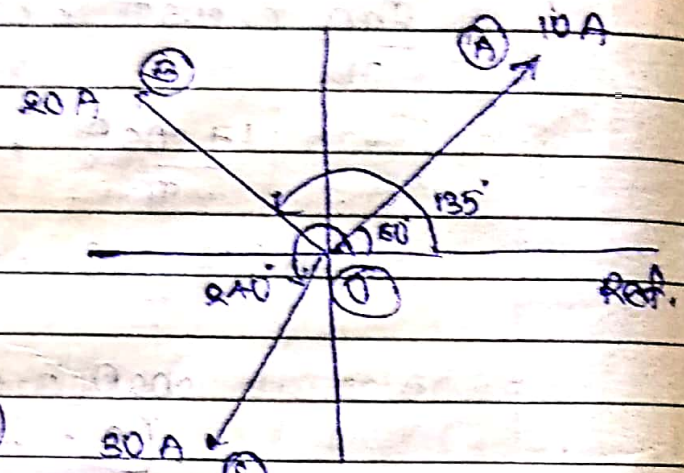


* Addition & Subtraction of Vectors

(i) Addition:

→ Suppose we are given the following three alternating currents & it is required to find the equation of resultant current.



$$i_1 = 10 \sin(\omega t + \pi/3)$$

$$i_2 = 20 \sin(\omega t + 3\pi/4)$$

$$i_3 = 30 \sin(\omega t + 4\pi/3)$$

[Fig. 18] vectors of i_1, i_2 & i_3

→ The current can be written in vector form as under.

$$i_1 = 10 \sin(\omega t + 60^\circ) = 10 \angle 60^\circ = OA$$

$$i_2 = 20 \sin(\omega t + 135^\circ) = 20 \angle 135^\circ = OB$$

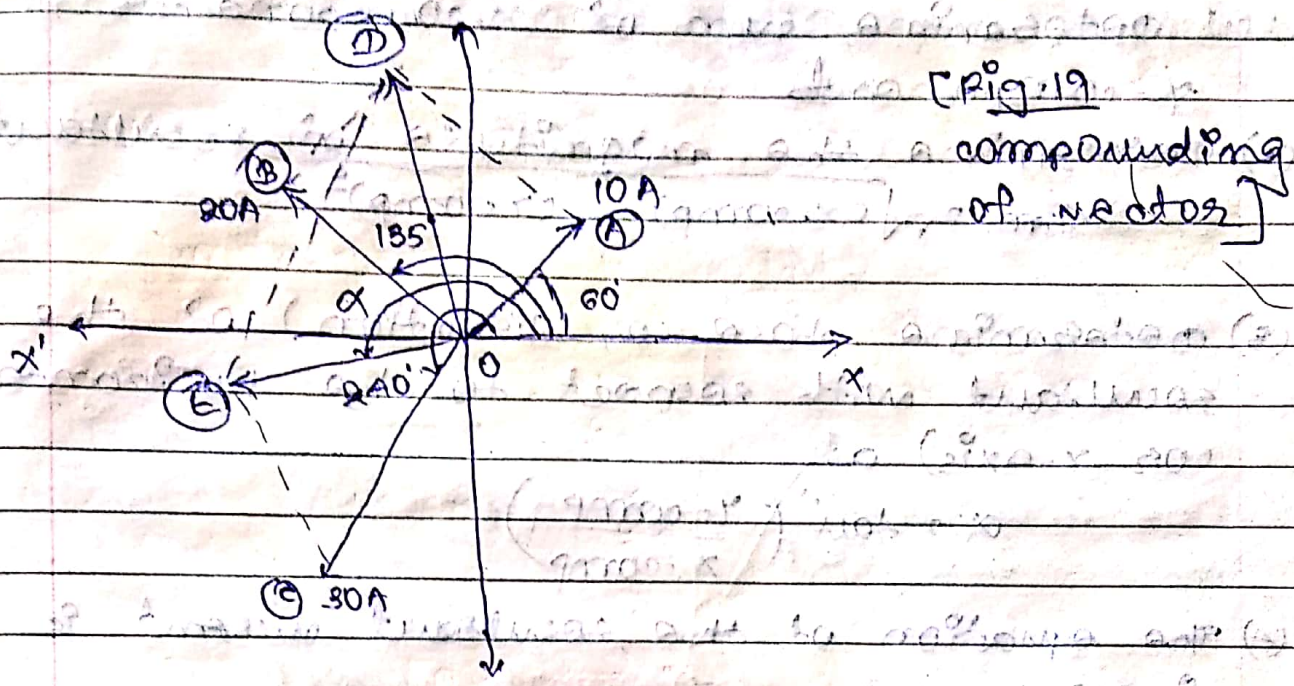
$$i_3 = 30 \sin(\omega t + 240^\circ) = 30 \angle 240^\circ = OC$$

