Kaushal R. Patel

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chap: AC circuits

→ The flow of electrons in an electric circuit constitutes the electric current.

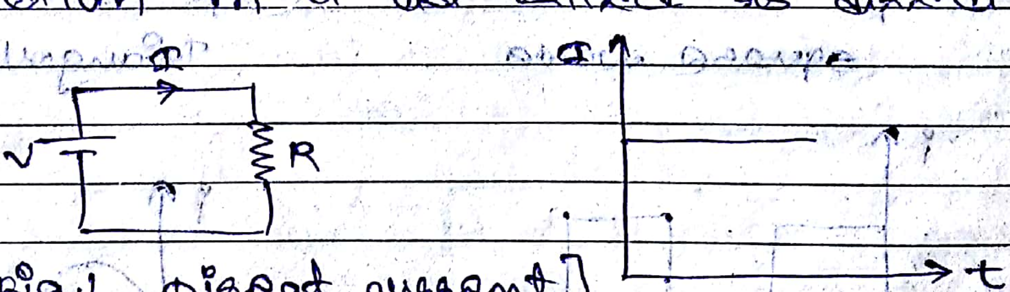
→ Flow of electron in a circuit may be in one direction only or there may be change in direction of flow of electrons in circuit.

→ Electric currents are mainly classified in two classes:
- Direct current (DC or dc)
- Alternating current (AC or ac)

* Definitions:

(1) Direct current:

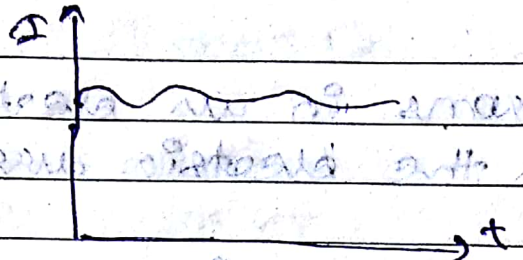
The current which always flows in one direction in a ckt called as direct current



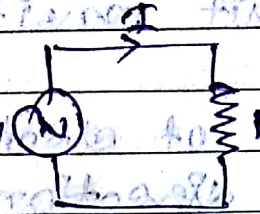
[Fig. 1 Direct current]

(2) Pulsating current:

The current whose magnitude changes but flows in the same direction is called as pulsating current.

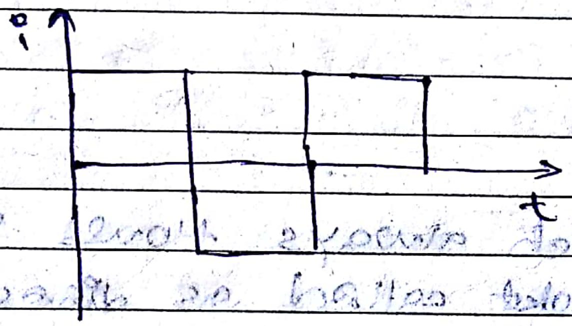


[Fig. 2 Pulsating current]

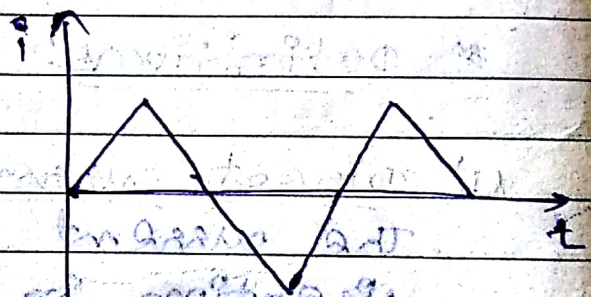


(3) Alternating current:

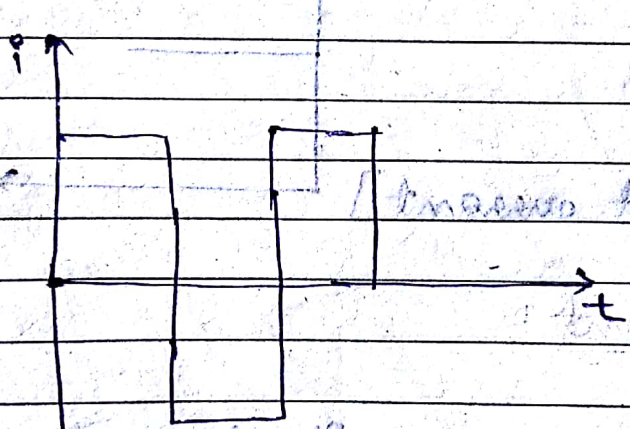
The current which changes its direction and magnitude periodically at regular intervals ~~and~~ of time in a circuit is called alternating current.



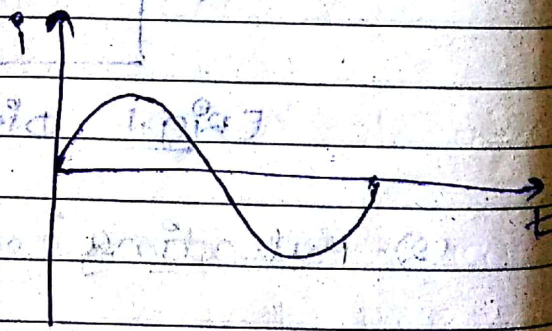
Square wave



Triangular wave

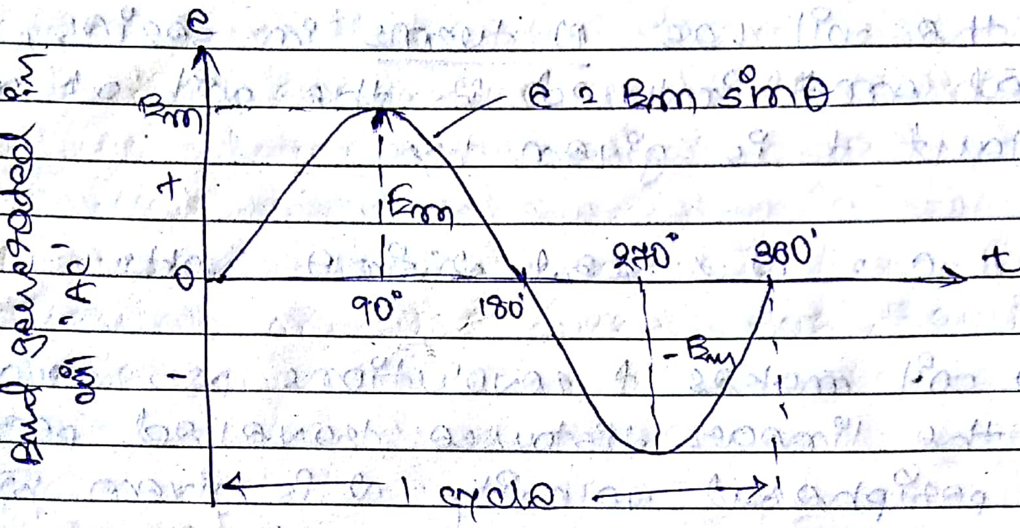


Rectangular wave



Sine wave

[Fig. 3 Different type of waveforms]



[Fig. 7 sinusoidal waveform of emf]

⇒ When θ varies from 0° to 180°, the emf is considered positive and it is negative when θ varies between 180° to 360°.

