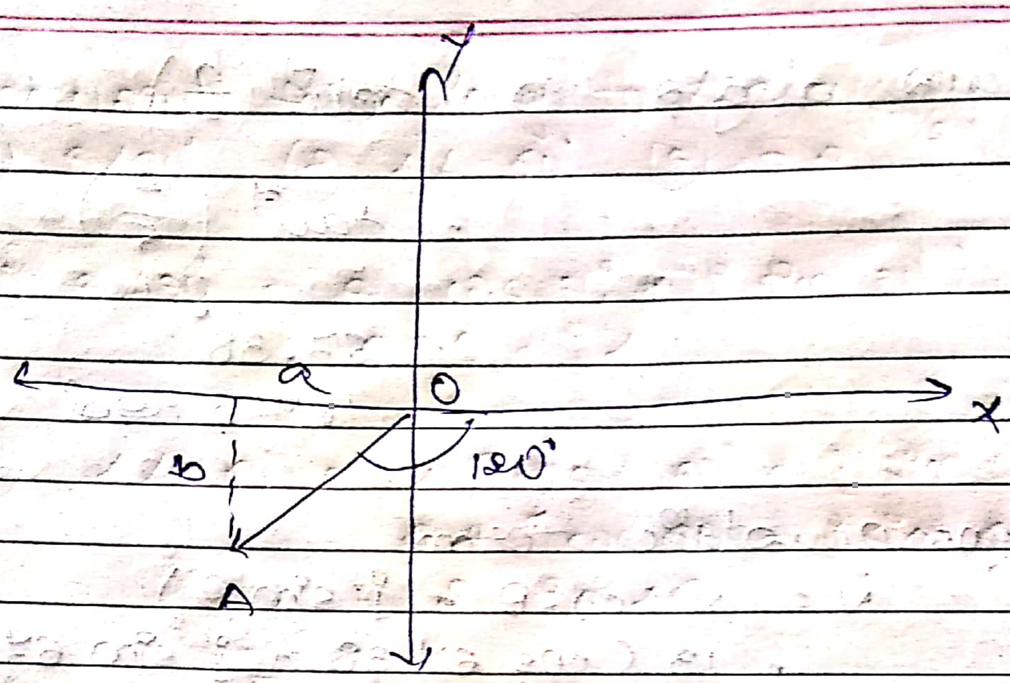
$= 13e^{j67.38^\circ}$ V

Ex.2 A phasor is expressed by $= 20e^{-j\frac{2\pi}{3}}$
write the various equivalent forms of phasor and illustrate by means of a phasor diagram, the magnitude and position of the above phasor.

A.

$\bar{A} = 20e^{-j2\pi/3}$

$2\pi/3 = 120^\circ$. so, -120° will be in clockwise.



① Trigonometric form

$\bar{A} = A\,[\cos\theta + j\sin\theta]$

$= 20\,[\cos(-120) + j\sin(-120)]$

$= 20\,[\cos 120 - j\sin 120]$

② Rectangular form $\quad \bar{A} = a + jb$

$a = A\cos\theta \qquad b = A\sin\theta$

$= 20\cos(-120) \qquad = 20\sin(-120)$

$= 20\cos 120 \qquad = -20\sin 120$

$\boxed{a = -10} \qquad \boxed{b = -17.32}$

$\boxed{\bar{A} = -10 - j17.32}$

③ Polar form, $\bar{A} = A\angle\theta$

$= 20\angle -120°$

Q.5 convert the following in polar form
(1) $5-j5$ (2) $-2+j2$ (3) $j8$

A. (1) $5-j5$

$a=5, \quad b=-5$

magnitude $= \sqrt{a^2+b^2}$	angle $\theta = \tan^{-1}\left(\frac{b}{a}\right)$
$= \sqrt{5^2+5^2}$	$= \tan^{-1}\left(\frac{-5}{5}\right)$
$= \sqrt{50}$	$= \tan^{-1}(-1)$
$= 7.07$	$= -45°$

so, $\boxed{5-j5 = 7.07\angle -45°}$

(2) $-2+j2$

$a=-2, \quad b=2$

magnitude $= \sqrt{a^2+b^2}$	angle $\theta = \tan^{-1}\left(\frac{b}{a}\right)$
$= \sqrt{(-2)^2+2^2}$	$= \tan^{-1}\left(\frac{2}{-2}\right)$
$= \sqrt{8}$	$= 135°$
$= 2.828$	

$\boxed{-2+j2 = 2.828\angle 135°}$

(3) $j8$

magnitude $= \sqrt{0^2+8^2}$	$\theta = \tan^{-1}\left(\frac{8}{0}\right)$
$= 8$	$= 90°$

$\boxed{j8 = 8\angle 90°}$

Ex. 1: convert following to rectangular

(1) $8\angle 30^\circ$ (2) $4\angle -60^\circ$ (3) $5e^{-j\pi/2}$

A: (1) $8\angle 30^\circ = 8(\cos 30^\circ + j\sin 30^\circ)$

$= 6.93 + j4$

(2) $4\angle -60^\circ = 4[\cos(-60) + j\sin(-60)]$

$= 4[\cos 60 - j\sin 60]$

$= 2 - j3.46$

(3) $5e^{-j\frac{\pi}{2}} = 5\angle -\pi/2$

$= 5\angle -90$

$= 5[\cos(-90) + j\sin(-90)]$

$= 5[\cos 90 - j\sin 90]$

$= 5(0 - j1)$

$= -j5$

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