→ Now, the resultant as addition of the three currents can be obtained by any of the following methods:

(A) By compounding according to parallelogram law:

Steps:

- (1) Obtain the resultant of vectors OA & OB as OD
- (2) Obtain the resultant of OD & OC as OE which is the resultant of vectors OA, OB & OC.

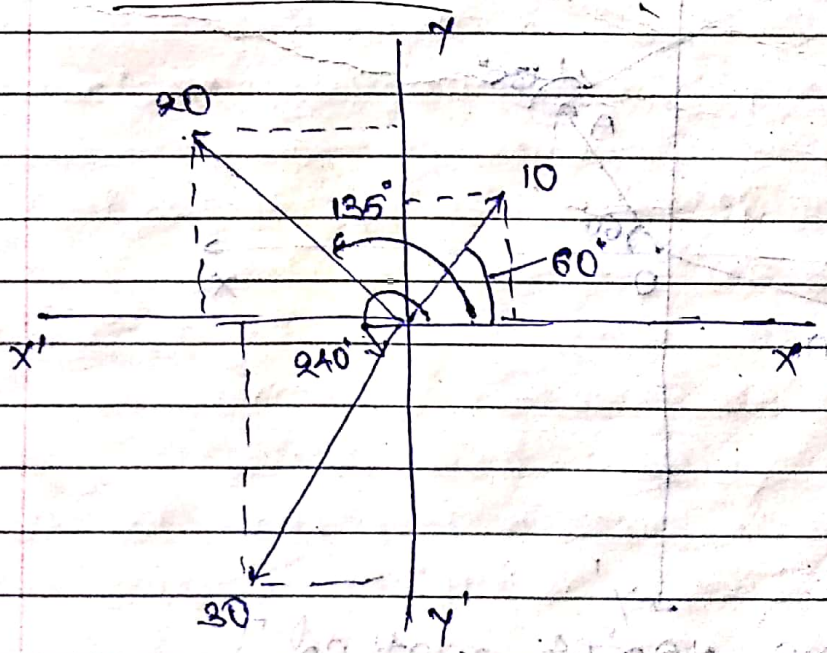


[Fig. 19
compounding
of vectors]

(3) Measure α which gives its phase with horizontal axis.

(4) The equation of resultant current is $i_R = I_m \sin(\omega t + \alpha)$

(5) By resolving the various vectors into their x-components & y-components



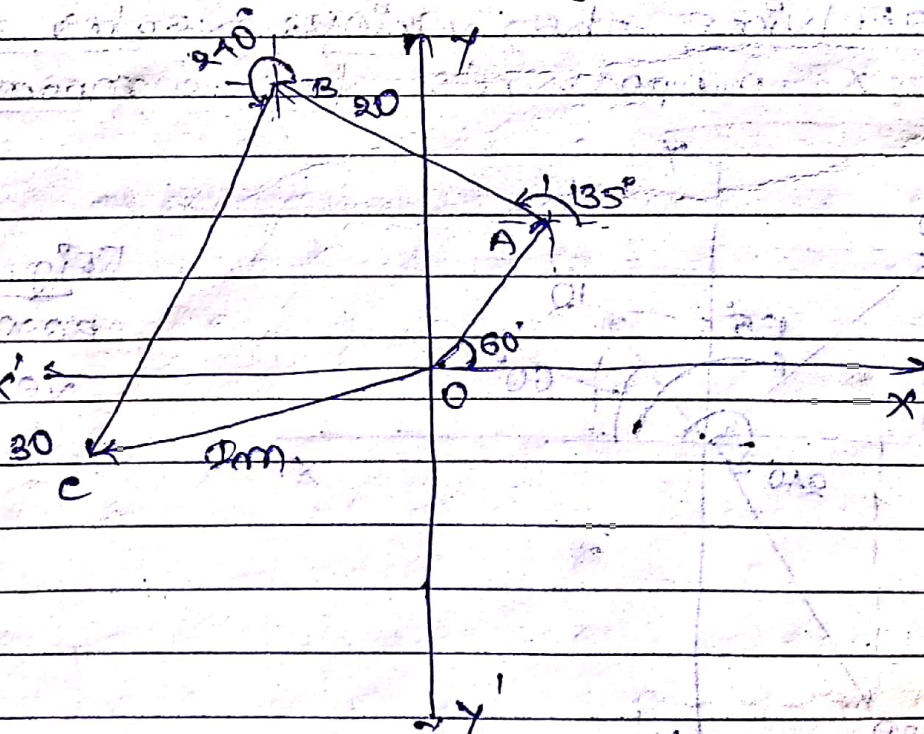
[Fig. 20
resolving
vectors]