* Definitions:

(1) Waveform:

The shape of the curve obtained by plotting the instantaneous value of the voltage or current as ordinate against time is called waveform.

(2) Alternation:

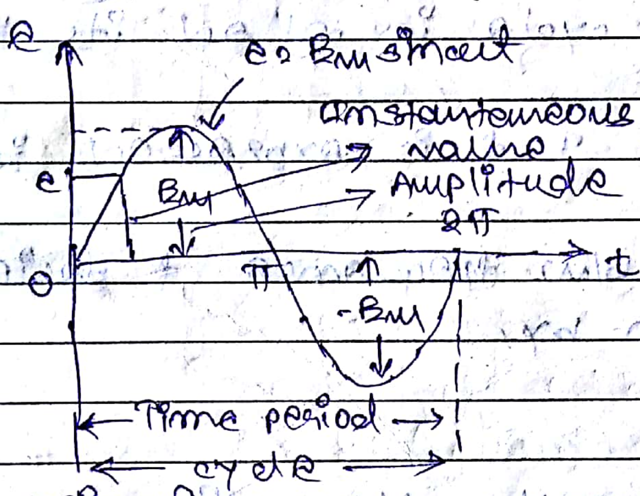
A complete set of positive value or negative values plotted against time is known as Alternation.

- One Alternation is equal to half-cycle.

since a cycle has two alternation, one positive and the other negative.

(3) sinusoidal waveform:
A wave in which the value of voltage or current at any instant (or time) depends on the sine of the angle at that instant is known as sinusoidal waveform.

(4) Instantaneous value:



(Fig. 8)

→ The value of alternating quantity at any instant is called instantaneous value.

→ The instantaneous value is equal to the product of the maximum value and the sine of the angle corresponding to the instant in case of sinusoidal wave.

(5) Amplitude:
→ The peak (or maximum) value, positive or negative of an alternating quantity is known as its amplitude.

(6) Cycle:
one complete set of positive & negative values of an alternating quantity is known as cycle.
- A cycle is normally specified in terms of angular measure spread over 360° or 2π radian.

(7) Time period (Periodic time):
- The time taken by an alternating quantity to complete one cycle is called its time period.
- It is denoted by T & is expressed in seconds.
- The relationship b/w frequency & periodic time (T) is given by,

$$T = 1/f$$

For example, a 50 Hz alternating quantity has a time period of $1/50 = 0.02$ second.

(8) Frequency:
- The number of cycles completed by an alternating quantity per second is known as frequency.
- It is denoted by f and is expressed in hertz (Hz) or cycle/second

$$f = \frac{PN}{120}$$

where, f = frequency
P = no. of pole of Alternator
N = speed of alternator in rpm

* **Different forms of EMF Equation:**

→ The standard form of an alternating voltage (sinusoidal) is given by

$e = E_m \sin\theta$ —— (1)

$= E_m \sin \omega t$

$= E_m \sin (2\pi f) t \quad (\omega = 2\pi f)$

$= E_m \sin \left(\frac{2\pi}{T}\right) t \quad \left(f = \frac{1}{T}\right)$

→ Similarly we can write different equations for an alternating current

$i = I_m \sin\theta$ —— (2)

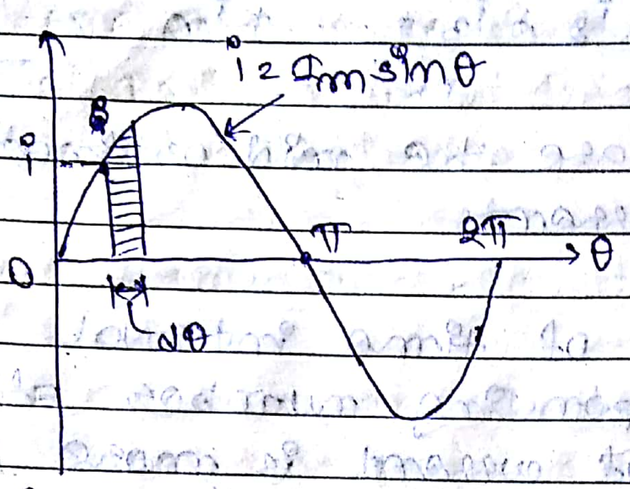
$= I_m \sin \omega t$

$= I_m \sin (2\pi f) t$

$= I_m \sin \left(\frac{2\pi}{T}\right) t$

* **Average value or Mean value:**

→ The average value of an alternating current is expressed by that value of direct current which transfer across a circuit the same charge as is transferred by the alternating current across the same circuit in the same time.



$i = I_m \sin\theta$, $0 \le \theta \le \pi$

[Fig.10 sinusoidal waveform]

→ $i = I_m \sin\theta$, $0 \le \theta \le \pi$

→ By definition,

$$I_{av} = \frac{\text{area under half cycle}}{\text{length of the base over half cycle}}$$

$$= \frac{\int_0^{\pi} i \, d\theta}{\pi}$$

$$= \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta \, d\theta \quad [I = I_m \sin\theta]$$

$$= \frac{I_m}{\pi}\left[-\cos\theta\right]_0^{\pi}$$

$$= -\frac{I_m}{\pi}\left[\cos\pi - \cos 0\right]$$

$$= -\frac{I_m}{\pi}\left[-1-1\right]$$

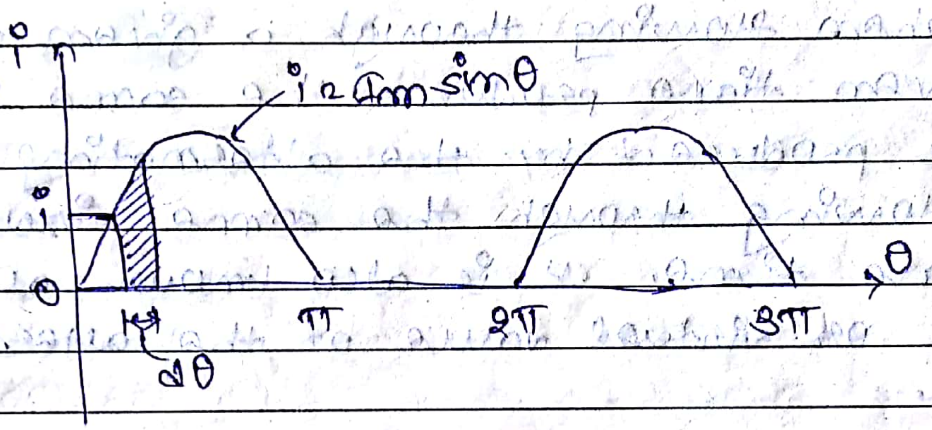
$$= \frac{2I_m}{\pi} = \frac{I_m}{\pi/2} = 0.637\, I_m$$