- steps: (1) Determine sum of x-components & y-components
- (2) Determine the magnitude of resultant
 $R_m = \sqrt{(x\text{-comp})^2 + (y\text{-comp})^2}$
- (3) Determine phase α (direction) of the resultant with respect to the horizontal (or x-axis) as
 $\alpha = \tan^{-1} \left(\frac{y\text{-comp}}{x\text{-comp}} \right)$

(4) The equation of the resultant current is given by,

$$i = R_m \sin(\omega t + \alpha)$$

- (c) By laying the various vectors end-on-end at their proper phase angles and then measuring the closing vector,

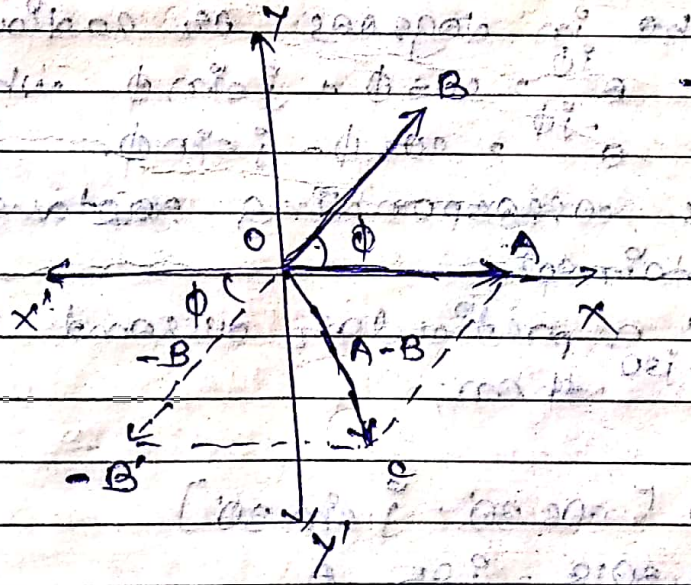


(Fig. 2) closing vector method

- steps
- (1) Draw phasor OA at angle 60° with x-axis.
 - (2) At A, draw the vector AB at an angle 135° with x-axis.
 - (3) At B, draw the vector BC at angle 240° with x-axis.
 - (4) Join C with origin to get the resultant vector OC. Its inclination with x-axis is given by α .
 - (5) Here, the equation of resultant current is given by, $i_r = I_m \sin(\omega t + \alpha)$ where I_m is the resultant current.

(2) subtraction of vectors

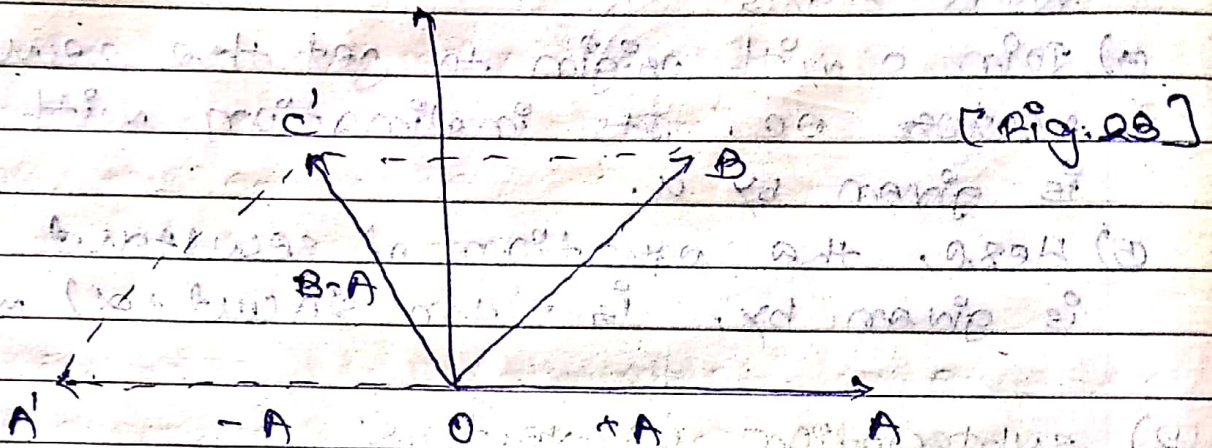
→ If difference of two vectors is required, one of the vectors is reversed and this reversed vector is then compounded with the other vector as usual.