The avg. value of current for a sinusoidal wave = 0.637 x max. value of current

* Average value of half wave rectified current :

→ Since it is unsymmetrical, we will consider full cycle.
Therefore,

$i = I_m \sin\theta$, $0 \le \theta \le \pi$
$i = 0$, $\pi \le \theta \le 2\pi$



[Fig. 1]

→ By definition,

$$I_{av} = \frac{\text{area under full-cycle}}{\text{length of the base over full-cycle}}$$

$$= \frac{\int_0^{\pi} i\, d\theta + \int_{\pi}^{2\pi} 0 . d\theta}{2\pi}$$

$$= \frac{1}{2\pi}\int_0^{\pi} I_m \sin\theta\, d\theta$$

$$= \frac{I_m}{2\pi}\left[-\cos\theta\right]_0^{\pi}$$

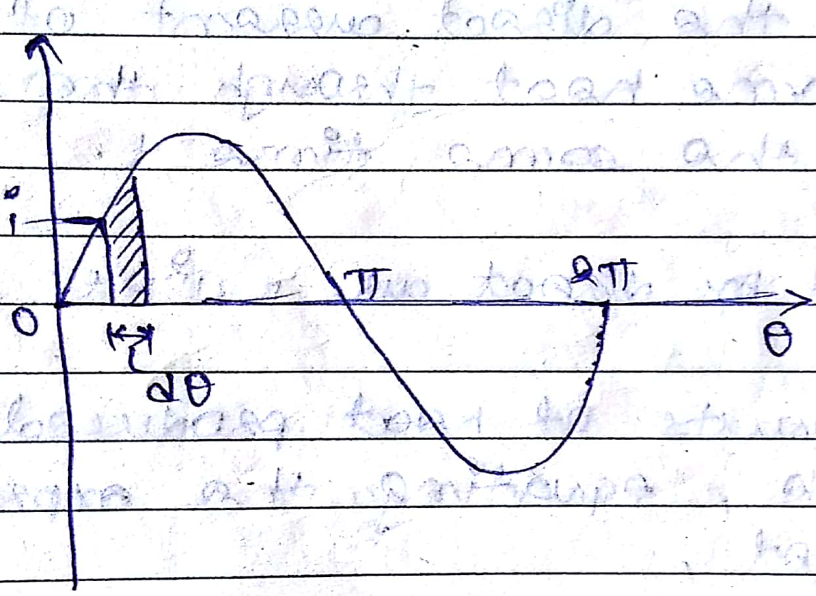
$$= \frac{-I_m}{2\pi}\left[-1-1\right] = \frac{2I_m}{2\pi} = \frac{I_m}{\pi}$$

∴ Average value of half wave rectified current $= \frac{1}{\pi} \times$ max. value of current

* **Root Mean Square Value (RMS value):-**

→ The RMS value of that alternating current is given by that value of direct current which when flowing through a given circuit for a given time produces the same heating effect as produced by the alternating current when flowing through the same circuit for the same time. It is also known as effective or virtual value of the alternating quantity.

Analytical method:



[Fig. 13 sinusoidal waveform]

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(3) → symmetrical wave

→ The instantaneous value of the current is given by,

$$i = I_m \sin\theta \quad , \quad 0 \le \theta \le 2\pi$$ ①

$$\text{Mean of } (i^2) = \frac{\text{area under } (i^2) \text{ curve a cycle}}{\text{length of base over a cycle}}$$

$$= \frac{\int_0^{2\pi} i^2 \, d\theta}{2\pi}$$

$$= \frac{1}{2\pi} \int_0^{2\pi} (I_m \sin\theta)^2 \, d\theta$$

$$= \frac{I_m^2}{2\pi} \int_0^{2\pi} \sin^2\theta \, d\theta$$

$$= \frac{I_m^2}{2\pi} \int_0^{2\pi} \left(\frac{1-\cos 2\theta}{2} \right) d\theta$$

$$= \frac{I_m^2}{2 \times 2\pi} \int_0^{2\pi} (1-\cos 2\theta) \, d\theta$$

$$= \frac{I_m^2}{4\pi} \left[\theta - \frac{\sin 2\theta}{2} \right]_0^{2\pi}$$

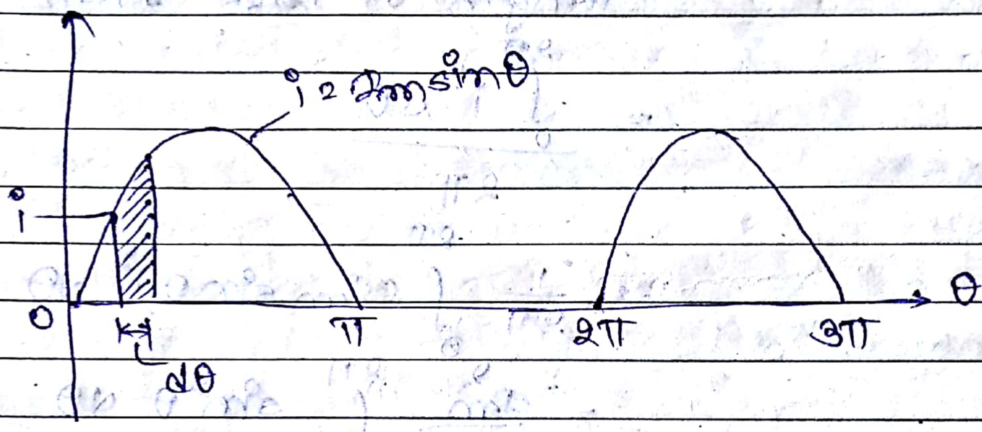
$$= \frac{I_m^2}{4\pi} (2\pi - 0)$$

$$\text{Mean of } (i^2) = \frac{I_m^2}{2}$$

∴ RMS value of current $I = \sqrt{\text{mean of } (i^2)}$

$$= \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m$$

iv) RMS value of half wave rectified current:



[Fig. 14 waveform of half wave rectified current]

→ $i = I_m \sin\theta \qquad 0 \leq \theta \leq \pi$ —— (1)

$i = 0 \qquad \pi \leq \theta \leq 2\pi$ —— (2)

→ The wave is unsymmetrical. so, we must consider full cycle.

$$\text{Mean of } (i^2) = \frac{\int_0^{\pi} i^2\, d\theta + \int_{\pi}^{2\pi} 0 \cdot d\theta}{2\pi}$$

$$= \frac{1}{2\pi} \int_0^{2\pi} i^2\, d\theta$$

$$= \frac{1}{2\pi} \int_0^{\pi} (I_m \sin\theta)^2\, d\theta \quad (\text{from } (1))$$

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$$= \frac{I_m^2}{2\pi} \int_0^{\pi} \sin^2\theta \, d\theta$$

$$= \frac{I_m^2}{2\pi} \int_0^{\pi} \left(\frac{1-\cos 2\theta}{2} \right) d\theta$$

$$= \frac{I_m^2}{4\pi} \left[\theta - \frac{\sin 2\theta}{2} \right]_0^{\pi}$$

$$= \frac{I_m^2}{4\pi} [\pi - 0]$$

$$\text{mean of } (i^2) = \frac{I_m^2}{4}$$

$\therefore$ RMS value of current $= \sqrt{\text{mean of } (i^2)}$

$$= \sqrt{\frac{I_m^2}{4}} = \frac{I_m}{2}$$

$$\boxed{I = 0.5 \, I_m}$$

$\therefore$ RMS value of current for half wave rectified wave $= 0.5 \times$ max. value of current

* Form Factor:

→ It is defined as the ratio of RMS value to the average value of alternating quantity (voltage or current). It is denoted by K_f.

$$\therefore K_f = \frac{\text{RMS value}}{\text{Average value}}$$

For sinusoidal current (or voltage)

$$K_f = \frac{0.707 \times I_m}{0.637 \times I_m} = 1.11$$

* Crest factor or Peak factor or Amplitude factor:

→ It is defined as the ratio of the maximum value to the RMS value of the alternating quantity (current or voltage). It is denoted by K_p.

$$K_p = \frac{\text{maximum value}}{\text{RMS value}}$$

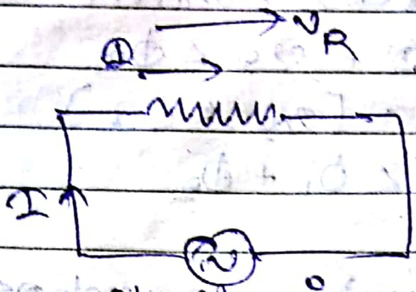
$$= \frac{I_m}{I_m/\sqrt{2}}$$

$$\therefore K_p = \sqrt{2}$$

Peak factor $K_p = 1.414$

Analysis of AC circuit:

* Purely Resistive circuit:



$v = V_m \sin \omega t$

[Fig. 24. Purely R-circuit]

→ consider a circuit having a resistor of Resistance R ohms across the terminals of an alternating voltage source as shown in fig. 24.

→ If the voltage has a sinusoidal waveform and $v = V_m \sin \omega t$ is the instantaneous value of the voltage at any given time 't', the current at that instant is given by ohm's law.

$$i = v/R$$

$$= \frac{V_m \sin \omega t}{R}$$

$$i = \frac{V_m}{R} \sin \omega t \longrightarrow (1)$$

→ The current i is max. (peak) when $\sin \omega t$ is unity.

$$\therefore I_m = \frac{V_m}{R} \longrightarrow (2)$$

→ Hence eq^n (1) becomes,

$$i = I_m \sin \omega t \longrightarrow (3)$$

Hence, current flowing in the ckt follows the sine law.

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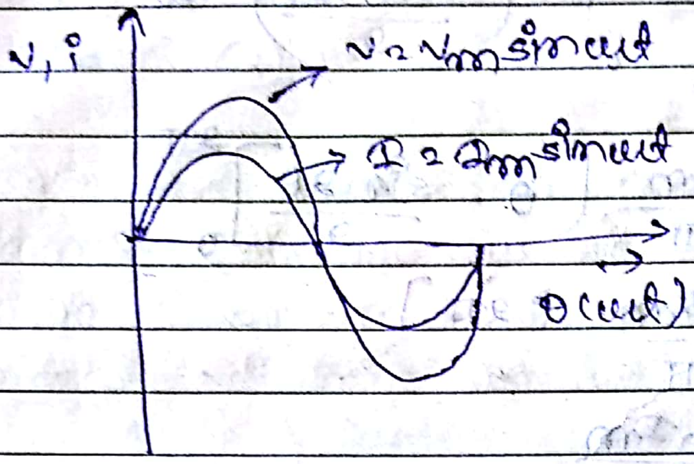
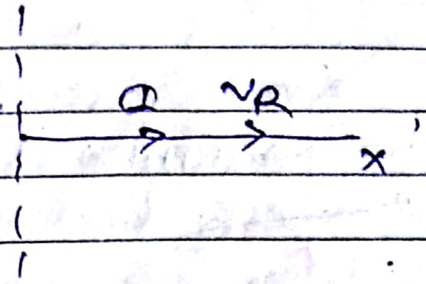
→ The graphical & phasor representation of these quantities are shown in fig 25 & 26.