→ Suppose two vectors OA & OB are given as shown in fig. Suppose, it is required to subtract a vector OB from OA. → So, reverse the vector OB as shown in fig. and obtain the resultant of vector OA & OB' according to parallelogram law.

[Fig. 23 subtraction of vectors]

→ The vector difference $(OA - OB)$ is given by the vector OC .
 → Similarly, vector difference $(OB - OA)$ can be obtained as OC' as shown in Fig. 22.1 (a)



* Another way of complex no. representation:

→ In complex number analysis, the complex number may be expressed as $|z| e^{j\phi}$

where ϕ may be in degrees, or radians.

Mathematically, $e^{j\phi} = \cos \phi + j \sin \phi$ while $e^{-j\phi} = \cos \phi - j \sin \phi$.

Using these eqⁿs. corresponding rectangular form can be obtained.

→ For example, if a particular current i is given as $50 e^{j30}$ then,

$$50 e^{j30} = 50 [\cos 30^\circ + j \sin 30^\circ]$$

$$= 43.3012 + j 25 \text{ A}$$

→ while particular voltage is given by $150 e^{j100}$

$$150 e^{j100} = 150 [\cos 100^\circ + j \sin 100^\circ]$$

$$= -26.047 + j 147.721 \text{ V}$$

→ If ϕ is given in radians, \sin & \cos must be calculated in radian mode.

→ In fact, $121 e^{j\phi}$ can be directly expressed in polar form as $121 \angle \phi$ where ϕ may be expressed in degrees or radians

$$\therefore 50 e^{j30} = 50 \angle 30^\circ$$

$$\text{while } 150 e^{j100} = 150 \angle 100^\circ$$

→ This can be cross checked by using rectangular to polar conversion.

$$121 e^{j\phi} = 121 \angle \phi$$

Exa. Two currents $I_1 = 10 e^{j50}$ and $I_2 = 5 e^{j100}$

flow in a $1 \angle \phi$ AC circuit.

Estimate (i) $I_1 + I_2$ (ii) $I_1 - I_2$ & (iii) I_1 / I_2 in complex form.

Ans. $I_1 = 10 [\cos 50^\circ + j \sin 50^\circ]$

$$= 6.4278 + j7.66 \text{ A}$$

$$I_1 = 10 \angle 50^\circ \text{ A}$$

$$I_2 = 5 [\cos 100^\circ + j \sin 100^\circ]$$

$$= -0.8682 + j4.924 \text{ A}$$

$$I_2 = 5 \angle 100^\circ \text{ A}$$

$$(i) I_1 + I_2 = [6.4278 + j7.66] + [-0.8682 + j4.924]$$

$$= 5.5596 + j12.584 \text{ A}$$

(2) $I_1 - I_2 = [6.4278 + j 9.68] - [-0.8682 - j 1.921]$
 $= 7.296 + j 12.581 \text{ A}$
 $= 14.51 \angle 60^\circ \text{ A}$

(3) $I_1 / I_2 = \frac{10 \angle 50^\circ}{5 \angle -100^\circ} = \frac{10}{5} \angle 50^\circ + 100^\circ$
 $= 2 \angle +150^\circ \text{ A}$

* Polar system: $r \angle \pm \phi$

* Rectangular system: $x + jy$

* Convert polar to rectangular system

$x = r \cos \phi$, $y = r \sin \phi$

* Convert rectangular to polar system

$r = \sqrt{x^2 + y^2}$

$\phi = \tan^{-1} \left(\frac{y}{x} \right)$

* Multiplication & Division of Phasors:

→ Addition & subtraction of phasors is performed, which is to be carried out using rectangular form of phasors. But the rectangular form is not suitable to perform multiplication & division of phasors. Hence multiplication & division must be performed using polar form of phasors.