[Fig.25 waveforms]



[Fig.26 phasor diagram]

→ In fig. 26, V_R = voltage across R (rms)
I = curr. through resistor (rms)

→ Power drawn by this ckt at any instant is the product of the instantaneous voltage and instantaneous current, i.e,

$$p = v \cdot i \quad \text{——— (4)}$$
$$= (V_m \sin\omega t) \cdot (I_m \sin\omega t)$$
$$= V_m I_m \sin^2\omega t$$
$$= V_m I_m \sin^2\theta \quad (\theta = \omega t)$$

→ Average power for one cycle,

$$P = \frac{1}{2\pi}\int_0^{2\pi} p \cdot d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi} V_m I_m \sin^2\theta \, d\theta$$

$$= \frac{V_m I_m}{2\pi}\int_0^{2\pi} \sin^2\theta \, d\theta$$

$$= \frac{V_m I_m}{2\pi}\int_0^{2\pi} \left(\frac{1-\cos 2\theta}{2}\right) d\theta$$

$$= \frac{V_m I_m}{2\times 2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi}$$

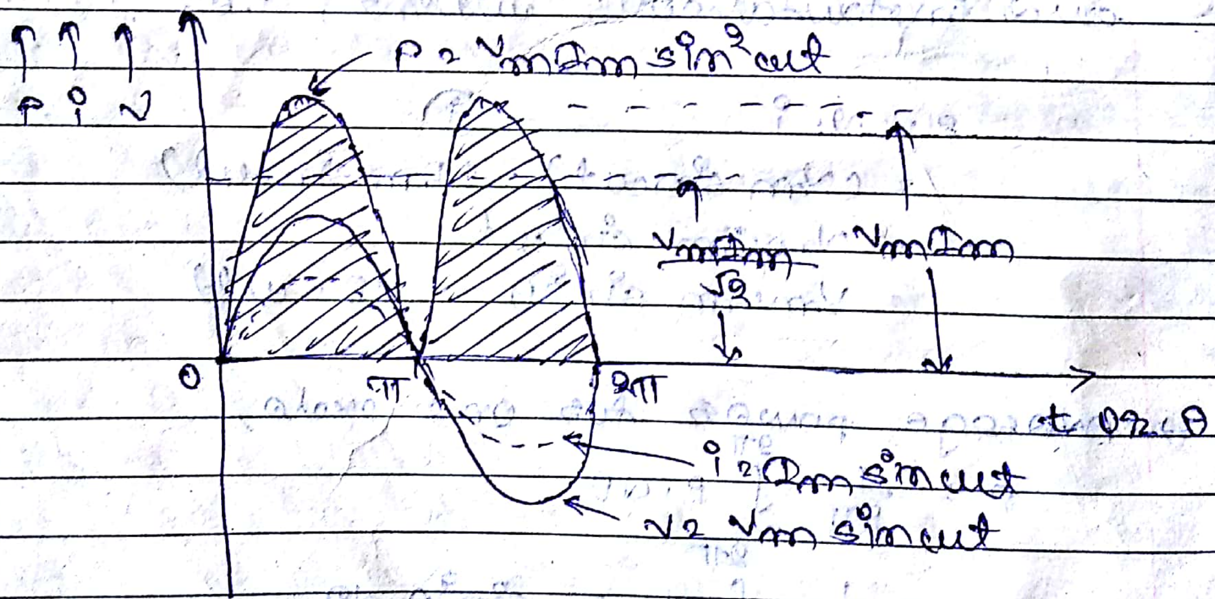
$$= \frac{V_m I_m}{4\pi}\left[2\pi\right]$$

$$= \frac{V_m I_m}{2}$$

$$= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}$$

$$P = V_{rms} \cdot I_{rms}$$

where V_{rms} = rms value of the applied volt.
I_{rms} = rms value of the current



[Fig. 27 waveforms of v, i & p]

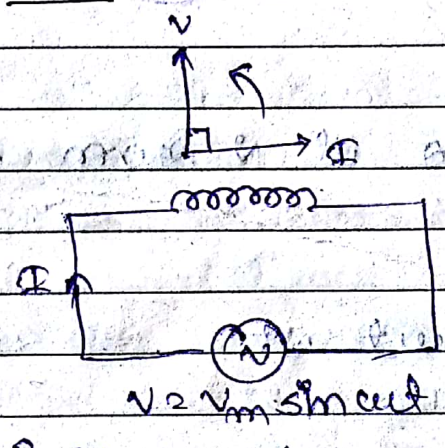
* Power factor:
→ It is the cosine angle of the phase angle between applied voltage and resulting current in the circuit.

→ In purely resistive circuit, V & I are both in phase with each other. So the phase angle b/w V & I is zero.

∴ Power factor = $\cos\phi$
= $\cos 0°$
= 1 (unity)

The p.f. of a purely resistive ckt is unity.

* Purely Inductive circuit:



[Fig. 28 Purely Inductive ckt]

→ In this, an alternating voltage is applied across a pure inductance of self inductance L Henry. Let the applied alternating voltage be
$v = V_m \sin\omega t$ — (1)

→ Alternating current flows in the circuit which will set up a magnetic field that is alternating in nature. This will give rise to an alternating induced emf often called as emf of self inductance.

→ Hence, self induced emf is given by,

$$e = -L\frac{di}{dt} \quad \text{--- (2)}$$

The negative sign in the above expression indicates that it is an opposing emf. Hence the applied voltage is equal and opposite to the self induced emf in the inductor at every instant.

→ The applied voltage

$$v = -e = -\left(-L\frac{di}{dt}\right)$$

$$v = L\frac{di}{dt} \quad \text{--- (3)}$$

→ Substituting the value of v from eqn (1),

$$V_m \sin\omega t = L\frac{di}{dt}$$

$$di = \frac{V_m}{L}\sin\omega t \, dt$$

→ Integrating both sides of this eqn

$$i = \frac{V_m}{L}\int \sin\omega t \, dt$$

$$= -\frac{V_m}{L}\frac{\cos\omega t}{\omega}$$

$$= -\frac{V_m}{\omega L}\cos\omega t$$

$$= \frac{-V_m}{\omega L} \sin\left[\left(\frac{\pi}{2} - \cancel{\omega t}\, \omega t\right)\right]$$

$$= \frac{V_m}{\omega L} \sin\left[-\left(\frac{\pi}{2} - \omega t\right)\right]$$

$$i = \frac{V_m}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right) \quad \text{--- (4)}$$

→ From eqⁿ (4), the current attains its max. value when $\sin(\omega t - \pi/2)$ becomes unity & max. value of current is