→ Let P and Q be the two phasors such that,
 $P = x_1 + jy_1$ and $Q = x_2 + jy_2$

→ To obtain the multiplication $P \times Q$, both must be expressed in polar form:

Let $P = r_1 \angle \phi_1$ and $Q = r_2 \angle \phi_2$
 then $P \times Q = [r_1 \angle \phi_1] \times [r_2 \angle \phi_2]$
 $= [r_1 \times r_2] \angle \phi_1 + \phi_2$

note: In multiplication of complex numbers in polar form, the magnitudes get multiplied while their angles get added.

→ The result then can be expressed back to rectangular form, if required.

Now, consider the division of the phases

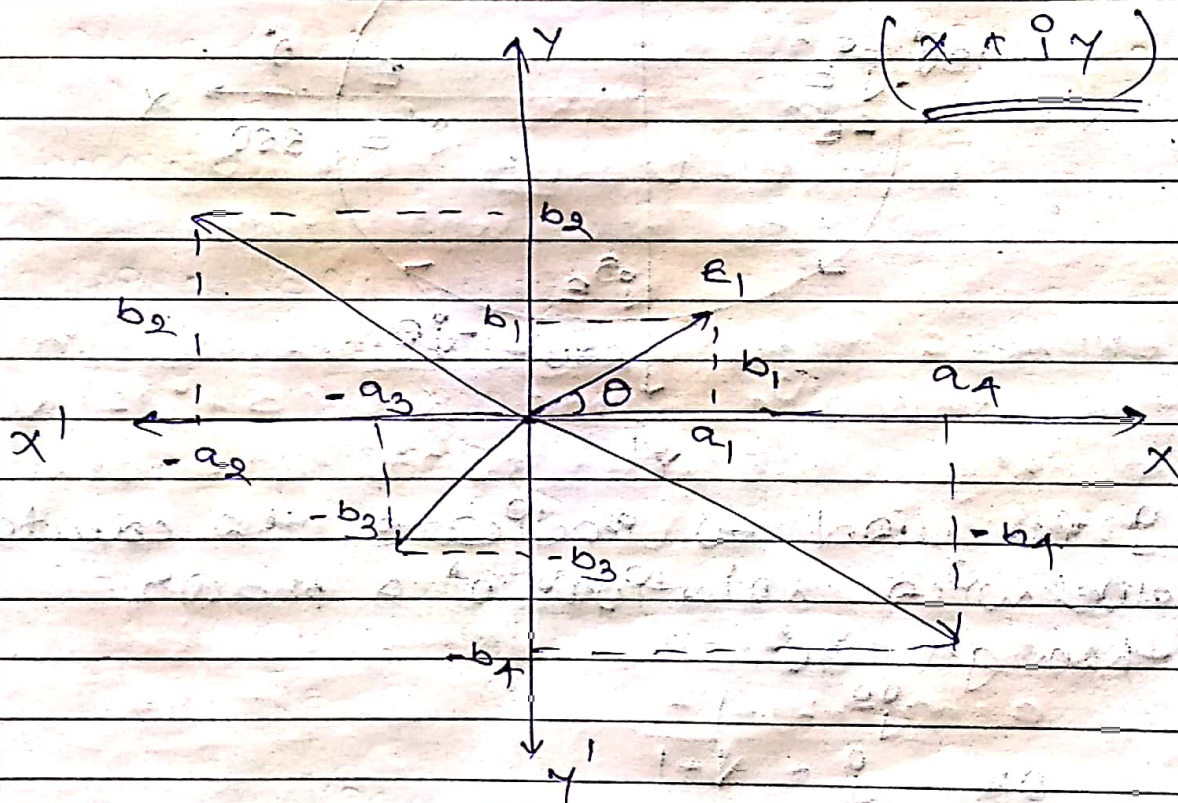
P and Q both in polar form

$$\frac{P}{Q} = \frac{r_1 \angle \phi_1}{r_2 \angle \phi_2} = \left| \frac{r_1}{r_2} \right| \angle \phi_1 - \phi_2$$

note: In division of complex numbers in polar form, the magnitudes get divided while their angles get subtracted.

* 4. ways of representation of vectors

- (1) symbolical notation OR rectangular OR cartesian form
- (2) Trigonometric form
- (3) exponential form
- (4) Polar form

(1) symbolical OR rectangular:

$$OB_1 = a_1 + j b_1 \quad \text{--- (1)}$$

$$OB_2 = -a_2 + j b_2 \quad \text{--- (2)}$$

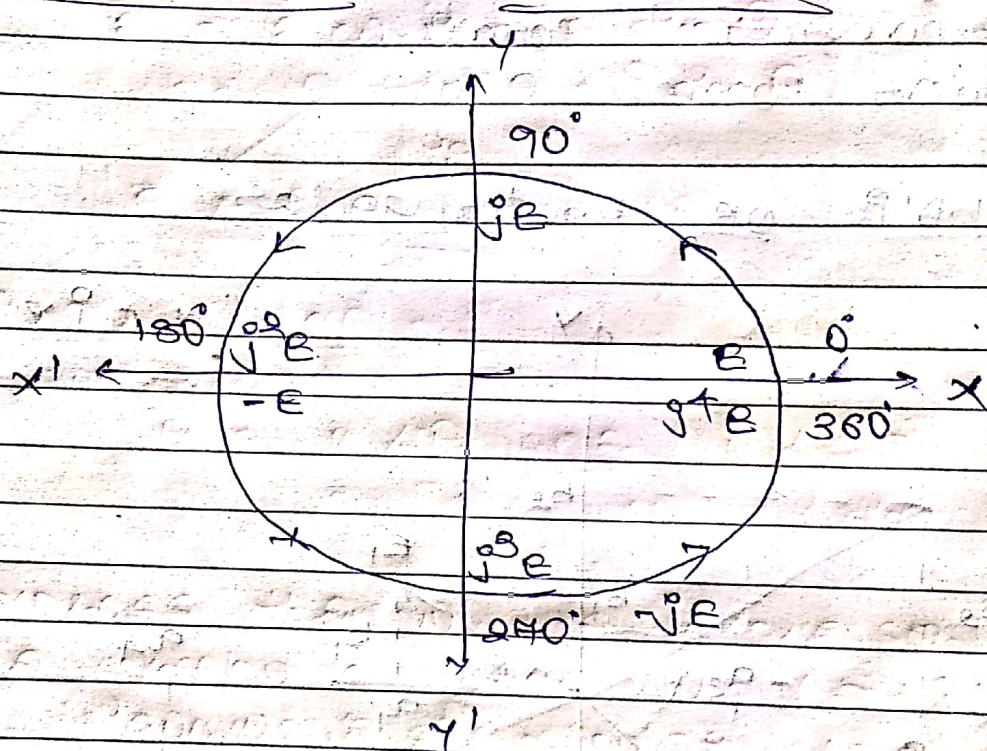
$$OB_3 = -a_3 - j b_3 \quad \text{--- (3)}$$

$$OB_4 = a_4 - j b_4 \quad \text{--- (4)}$$

→ length of vector = $\sqrt{a^2 + b^2}$

→ Angle with the x-axis = $\theta = \tan^{-1}\left(\frac{b_1}{a_1}\right)$

* significance of j-operator:



→ j is used to indicate the counter clockwise rotation of a phasor through 90° .

OR $j^2 = -1$

$\therefore j^2 E = -E$

and $\frac{1}{j} = \frac{j}{j^2} = \frac{j}{-1} = -j$

→ complex no. $\bar{A} = a + jb$

~~Real~~ $\text{Re}(\bar{A}) = a$ (Real)

$\text{Im}(\bar{A}) = jb$ (imaginary)

→ $(a + jb) + (a - jb) = 2a$ = active comp
 $(a + jb) - (a - jb) = 2jb$ = reactive comp.