$$I_m = \frac{V_m}{\omega L} \quad \text{--- (5)}$$

→ So, we find that if the applied voltage is represented by

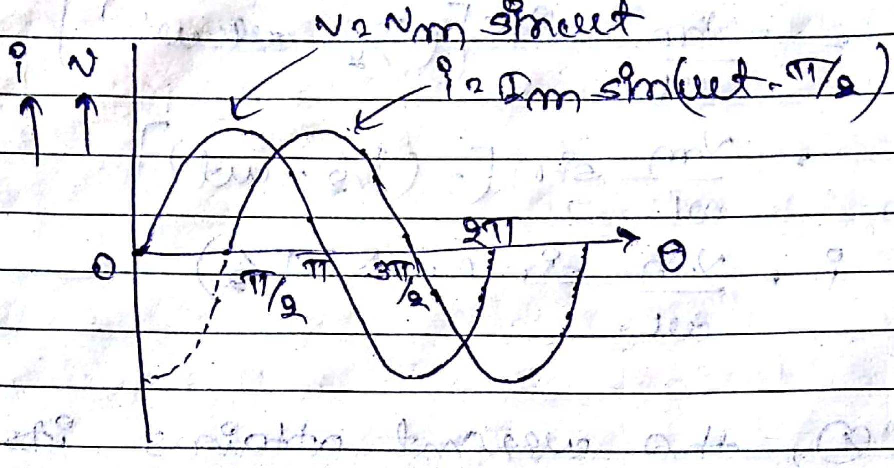
$$v = V_m \sin \omega t$$

then current flowing in a ~~partly~~ purely inductive ckt is given by

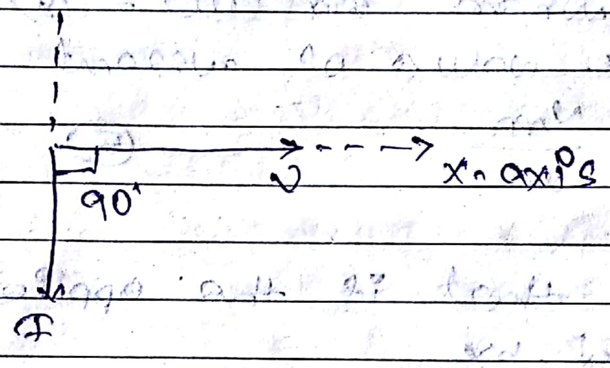
$$i = I_m \sin(\omega t - \pi/2)$$

→ From the above expression, it is clearly seen that current lags behind the applied voltage by the angle $\pi/2$ radian or $90°$

→ Graphical & phasor representation of these quantities are shown in fig. 29 & 30, respectively.



[Fig. 29 waveforms of v & i]



[Fig. 30 phasor diagram]

→ From eqⁿ (5),

$$I_m = \frac{V_m}{\omega L}$$

In this expression, the term 'ωL' plays the role of opposition similar to resistance. This opposition to the flow of current offered by an inductance (L) is called the inductive reactance denoted by X_L. Its unit is ohm (Ω).

Hence $X_L = \omega L$

$$\therefore I_m = \frac{V_m}{X_L} \quad \text{--- (6)}$$

or $X_L = \frac{V_m}{I_m} = \left(\frac{V_m/\sqrt{2}}{I_m/\sqrt{2}}\right) = \frac{V_{rms}}{I_{rms}}$

Thus, $\frac{V_m}{I_m} = \frac{V_{rms}}{I_{rms}} = X_L$ —— (7)

→ Hence inductive reactance (X_L) is the ratio of $\frac{V_m}{I_m}$ or $\frac{V_{rms}}{I_{rms}}$

→ Again $X_L = \omega L$

$= 2\pi f L$

$X_L \propto f$ —— (8)

→ We conclude that the inductive reactance is directly proportional to the frequency of sinusoidal emf applied to the ckt.

POWER:

→ Instantaneous power

$P = v \cdot i$

$= V_m \sin \omega t \times I_m \sin(\omega t - \pi/2)$

$= V_m I_m \sin \omega t \cdot \sin(\omega t - \pi/2)$

$= V_m I_m \sin \omega t \cdot [-\sin(\pi/2 - \omega t)]$

$= -V_m I_m \sin \omega t \cdot \sin(\pi/2 - \omega t)$

$= -V_m I_m \sin \omega t \cdot \cos \omega t$

$= -\frac{V_m I_m}{2} \sin(2\omega t)$

$= -\frac{V_m I_m}{2} \sin 2\theta$ $(\because \omega t = \theta)$

→ Average power for the cycle

$$P_{avg} = \frac{1}{2\pi}\int_0^{2\pi} p \cdot d\theta$$

$$= \frac{-1}{2\pi}\int_0^{2\pi} \frac{V_m I_m}{2} \sin 2\theta \, d\theta$$

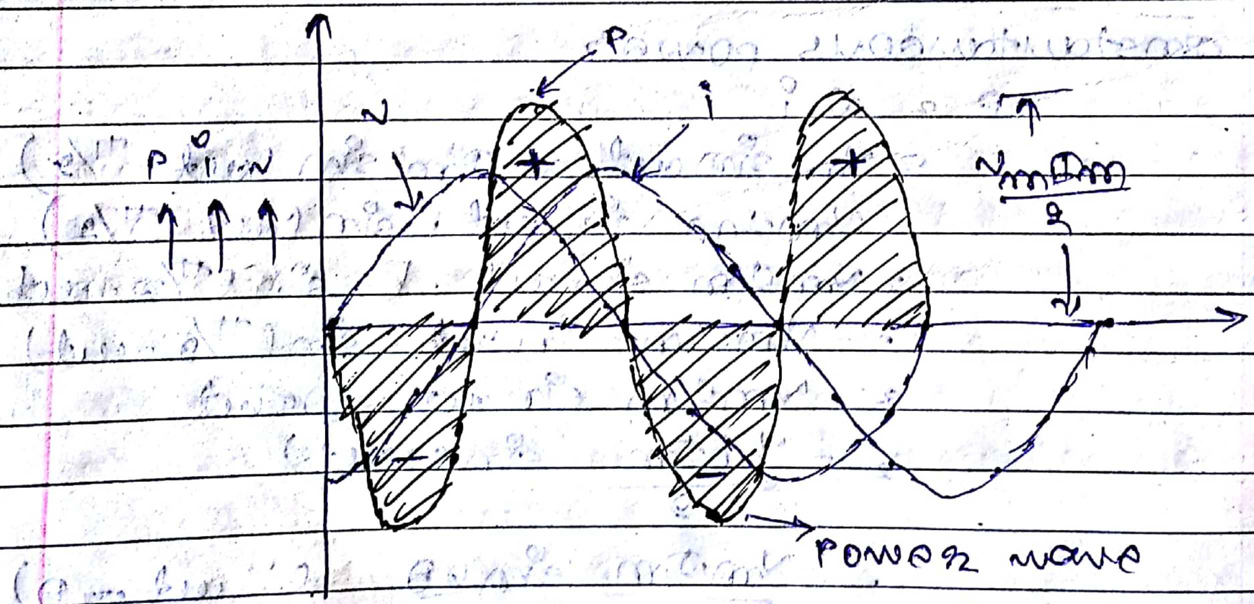
$$= \frac{-V_m I_m}{4\pi}\left[\frac{-\cos 2\theta}{2}\right]_0^{2\pi}$$

$$= \frac{V_m I_m}{8\pi}\left[\cos 2\theta\right]_0^{2\pi}$$

$$= \frac{V_m I_m}{8\pi}\left[\cos 4\pi - \cos 0\right]$$

$$P_{avg} = 0 \quad \text{--- (9)}$$

Hence the average power consumed in a purely inductive ckt is zero.



(Fig. 3) waveforms of v, i & p)

→ It is also clear from fig. 3) that the average power demand from the supply for a complete cycle is zero.

Power factor:

Power factor = $\cos\phi$

$= \cos 90°$

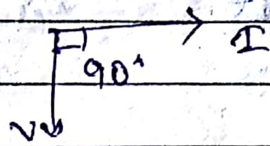
$= 0$ (lagging)

→ Thus power factor of a purely inductive circuit is zero lagging.

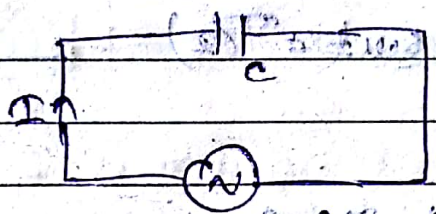
$Z = jX_L$

$Z = X_L \angle 90°$

* Purely capacitive circuit:



→ Fig. 82 shows an a.c. circuit containing a capacitor of capacitance C farads.



[Fig. 82 Purely capacitive ckt]

→ Let the alternating voltage applied be

$v = V_m \sin\omega t$ ——— (1)

→ We know that,

$q = Cv$ ——— (2)

where, q = charge on the plate at any instant of time 't'

v = voltage across the plate at any instant of time 't'

→ substituting the value of v from eq^n (1) in eq^n (2),

$$q = C V_m \sin \omega t \quad \text{—— (3)}$$

Also we know that,

$$i = \frac{dq}{dt} \quad \text{—— (4)}$$

$$= \frac{d}{dt} (C V_m \sin \omega t)$$

$$= C V_m \omega \cos \omega t$$

$$= \frac{V_m}{1/\omega C} \cos \omega t$$

$$= \frac{V_m}{1/\omega C} \sin(\omega t + \pi/2) \quad \text{—— (5)}$$

→ From the expression (5), it is observed that current attains its max. value when $\sin(\omega t + \pi/2)$ becomes unity & hence the max. value of current is given by

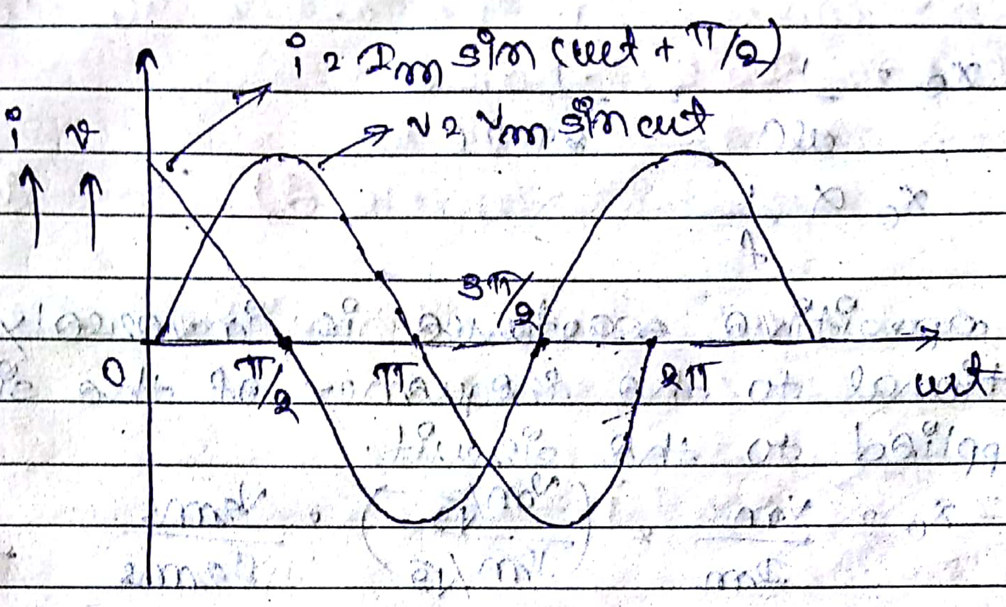
$$I_m = \frac{V_m}{1/\omega C} = \omega C \cdot V_m \quad \text{—— (6)}$$

and the expression for the current becomes

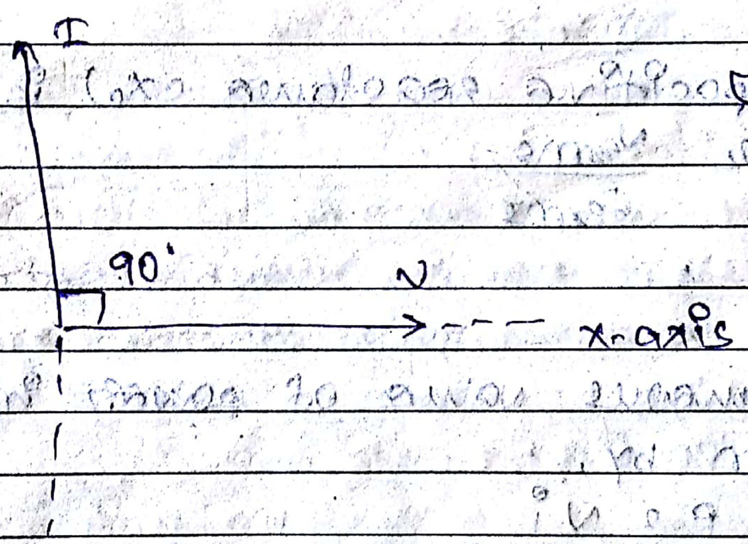
$$i = I_m \sin(\omega t + \pi/2)$$

→ The current in a purely capacitive ckt is leading the applied voltage by an angle $\pi/2$ radian or $90°$.

→ Graphical & phasor representation of these quantities are shown in fig. 33 & 34 respectively.