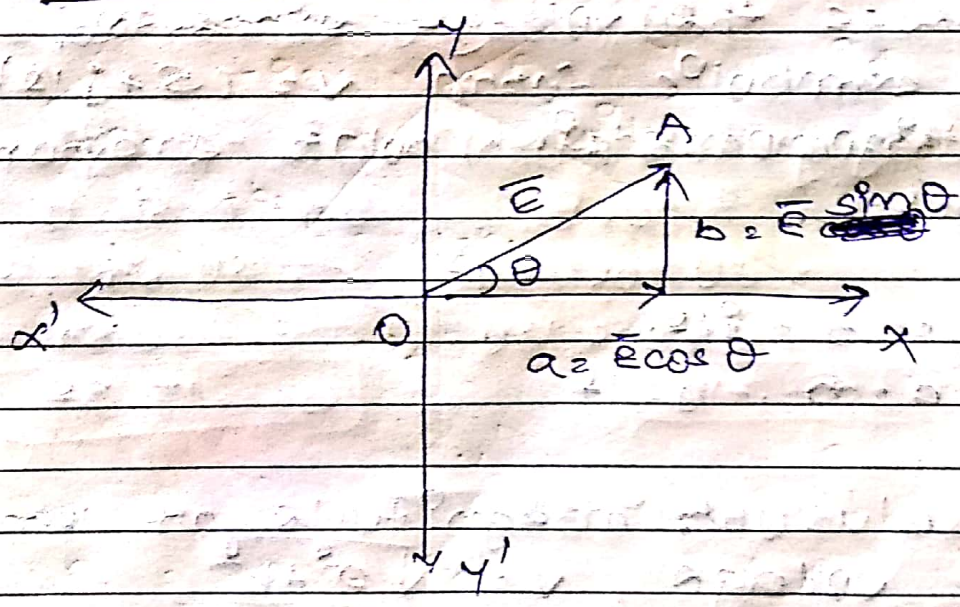
$$(a + jb) \times (a - jb) = a^2 - (jb)^2$$

$$= a^2 - j^2 b^2$$

$$= a^2 + b^2$$

= active comp.

* Trigonometric form of phasor representation:



$$\bar{E} = E \cos \theta + j E \sin \theta \quad (a + jb)$$

* Exponential Form:

$$e^{j\theta} = \cos \theta + j \sin \theta$$

$$\bar{e} = E \cos \theta + j E \sin \theta$$

$$\bar{e} = E e^{j\theta}$$

* Polar Form:

$$\bar{e} = E \angle \theta \quad \text{OR} \quad E \angle \theta$$

Exa. 1 Express the voltage expressed in the symbolic form $V = (5 + j12)$ volts in trigonometric, polar & exponential forms.

A. $V = 5 + j12$
 $a = 5, b = 12$

modulus or magnitude or length of voltage, $V = \sqrt{a^2 + b^2}$

$$= \sqrt{5^2 + 12^2}$$

$$= 13 \text{ V}$$

phase angle $\theta = \tan^{-1} \left(\frac{b}{a} \right)$

$$= \tan^{-1} \left(\frac{10}{5} \right)$$

$$= 67.38^\circ$$

$$= 1.176 \text{ rad}$$

(1) Trigonometric form

$$\bar{V} = V \cos \theta + j \sin \theta$$

$$= 13 (\cos 67.38 + j \sin 67.38) \text{ V}$$

(2) Polar form

$$\bar{V} = V \angle \theta$$

$$= 13 \angle 67.38^\circ \text{ V}$$

(3) Exponential form

$$\bar{V} = V e^{j\theta}$$

$$= 13 e^{j1.176} \text{ V}$$

$$= 13 e^{j67.38^\circ} \text{ V}$$

$$= 13 \angle 67.38^\circ \text{ V}$$

Ex. 9 A phasor is expressed by $20 e^{-j\frac{2\pi}{3}}$

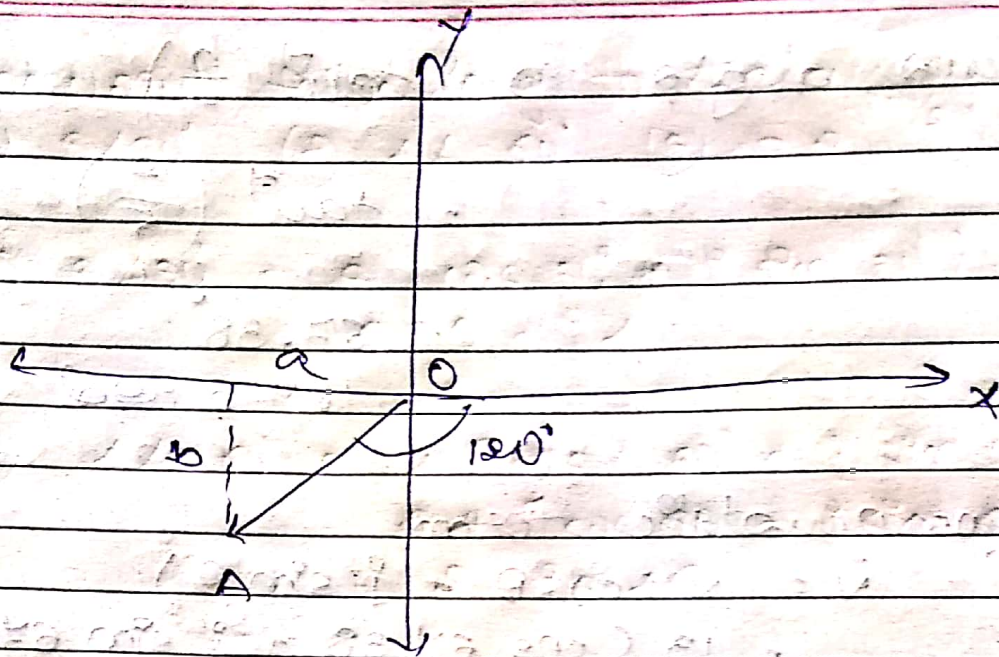
write the various equivalent forms of phasor and illustrate by means of a phasor diagram, the magnitude and position of the above phasor.