[Fig. 33 waveforms of v & i]



[Fig. 34 phasor diagram]

→ From expression (6)

$$I_m = \frac{V_m}{1/\omega C}$$

→ In this expression, the term $1/\omega C$ plays the role of opposition similar to resistance. This opposition to the flow of current offered by a capacitance C is called the capacitive reactance. It is denoted by x_c. Its unit is ohm (Ω)

→ Again,

$$x_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$x_c \propto \frac{1}{f}$$

→ Thus, capacitive reactance is inversely proportional to the frequency of the sinusoidal emf applied to the circuit.

$$\therefore x_c = \frac{V_m}{I_m} = \left(\frac{V_m/\sqrt{2}}{I_m/\sqrt{2}}\right) = \frac{V_{rms}}{I_{rms}} \quad (7)$$

→ Thus capacitive reactance (x_c) is the ratio of $\frac{V_m}{I_m}$ or $\frac{V_{rms}}{I_{rms}}$

→ Power:

Instantaneous value of power in an ac ckt is given by,

$$P = Vi$$

$$= V_m \sin\omega t \cdot I_m \sin(\omega t + \pi/2)$$

$$= V_m \cdot I_m \sin\omega t \cdot \sin(\omega t + \pi/2)$$

$$= V_m I_m \sin\omega t \cdot \cos\omega t$$

$$= \frac{V_m I_m}{2} \sin 2\omega t$$

$$= \frac{V_m I_m}{2} \sin 2\theta \quad (\theta = \omega t)$$

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→ Average power for one complete cycle is given by

$$P_{avg} = \frac{1}{2\pi}\int_0^{2\pi} p \, d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \frac{V_m I_m}{2} \sin 2\theta \, d\theta$$

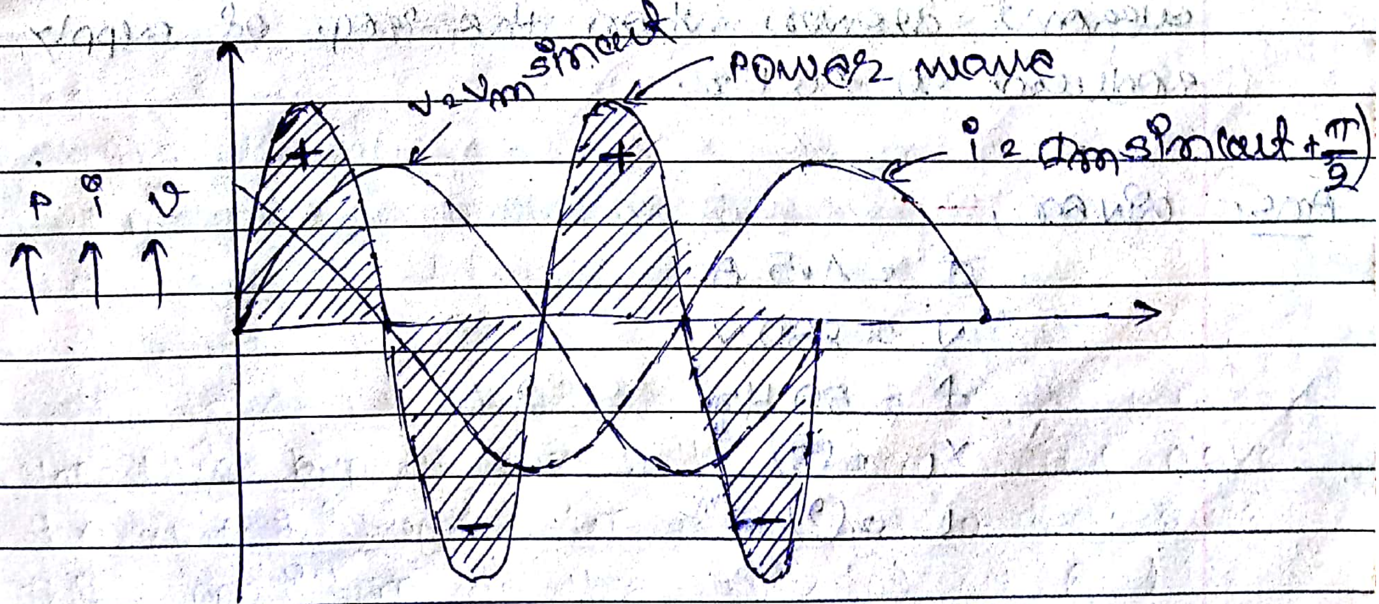
$$= \frac{V_m I_m}{4\pi}\left[\frac{-\cos 2\theta}{2}\right]_0^{2\pi}$$

$$= -\frac{V_m I_m}{8\pi}\left[\cos 2\theta\right]_0^{2\pi}$$

$$= -\frac{V_m I_m}{8\pi}\left[\cos 4\pi - \cos 0\right]$$

$$\boxed{P_{avg} = 0}$$

Thus, average power consumed by a purely capacitive ckt for one complete cycle is zero.



[Fig. 35 waveforms of v, i & p]

* Power factor:

In purely capacitive ckt, the phase angle (phase difference) b/w the applied voltage & current in the ckt is 90° (lead).

$\therefore$ Power factor $= \cos\phi$

$= \cos 90°$

$= 0$ (lead) ——— (8)

→ Power factor of a purely capacitive ckt is zero leading.

$$Z = -jX_C = -j\frac{1}{X_C}$$

$$Z = -jX_C \angle -90°$$

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Series circuits:

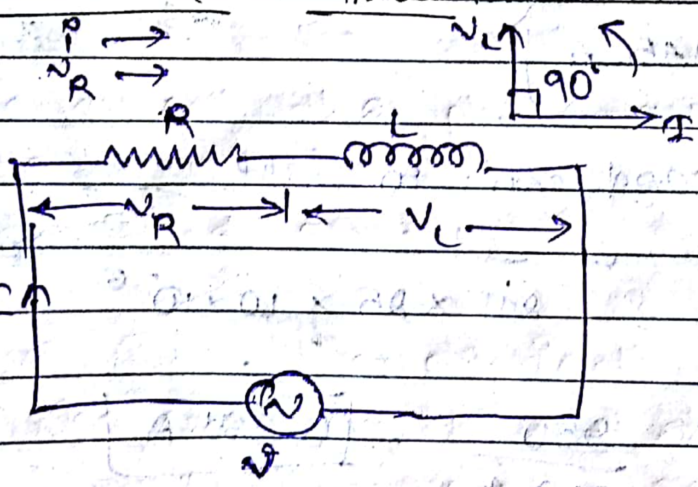
* Introduction:

→ In the previous sections, a detailed discussion has been dealt with the effects produced by each of the three fundamental elements R, L & C when a sinusoidal emf is applied.

→ However, practical ckt normally comprise of the above elements connected in combination. For example, a coil can be represented by its resistance & inductance connected in series. Thus coil can be considered as a series combination of the fundamental elements R & L.

→ We will consider various series combinations for analysis of ac circuits.

* R-L series circuit:



[Fig. 36 R-L series circuit]