A.

$$\bar{A} = 20 e^{-j\frac{2\pi}{3}}$$

$$\frac{2\pi}{3} = 120^\circ$$

So, -120° will be in clockwise,



① Trigonometric form

$$\bar{A} = A [\cos \theta + j \sin \theta]$$

$$= 20 [\cos (-120^\circ) + j \sin (-120^\circ)]$$

$$= 20 [\cos 120^\circ - j \sin 120^\circ]$$

② Rectangular form

$$\bar{A} = a + jb$$

$$a = A \cos \theta$$

$$b = A \sin \theta$$

$$= 20 \cos (-120^\circ)$$

$$= 20 \sin (-120^\circ)$$

$$= 20 \cos 120^\circ$$

$$= -20 \sin 120^\circ$$

$$\boxed{a = -10}$$

$$\boxed{b = -17.32}$$

$$\boxed{\bar{A} = -10 - j17.32}$$

③ Polar form, $\bar{A} = A \angle \theta$

$$= 20 \angle -120^\circ$$

Q.5 Convert the following in polar form

- (1) $5 - j5$ (2) $-2 + j2$ (3) $j8$

1. (1) $5 - j5$

$$a = 5, b = -5$$

$$\begin{aligned} \text{magnitude} &= \sqrt{a^2 + b^2} \\ &= \sqrt{5^2 + 5^2} \\ &= \sqrt{50} \\ &= 7.07 \end{aligned}$$

$$\begin{aligned} \text{angle } \theta &= \tan^{-1}\left(\frac{b}{a}\right) \\ &= \tan^{-1}\left(\frac{-5}{5}\right) \\ &= \tan^{-1}(-1) \\ &= -45^\circ \end{aligned}$$

$$\text{So, } \boxed{5 - j5 = 7.07 \angle -45^\circ}$$

(2) $-2 + j2$

$$a = -2, b = 2$$

$$\begin{aligned} \text{magnitude} &= \sqrt{a^2 + b^2} \\ &= \sqrt{(-2)^2 + 2^2} \\ &= \sqrt{8} \\ &= 2.828 \end{aligned}$$

$$\begin{aligned} \text{angle } \theta &= \tan^{-1}\left(\frac{b}{a}\right) \\ &= \tan^{-1}\left(\frac{2}{-2}\right) \\ &= 135^\circ \end{aligned}$$

$$\boxed{-2 + j2 = 2.828 \angle 135^\circ}$$

(3) $j8$

$$\begin{aligned} \text{magnitude} &= \sqrt{0^2 + 8^2} \\ &= 8 \end{aligned}$$

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{8}{0}\right) \\ &= 90^\circ \end{aligned}$$

$$\boxed{j8 = 8 \angle 90^\circ}$$

Ex. 1) convert following to rectangular

- (1) $8 \angle 30^\circ$ (2) $4 \angle -60^\circ$ (3) $5 \angle \frac{j\pi}{2}$

A. (1) $8 \angle 30^\circ = 8 (\cos 30^\circ + j \sin 30^\circ)$
 $= 8.98 + j4$

(2) $4 \angle -60^\circ = 4 [\cos (-60) + j \sin (-60)]$
 $= 4 [\cos 60 - j \sin 60]$
 $= 2 - j3.46$

(3) $5 \angle \frac{j\pi}{2} = 5 \angle -90^\circ$
 $= 5 \angle -90$
 $= 5 [\cos (-90) + j \sin (-90)]$
 $= 5 [\cos 90 - j \sin 90]$
 $= 5 [0 - j1]$
 $= -j5$